Abstracts – Invited Speakers

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1. Omer Angel, UBC, Vancouver, e-mail: angel@math.ubc.ca;

Title: Phase transition in Dyadic tilings.

Abstract: A dyadic tile in the unit square $[0,1]^2$ is a rectangle of the form $[i/2^a,(i+1)/2^a] \times [j/2^b, (j+1)/2^b]$ for some i, j, a, b. Such a tile is of order a + b. Suppose each tile of order n is available with some probability p. Is it possible to tile the square using a subset of 2^n of the available tiles? We show that this is possible with high probability if and only if $p > p_c$ for some $0 < p_c < 1$.

2. Vincent Beffara, ENS, Lyon, e-mail: vbeffara@ens-lyon.fr;

Title: Recent progress on the Random Cluster model.

Abstract: We review a few recent advances in the study if the (two-dimensional) Fortuyn-Kasteleyn Random – Cluster model.

3. Jeremie Bouttier, CEA Saclay, e-mail: jeremie.bouttier@cea.fr;

Title: Planar maps and continued fractions.

Abstract: Planar maps (graphs embedded in the sphere) form a natural model for discrete (tessellated) random surfaces, used in the context of two-dimensional quantum gravity. Many questions about the geometry of random maps can be rephrased as enumeration problems. In this talk, I will present an unexpected connection between two such problems.
In the first problem, we consider maps with one boundary, whose generating function is the so-called disk amplitude. This quantity is well-studied, it is for instance expressible as a matrix integral, and computable using Tutte´s/loop equations.
In the second problem, we consider maps with two marked points at a given distance, whose generating function is the so-called two-point function. Though it is one of the simplest metric-related observables, much less is known about it.
I will explain that, in a rather general class of maps, the disk amplitude and the two-point function are two facets of the same quantity, which has to be viewed respectively as a power series and as a continuous fraction.
I will then explain how the known solution to the first problem yields the solution to the second problem.

4. **Petter Brändén**, Stockholm University**,** pbranden@math.su.se;

Title: Negative dependence and zeros of polynomials

Abstract: Recently we proved (joint with Borcea and Liggett) that there is a connection between negative dependence in probability theory and the geometry of zeros of multivariate polynomials. We will explain this connection and its consequences, and also mention some new results.

5. **Guillaume Chapuy**, LIAFA Paris, e-mail: chapuy@ liafa.jussieu.fr;

Title: Bijections for (random) maps on surfaces.

Abstract: We consider graphs embedded on a fixed orientable surface of genus g ("maps"). In the planar case g=0, the central tool to study these objects is Schaeffer's bijection, which relates planar maps to certain models of plane trees. In this talk we discuss recent advances in the case g>0. We combine two layers of bijections (one similar to the planar case, due to Marcus & Schaeffer, another one specific to higher topologies) to give a description of maps of arbitrary genus in terms of more complicated models of trees. From the enumerative viewpoint, this leads us to a very explicit interpretation of the counting exponent $n^{5/2(g-1)}$. From the viewpoint of random maps of fixed genus g, we show that the correct scaling factor for the graph distance is $n^{1/4}$ as in the planar case, and we give an explicit characterization of the distribution of distances to a random vertex in the continuum limit, as a weighted version of Aldous's ISE measure.

6. **Nicolas Curien**, ENS Paris, e-mail: nicolas.curien@ens.fr;

Title: A view from infinity of the uniform infinite planar quadrangulation.

Abstract: The Uniform Infinite Planar Quadrangulation (UIPQ) is an infinite planar graph which is the local limit of uniform planar quadrangulations of size n. It has been introduced by Krikun in 2005 following a pioneer work of Angel and Schramm on triangulations. The geometry of this object is particularly interesting, for example the volume growth exponent is 4. In this talk we introduce a new construction of the UIPQ and deduce several geometric properties: For example an essential uniqueness of infinite geodesics reminiscent of the work of Le Gall on the Brownian Map. Joint work in progress with L. Ménard, G. Miermont.

7. **Hugo Duminil-Copin**, University of Geneva, e-mail: hugo.duminil@unige.ch;

Title: Nienhuis conjectures for two-dimensional self-avoiding walks.

