Model theoretic connected components of groups

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Model Theory
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Lascar group

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- T complete first order theory, $\mathfrak{C} \models T$ – a monster model
- group of Lascar strong automorphisms

$$\mathrm{Autf}_{\mathrm{L}}(\mathfrak{C}) = \left\langle \bigcup \{\mathrm{Aut}(\mathfrak{C}/M) : M \prec \mathfrak{C}\} \right\rangle \lhd \mathrm{Aut}(\mathfrak{C})$$

- (Lascar '82 JSL) Lascar group of T:

$$\mathrm{Gal}_{\mathrm{L}}(T) = \mathrm{Aut}(\mathfrak{C}) / \mathrm{Autf}_{\mathrm{L}}(\mathfrak{C})$$

- (Lascar, Pillay, Ziegler) $\mathrm{Gal}_{\mathrm{L}}(T)$ carries a natural quasi-compact topology (not necessary Hausdorff) and $|\mathrm{Gal}_{\mathrm{L}}(T)| \leq 2^{|T|}$

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$\mathrm{Gal}_{\mathrm{L}}(T)$ "descriptive set-theoretic" invariant of T

Definition (Lascar)

T is G-compact $\iff$ the topology on $\mathrm{Gal}_{\mathrm{L}}(T)$ is Hausdorff

Model theoretic connected components

$(G, \cdot, \dots)$ – a group with some first order structure ($\overline{\kappa}$-saturated),
$A \subset G$ some small set of parameters

Definition (Pillay, Shelah, ...)

- $G_A^0 = \bigcap \{H < G : H \text{ is } A\text{-def. and } [G:H] < \omega\}$
- $G_A^{00} = \bigcap \{H < G : H \text{ is } A\text{-type def. and } [G:H] < \overline{\kappa}\}$
- $G_A^{\infty} = \bigcap \{H < G : H \text{ is } \mathrm{Aut}(G/A)\text{-inv. and } [G:H] < \overline{\kappa}\}$ (also G_A^{000})

$G_A^{\infty} \subseteq G_A^{00} \subseteq G_A^0 \lhd G$

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$G^{\infty}_A \subseteq G^{00}_A \subseteq G^0_A \lhd G$

Example

Let $G = (S^1, \cdot, P_n)_{n \in \mathbb{N}}$, where $P_n = \{x \in S^1 : d(x, 0) < \frac{1}{n}\}$ and G^* – saturated extension of G. Then

$$G^* = {G^*}^0_A \neq {G^*}^{00}_A = {G^*}^{\infty}_A = \bigcap_{n \in \mathbb{N}} P^*_n \text{ – infinitesimals.}$$

Consider the following auxiliary structure

$$\mathcal{G} = (G, \cdot, \dots, X, *),$$

where X is an additional sort and $*\colon G \times X \to X$ is a regular (free and transitive) action of G on X i.e. X is an "affine" copy of G. Then

Fact

$\mathrm{Gal_L}(\mathrm{Th}(\mathcal{G})) = G/G^{\infty}_{\emptyset} \rtimes \mathrm{Gal_L}(\mathrm{Th}(G, \cdot, \dots))$ *(topologically)*

Connected components vs. Lascar group

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Hence, $\mathrm{Th}(\mathcal{G})$ is G compact $\iff \mathrm{Th}(G, \cdot, \ldots)$ is G-compact and $G^{\infty}_{\emptyset} = G^{00}_{\emptyset}$ (i.e. $G^{\infty}_{\emptyset}$ is type definable)

Motivating problem

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$G^{0}_{\emptyset}$, $G^{00}_{\emptyset}$ and $G^{\infty}_{\emptyset}$ correspond to the strong types on X in $\mathcal{G}$:

$G^{0}_{\emptyset}$	Shelah strong types (just strong types)
$G^{00}_{\emptyset}$	Kim-Pillay strong types (the compact strong types)
$G^{\infty}_{\emptyset}$	Lascar strong types (the invariant strong types)

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An example with $G^{00}_{\emptyset} \neq G^{\infty}_{\emptyset}$ gives us a new kind of non-G-compact theory, based on the group structure.

