Prediction of catastrophes in space and time

Anastassia Baxevani

anastass@chalmers.se

Department of Mathematical Statistics

University of Gothenburg

Chalmers University of Technology

Application in mind

We are interested in developing a methodology / framework so that:

given data from a network of buoys, satellite measurements or reanalysis output, over a certain region, to be able to predict the occurrence of certain rare events for some other area of interest, in order to give warnings about potential catastrophes.

The approach ...

... taken is to construct optimal alarm systems based on event prediction of level crossings for random fields. We consider the (potentially catastrophic) spatio-temporal process of interest as a family of spatial random fields indexed by time $\{\xi(\mathbf{s},t), \mathbf{s} \in \mathbf{R}^2, t \in \mathbf{R}\}$. In addition, we assume there is a multivariate alarm random field, $\{\boldsymbol{\eta}(\mathbf{s},t) = (\eta_1(\mathbf{s},t), \ldots, \eta_k(\mathbf{s},t)), \mathbf{s} \in \mathbf{R}^2, t \in \mathbf{R}\}$, for some $k \in \mathbf{N}$, which could be ξ and possibly its derivatives with a different time origin.

For simplicity, we also suppose that there are a number of fixed spatial locations from which the alarm field $\boldsymbol{\eta}$ can be observed in order to give an alarm at a number of fixed spatial locations for a catastrophe for the field ξ.

Alarms and Catastrophes

Suppose that

- a catastrophe occurs at time t if $\boldsymbol{\xi}^r(t) = (\xi(\mathbf{s}_{01}, t), \ldots, \xi(\mathbf{s}_{0r}, t)) \in C \in \mathcal{R}^r$
- an alarm is given at time t if $\boldsymbol{\eta}^m(t) = (\boldsymbol{\eta}(\mathbf{s}_{11}, t), \ldots, \boldsymbol{\eta}(\mathbf{s}_{1n}, t)) \in A \in \mathcal{R}^m$.

The context of interest would determine both sets of spatial locations and the choice of C. The form of C should reflect the occurrence of high levels for the catastrophe ξ at the locations chosen for ξ.
For example, C could be:

$$C = \{\mathbf{x} \in \mathbf{R}^r : max(x_1 - u_1, \ldots, x_r - u_r) \geq 0\},$$

where $u_1, \ldots, u_r$ are (usually high) levels which may vary according to the spatial location.
The choice of A would reflect information which gives a higher probability of a catastrophe and should be chosen according to some optimality criteria.

Optimal Alarm Systems

Problem: is to predict h- time units in advance whether a catastrophe will occur or not. The probabilities of interest in the optimal alarm setting are:

- The alarm size of A is given by $\alpha = P(\boldsymbol{\eta}^m(t) \in A)$ - the proportion of time the alarm process is in the alarm state.
- The catastrophe size of C is given by $\gamma = P(\boldsymbol{\xi}^r(t) \in C)$ - the proportion of time the catastrophe process is in the catastrophe state.
- The risk of a catastrophe C at lag h for an alarm region A , $\rho_h = P(\boldsymbol{\xi}^r(t+h) \in C \Big| A_t)$ - the proportion of time the catastrophe process is in the catastrophe state at lag h after the alarm process has entered the alarm state. A measure of how correct the alarm is.
- The detection probability with warning time h of an alarm A for a catastrophe C, $\delta_h = P(\boldsymbol{\eta}^m(t) \in A \Big| C_{t+h})$ - the proportion of time that the start of a catastrophe had an alarm associated with it exactly h time units earlier. A measure of how many catastrophes were detected.

Probabilities of Interest

As in any diagnostic setting is desirable to keep the detection probability close to one, i.e. that we detect as many catastrophes as possible while we keep the risk probability as high as possible, i.e. keeping the number of false alarms as low as possible . One way of doing so is by making the alarm region as large as possible, which obviously would lead to a larger number of false alarms.

Fixing the size of the alarm region (time the alarm process spends in the alarm state α) and then choose the best alarm region in the sense of maximising the probability to detect a catastrophe, is one way of achieving a compromise.

Comment: There are alternative ways of choosing the optimal alarm region but almost all of them involve the quantities that will be presented later ...

Optimal Alarm Systems, cont.

Lemma: The alarm system with

$$A_h = \left\{ \mathbf{x} \in \mathbf{R}^m : \frac{f^0_{\boldsymbol{\eta}^m(t)}(\mathbf{x} \,\big|\, C_{t+h})}{f_{\boldsymbol{\eta}^m(t)}(\mathbf{x})} \geq k_h \right\},$$

where

- $f^0_{\boldsymbol{\eta}^m}(\cdot \,\big|\, C_{t+h})$ is the Palm density given a catastrophe will commence at time $t+h$
- $f_{\boldsymbol{\eta}^m(t)}(\mathbf{x})$ is the unconditional multivariate density
- $k_h \geq 0$ are such that $P(\boldsymbol{\eta}^m(t) \in A_h) = \alpha_h$,

is optimal of size α_h,

Optimality is in the sense $P^0(\boldsymbol{\eta}^m(t) \in A_h \,\big|\, C(t+h)) = \sup_B P^0(\boldsymbol{\eta}^m(t) \in B \,\big|\, C(t+h))$ with sup taken over all Borel subsets $B \in \mathbf{R}^m$ such that $P(\boldsymbol{\eta}^m(t) \in B) \leq \alpha_h$.

Palm Distributions

We now turn to the problem of obtaining the densities required to determine the optimal alarm regions.