Abstract: We will prove one of the famous conjectures made by Nienhuis regarding the number of self-avoiding walks on the hexagonal lattice. More precisely, we will show that the number a_n of self-avoiding walks of length n (starting at the origin) satisfies:
$\lim_{n\rightarrow \infty}a_n^{\frac 1n}=\sqrt{2+\sqrt 2}$.
The proof uses a parafermionic observable for the self-avoiding walk, which satisfies a half of the discrete Cauchy-Riemann relations. Establishing the other half of the relations (which conjecturally holds in the scaling limit) would also imply convergence of the self-avoiding walk to SLE(8/3).
Joint work with S. Smirnov.

8. Alexandre Gaudillier, LATP Marseille, e-mail: gaudilli@cmi.univ-mrs.fr;

Title: From logarithmic to subdiffusive polynomial fluctuations for internal DLA and related growth models.

Abstract: We consider a family of cluster growth models on the d-dimensional lattice that generalize the internal diffusion limited aggregation (internal DLA). The latter model is the simplest to describe: random walks start at the origin, one at a time, and stop moving when reaching a site not occupied by previous walks. In this case, it is known that the asymptotic shape of the cluster is a sphere. For all our cluster growth models, we obtain upper and lower bounds for the inner and outer deviation probabilities from the asymptotic shape. Our growth models are all coupled to internal DLA and we obtain that the fluctuations for internal DLA are at most logarithmic in the radius of the sphere in dimension d larger than or equal to 2.
Joint work with Amine Asselah

9. Alan Hammond, Oxford, e-mail: hammond@stats.ox.ac.uk;

Title: Fluctuation in droplet boundaries.

Abstract: When a low-temperature Ising model in a box with negative boundary conditions is conditioned on the presence of a significant excess of plus signs, this excess tends to gather together in a droplet of the plus phase, surrounded by a sea of the minus phase. Between the droplet and its surroundings is a dual contour, the droplet boundary.
I will discuss the fluctuation of such boundaries from their convex hull, introducing local notions of deviation, and explain how the longitudinal and lateral deviations in the droplet boundary have exponents coinciding with those in the KPZ class of growing
interfaces. A key tool in the proofs are resampling and surgical techniques that may also be useful studying random objects in the KPZ class, such as the multi-line Airy process.

10. Remco van der Hofstad, TU-Eindhoven, e-mail: rhofstad@win.tue.nl;

Title: Recent developments in high-dimensional percolation.

Abstract: In this survey talk, we give an overview of recent developments in high-dimensional percolation. It is now 20 years ago that Hara and Slade published their seminal work on the mean-field behavior of percolation above the upper critical dimension, showing, for example, that at criticality there is no percolation. The main technique used is the lace expansion, a perturbation technique that allows us to compare percolation paths to random walks.
In the past few years, a number of novel results have been proved, of which I intend to highlight:
(1) the identification of arm exponents by Kozma and Nachmias, and their implications for random walk on the high-dimensional incipient infinite cluster;
(2) finite-size scaling for percolation on a torus, where the largest connected components shares many features to the Erdos-Renyi random graph;
(3) the upper critical dimension of long-range percolation and scaling limits of percolation paths.

11. Antal Jarai, Univ. of Bath, e-mail: A.Jarai@bath.ac.uk;

Title: Zero dissipation limit and ergodicity in Abelian sandpiles

Abstract: We consider a continuous height version of the Abelian sandpile model that allows arbitrarily small amount of bulk dissipation, that is, on every toppling, a small amount of height is dissipated. In this model correlations and avalanches decay exponentially, and as the dissipation approaches zero, the classical discrete sandpile is recovered. We give estimates on how fast the stationary measure of the dissipative model converges to the critical sandpile measure in dimensions d=2 and 3. We also study ergodic properties for finite volumes.

12. Gady Kozma, Weizmann Institute, e-mail: gady.kozma@weizmann.ac.il;

Title: The cycle structure of the interchange process.

Abstract: We examine certain functions on the symmetric group which are natural for studying the cycle structure of the interchange process. We decompose them explicitly as sums of characters of irreducible representations and derive some probabilistic corollaries. Joint work with Gil Alon, Omer Angel and Jim Propp.

13. Takashi Kumagai, RIMS, Kyoto, e-mail: kumagai@kurims.kyoto-u.ac.jp;

Title: Convergence of mixing times for sequences of simple random walks on graphs.