Example: groups with NIP

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Definition (Shelah)

T has NIP $\Leftrightarrow$ there is no formula $\varphi(\bar{x}, \bar{y}) \in L$ and $\{\bar{a}_i, \bar{b}_w : i < \omega, w \underset{finite}{\subset} \omega\}$ such that $\mathfrak{C} \models \varphi(\bar{a}_i, \bar{b}_w) \iff i \in w$.

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We say, that G^∞ exists, if for an arbitrary set of parameters $A \subset G$, $G^\infty_A = G^\infty_\emptyset$. Similarly for G^{00} and G^0.
Existence G^∞ $\Rightarrow$ existence G^{00} $\Rightarrow$ existence G^0

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Theorem

Assume that G is definable in the structure with NIP (e.g. o-minimal)

- *(Shelah) G^{00} exists (even for type definable G), if G is abelian, then G^∞ exists.*
- *(JG) G^∞ exists.*
- *(Hrushovski, Pillay) If G is definable amenable, then $G^{00} = G^\infty$.*

Another Description of G_A^∞

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Definition

$(G, \cdot)$ – an arbitrary group, $P \subseteq G$, $n < \omega$

- P is n-thick $\Leftrightarrow$ $P = P^{-1}$ and for every $g_0, \ldots, g_{n-1} \in G$ there are $i < j < n$ such that

$$g_i^{-1} g_j \in P,$$

- P is thick $\Leftrightarrow$ P is n-thick for some natural n.

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Lemma

$$G_A^\infty = \left\langle \bigcap \{P \subseteq G : P \textit{ is } A\textit{-def. and thick }\}\right\rangle$$

Example: additive and multiplicative group of a field

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V. Bergelson and D. B. Shapiro proved [PAMS '92] that if K is an infinite field and $G < K^{\times}$ is with finite index, then $G - G = K$.

Their proof generalizes to the thick subsets of $K^{\times}$:

Theorem

Let K be an infinite field with some structure (saturated). Then

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If $(K,+)^\infty$ exists (e.g. K has NIP), then

$$(K,+)^\infty = K,$$

because $(K,+)^\infty$ is an ideal of K.

When $G^{\infty} = G$?

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Proposition

$(G, \cdot, \ldots)$ – a group with some first order structure, G^* – saturated extension. TFAE

- $G^{*\infty}$ exists and $G^{*\infty} = G^*$
- there is a natural number N such that for **every definable** thick $P \subseteq G^*$

$$P^N = G^*.$$

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Proposition

$(G, \cdot)$ – a group. TFAE

- $G^{*\infty}$ exists and $G^{*\infty} = G^*$, where G^* is a monster model of an arbitrary first order expansion of G
- there is a natural number N such that for **an arbitrary** thick $P \subseteq G$
$$P^N = G.$$

Absolutely connected groups

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Definition

- G is N-absolutely connected (N-ac) if for every thick $P \subseteq G$
$$P^N = G.$$
- G is absolutely connected if G is N-absolutely connected for some natural N.
- Let $\mathcal{C}_N = \{N\text{-absolutely connected groups}\}$ and $\mathcal{C}_\infty = \bigcup_{N<\omega} \mathcal{C}_N = \{\text{absolutely connected groups}\}$.

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The strategy for solving the main problem:

Proposition

If for every natural N, $\mathcal{C}_\infty \neq \mathcal{C}_N$, then there is a group G with
$$G^\infty_\emptyset \neq G^{00}_\emptyset.$$

Examples of absolutely connected groups

Example

1. $(\kappa > \omega)$ $\mathrm{Sym}^\kappa(\Omega) = \{\sigma \in \mathrm{Sym}(\Omega) : |\mathrm{supp}(\sigma)| < \kappa\}$ is 16-ac
2. if V – a vector space over a division ring with $\dim(V) = \infty$, then $\mathrm{GL}(V)$ is 128-ac
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Proof.