Abstract: We give a general criteria for the convergence of L^p-mixing times when a sequence of finite connected graphs converges to some compact metric measure space in a generalized Gromov-Hausdorff sense. As an application, we consider mixing times of random walks on finite random graphs which typically arise in the critical window of a system. In particular, we give almost sure mixing time bounds for random walk on the Erdos-Rényi random graphs in the critical window, which is in contrast to those obtained by Nachmias and Peres (AOP 2008). This is a on-going joint work with D.A. Croydon (Warwick) B.M. Hambly (Oxford).

14. Fabio Martinelli, Roma Tre, e-mail: martinelli.fabio@gmail.com;

Title: Glauber dynamics for the 2D Ising model at low temperature: quasi-polynomial bound on the mixing time.

Abstract: Consider the standard Ising model on a finite n x n grid at low temperature. If the boundary spins are held fixed equal to +1 it is believed that the mixing time of the corresponding Glauber dynamics (Gibbs sampler) is poly(n). Although such a result is still far from being proved, recently there has been some exciting progress using the censoring inequality by Peres and P. Winkler together with inductive schemes. The final outcome is a quasi-poly(n) bound valid for all temperatures below the critical one. Based on joint work with F.L.Toninelli, and F.L.Toninelli, E. Lubetzki and A. Sly.

15. Ron Peled, Tel-Aviv University, e-mail: peledron@gmail.com;

Title: High-dimensional Lipschitz functions are typically flat.

Abstract: We consider integer-valued functions on the discrete torus Z_n^d constrained to change by at most 1 between adjacent vertices. This model can be viewed as a discrete model for Lipschitz functions. We will explain why in high dimensions a typical function from this class, suitably normalized, exhibits long range order and is nearly constant on either the even or odd sub-lattice. Thus the function is very flat, having bounded variance at each point and taking few values overall. Our model is closely related to 3-colorings and we deduce that in high dimensions, a uniformly sampled proper 3-coloring will take predominantly one color on either the even or odd vertices. Consequently, we establish for the first time the existence of a phase transition in high dimensions for the anti-ferromagnetic 3-state Potts model. The results have consequences also in 2 dimensions, where they give one side of a conjectured roughening transition. Our work generalizes results of Kahn and Galvin, refutes a conjecture of Benjamini, Yadin and Yehudayoff and answers a question of Benjamini, H\"aggstr\"om and Mossel.

16. Robin Pemantle, University of Pennsylvania, e-mail: pemantle@math.upenn.edu;

Title: An anisotropoic percolation arising from a deletion channel.

Abstract: We consider an an-isotropic oriented percolation. Horizontal edges are always present, while vertical edges are present with probability 1/2. Such a percolation arises in the context of information in a deletion channel. The percolation as it arises is not Bernoulli, but we consider as well the Bernoulli percolation with the same marginals. We would like to be able to distinguish the quenched and annealed growth rate of highest paths; we cannot yet do this; instead, we give a number of admittedly less satisfying preliminary results.These apply to the dependent model as well as the Bernoulli one.

17. Yuval Peres, Microsoft Research Theory Group, e-mail: peres@microsoft.com;

Title: Cover times, blanket times, and the Gaussian free field.

Abstract: The cover time of a finite graph G (the expected time for simple random walk to visit all vertices) has been extensively studied, yet a number of fundamental questions concerning cover times have remained open: Aldous and Fill (1994) asked whether there is a deterministic polynomial-time algorithm that computes the cover time up to an O(1) factor; Winkler and Zuckerman (1996) defined the blanket time (when the empirical distribution of the walk is within a factor of 2, say, of the stationary distribution) and conjectured that the blanket time is always within O(1) of the cover time. The best approximation factor found earlier for both these problems was (log log n)^2 for n-vertex graphs, due to Kahn, Kim, Lovasz, and Vu (2000).

We show that the cover time of G, normalized by the number of edges, is equivalent (up to a universal constant) to the square of the expected maximum of the Gaussian free field on G. We use this connection and Talagrand's majorizing measure theory to deduce positive answers to the questions of Aldous-Fill and Winkler-Zuckerman. No prior knowledge of Talagrand's theory or of cover times will be assumed. I will emphasize the remaining open problem: Estimating the cover time (and the maximum of a Gaussian process) more precisely. Joint work with Jian Ding and James Lee.

18. Thomas Richthammer, TU-Munich, e-mail: richthammer@ma.tum.de;

Title: The spectral gap of the interchange process - a proof of Aldous' conjecture.

Abstract: We consider a finite, connected, undirected graph with weighted edges. Labeled particles are assigned to its vertices (one to each vertex), and any two particles connected by an edge may interchange their positions at a rate given by the corresponding edge weight. The resulting continuous time Markov chain is called interchange process. The spectral gap of this process determines its rate of convergence to the uniform distribution. In about 1990 David Aldous conjectured that the spectral gap of the interchange process is the same as that of the random walk on the same graph, but so far this has been proved only in some special cases. We prove the conjecture using a recursive strategy. The approach is a natural extension of the method already used to prove the validity of the conjecture on trees. The novelty is an idea based on electric network reduction, where the edge weights play the role of conductances. The main technical obstacle is to show that this reduction decreases the total energy of the network corresponding to the (weighted) Cayley graph of the group of vertex permutations, generated by edge transpositions. Our method gives a proof of Aldous' conjecture in full generality. As a consequence we are able to relate the spectral gaps of various processes (such as subgraph processes or exclusion processes) to that of the random walk on the same graph. This is joint work with Pietro Caputo (Roma Tre) and Tom Liggett (UCLA).

19. Alister Sinclair, UC Berkeley, e-mail: Sinclair@cs.Berkeley.edu;

Title: Liftings of Tree-Structured Markov Chains.

Abstract: A "lifting" of a Markov chain is a larger chain obtained by replacing each state of the original chain by a set of states, with transition probabilities defined in such a way that the lifted chain projects down exactly to the original one. It is well known that lifting can potentially speed up the mixing time substantially. Essentially all known examples of efficiently implementable liftings have required a high degree of symmetry in the original chain.
Addressing an open question of Chen, Lovasz and Pak, we present the first example of a successful lifting for a complex Markov chain that has been used in sampling algorithms. This chain, first introduced by Sinclair and Jerrum, samples a leaf uniformly at random in a large tree, given approximate information about the number of leaves in any subtree, and has applications to the theory of approximate counting and to importance sampling in Statistics. Our lifted version of the chain (which, unlike the original one, is non-reversible) gives a significant speedup over the original version whenever the error in the leaf counting estimates is o(1). Our lifting construction, based on flows, is systematic, and thus may be applicable to other Markov chains used in sampling algorithms. Joint work with T. Hayes.

20. Stas Smirnov, University of Geneva, e-mail: stanislav.smirnov@unige.ch;

Title: ***The energy density field of the Ising model***
Abstract:

21. Jeff Steif, Chalmers University of Technology, e-mail: steif@chalmers.se;

Title: Noise and Exclusion Sensitivity.

Abstract: In the first half of the talk, I will give a very brief overview of noise sensitivity of Boolean functions. This will be based on lecture notes being written together with Christophe Garban which hopefully will be completed by the time of the talk. In the second half of the talk, I will describe ongoing joint work with Erik Broman and Christophe Garban concerning the notion of sensitivity under a different dynamics, namely running the exclusion process for a short amount of time.

22. Pierre Tarres, Oxford, e-mail: tarres@maths.ox.ac.uk;

Title: Dynamics of vertex-reinforced random walks.

Abstract: Vertex-reinforced random walks (VRRWs) are processes reinforcing themselves on the sites they have more visited. We generalize a result of Volkov (2001) and show that, on a large class of graphs and with symmetric reinforcement coefficients, the walk eventually localizes with positive probability on subsets which consist of a complete d-partite subgraph with possible loops and its outer boundary, for some positive integer d.
The proof of the result describes a property of the structure of surviving species in the population genetics "replicator" model introduced by Fisher, Wright and Haldane in the 1920s. Joint work with M. Benaim.

23. Balint Toth, Budapest University of Technology, e-mail: balint@math.bme.hu;

Title: Superdiffusive bounds on random walks and diffusions with long memory in the critical dimension.

Abstract: We prove super-diffusivity with multiplicative logarithmic corrections for a class of models of random walks and diffusions with long memory. The family of models includes the “true” (or “myopic”) self-avoiding random walk, self-repelling Durrett-Rogers polymer model and diffusion in the curl-field of (mollified) massless free Gaussian field in 2D. We adapt methods developed in the context of bulk diffusion of ASEP by Landim-Quastel-Salmhofer-Yau (2004). This is joint work in progress with Benedek Valkó (U Wisconsin, Madison).