We use an auxiliary class of *weakly simple groups*. Let

$$\mathcal{G}_N(G) = \{g \in G : \left(g^G \cup {g^{-1}}^G\right)^{\leq N} = G\}.$$

A group G is N-weakly simple if $\mathcal{G}_N(G)$ is "big" in some sense:

$$G \setminus \mathcal{G}_N(G) \text{ is not thick.}$$

It can be proved that N-weak simplicity $\Rightarrow$ $4N$-ac.

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Now use description of the conjugacy classes: in 1. results of Bertram '73 and Moran '76; in 2. — Tolstykh '06; in 3. — Lev '96. □

Chevalley groups

Absolutely connectedness of $\mathrm{SL}_n(K)$ suggests to look at Chevalley groups. Ellers, Gordeev and Herzog determined "covering numbers" for a quasisimple Chevalley groups:

Definition (Arad, Herzog '85)

G – simple nonabelian group. The covering number $\mathrm{cn}(G)$ of G is the smallest natural N (or ∞), such that $C^N = G$ for every nontrivial conjugacy class C of G.

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Theorem (Ellers, Gordeev, Herzog, Israel J. of Math. '99)

There is a positive integer d such that $\mathrm{cn}(G) \leq d \cdot \mathrm{rank}(G)$, for every quasisimple proper Chevalley group G.

Chevalley groups cd.

It means, that

$$\forall C \text{ conjugacy class of } G, \quad C^{d \cdot \mathrm{rank}(G)} = G.$$

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In my case, G is N-weakly simple (so $4N$ absolutely connected) iff $G \setminus \mathcal{G}_N(G) = \{g \in G : \left(g^G \cup {g^{-1}}^G\right)^{\leq N} = G\}$ is not thick. i.e.

$$\forall^{\text{almost all}} C \text{ conjugacy class of } G, \quad (C \cup C^{-1})^N = G,$$

where "almost all" means that the set of failures is not thick (i.e. is rather small).

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where "almost all" means that the set of failures is not thick (i.e. is rather small). It seems that methods of Ellers, Gordeev, Herzog give that all Chevalley groups are 192-weakly simple (so 768-absolutely connected).

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Muranov ['07] constructed (using small cancellation theory) a collection of simple torsion free groups $\{M_n\}_{n<\omega}$ satisfying

- M_n is $(2n + 2)$-boundedly simple (so $(8n + 8)$-ac),
- the commutator width of M_n is between $(n + 1)$ and $(2n + 2)$.

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Using Muranov's groups we can prove:

Proposition

Either $\forall N$, $\mathcal{C}_\infty \neq \mathcal{C}_N$ (so there is a group G with $G^\infty_\emptyset \neq G^{00}_\emptyset$) or there is an absolutely connected group an with infinite commutator width.

G^0 in finitely generated virtually nilpotent groups

Let G be a finitely generated group with nilpotent subgroup of finite index (virtually nilpotent group). Martinez [TAMS '94] proved (using the positive solution of the restricted Burnside problem) that the set of nth powers $\mathrm{Val}_{X^n}(G) = \{g^n : g \in G\}$ generates the subgroup G_n of G in finitely many steps. From model theoretical point of view, this gives us an existence and description of G^0.

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$$G^0 = \bigcap_{n \in \mathbb{N}} G_n.$$

The next step would be to find a similar description of G^{00} G^{∞}.

Problem

Describe G^{00} and G^{∞} for finitely generated virtually nilpotent group G.

$\mathbb{Z}$ case

This problem is unclear even for integers $\mathbb{Z}$ and has connections to additive combinatorics. The natural candidate for $(\mathbb{Z}, +, \ldots)^\infty$ is

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The answer is closely related to the solution of the following:

Problem

Is there a universal constant C, such that for an arbitrary partition $\mathbb{Z} = A_1 \cup \ldots \cup A_n$, there is natural number m such that, the set $P = \{a - b : a, b \in A_i, \text{ for } 1 \leq i \leq n\}$ has the property that $\underbrace{P + \ldots + P}_{C}$ contains $m \cdot \mathbb{Z}$?

By van der Waerden theorem, some A_i contains an arbitrary long arithmetic progression. Hence P contains arbitrary (big) part of $m\mathbb{Z}$s, but m may tend to infinity.

References

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