We want $(\xi, \boldsymbol{\eta})$, C and A be such that points at which these events occur are isolated in time and that the conditioning events $A(t)$ - the time the process $\boldsymbol{\eta}$ enters A- and $C(t)$ - the time the process ξ enters C - have zero probability for a fixed time t.

Hence, the distributions for $\boldsymbol{\eta}$ and ξ conditionally on $A(t)$ and $C(t)$, will be defined by means of Palm distributions and under the assumption $(\boldsymbol{\eta}, \xi)$ is ergodic.

For this, we assume that the fields $\xi, \boldsymbol{\eta}$ and the sets A and C satisfy about a page and a half of conditions which can be found in Corollary 11.2.2 and Theorem 11.2.1 of Adler and Taylor (2007).

These conditions, technical in nature, just make sure that: the fields are smooth enough for the appropriate point processes to have nice intensities, so points are isolated, and the boundaries of the alarm and catastrophe are smooth with respect to Lebesgue measures.

These conditions simplify considerably the moment we assume the fields to be Gaussian and actually translate to conditions on the covariances of the fields and their derivatives!

Palm Distributions , cont.

To compute the Palm distribution for $\boldsymbol{\eta}$ given the event $C(0)$ occurs, we define:

$$N_T = \#\{t \in [0,T] : C(t) \text{ occurs}\} = \# \text{ of entries into } C \text{ by } \boldsymbol{\xi} \text{ through } \partial C \text{ over } [0,T]$$

and

$$\begin{aligned} N_T(B_{\boldsymbol{\tau}}) &= \#\{t \in [0,T] : C(t) \text{ occurs and } \boldsymbol{\eta}(\mathrm{t}+\cdot) \in \mathrm{B}_{\boldsymbol{\tau}}\} = \\ &= \# \text{ of entries into} C \text{ by } \boldsymbol{\xi} \text{ through } \partial C \text{ over } [0,T] \text{ so that } \boldsymbol{\eta}(t+\cdot) \in B_{\boldsymbol{\tau}}, \end{aligned}$$

where $B_{\boldsymbol{\tau}}$ is a finite dimensional set.

Assuming the expectations exist, the conditional f.d.d of $\boldsymbol{\eta}$ given the event $C(0)$ are defined as

$$P\Big((\boldsymbol{\eta}(\tau_1),\ldots,\boldsymbol{\eta}(\tau_n)) \in B_{\boldsymbol{\tau}} \Big| C(0)\Big) = \frac{E\left[N_1(B_{\boldsymbol{\tau}})\right]}{E[N_1]}.$$

Palm distributions, cont.

In order to find the expectations in the previous slide, we suppose that the boundary ∂C can be expressed in terms of a real valued function $\mathcal{M}$ satisfying

$$\mathbf{x} \in \partial C \iff \mathcal{M}(\mathbf{x}) = 0$$

$$\mathbf{x} \in C \iff \mathcal{M}(\mathbf{x}) > 0,$$

and $\mathcal{M}(\mathbf{x}) < 0$ otherwise, with $\mathbf{x} \in \mathbf{R}^r$. It can be assumed that $\mathcal{M}(\mathbf{x})$ is continuously differentiable w.r. to all components of $\mathbf{x}$, at least near the boundary ∂C and except possibly for a set of ($(r-1)$-dim) Lebesgue measure zero on ∂C, (due to ∂C being smooth a.e.).
This would be true, for instance, if C is the region similar to that given by

$$C = \{\mathbf{x} \in \mathbf{R}^r : max(x_1 - u_1, \ldots, x_r - u_r) \geq 0\}.$$

Palm Distributions, (cont.)

For the restriction of the process on the boundary, $\mathcal{M}_t = \mathcal{M}(\boldsymbol{\xi}(t))$, which is also stationary and ergodic, by an appropriate choice of $\mathcal{M}(\cdot)$, $\mathcal{M}_t$ are such that the one and a half-page conditions of Theorem 11.2.1 of Adler and Taylor (2007) are satisfied for $\{(\mathcal{M}_t, \dot{\mathcal{M}}_t, \boldsymbol{\eta}(t)), t \in \mathbf{R}\}$. These restrictions are not onerous and, for Gaussian processes, are satisfied by the earlier conditions and appropriate conditions on their covariance functions (such as (11.2.5) on p. 268 of Adler and Taylor(2007)).

Hence, the entries into C through ∂C by $\boldsymbol{\xi}(t)$ can be expressed by means of zero-upcrossings by $\mathcal{M}_t$.

Thus, the event that $\boldsymbol{\xi}(\cdot)$ enters C at time t is given by

$$C(t) = \{\mathcal{M}_t = 0, \dot{\mathcal{M}}_t > 0\}.$$

This is a quite neat construction since by reducing a one variable restriction for a multivariate process to a one variable restriction for a univariate process, allows use of the Rice formula.

Rice formula

Theorem: The mean number of entries into C from C^c across the boundary ∂C per time unit is given by:

$$\begin{aligned}\gamma_{\boldsymbol{\xi}} &= E(N_1) = f_{\mathcal{M}_0}(0) E\left[(\dot{\mathcal{M}}_0)^+ \Big| \mathcal{M}_0 = 0 \right] \\ &= \int_{z=0}^{\infty} z f_{\mathcal{M}_0, \dot{\mathcal{M}}_0}(0, z) dz \\ &= \int_{\mathbf{x} \in \partial C} q(\mathbf{x}) f_{\boldsymbol{\xi}(\mathbf{0})}(\mathbf{x}) ds(\mathbf{x})\end{aligned}$$

where

$$q(\mathbf{x}) = |\dot{\mathcal{M}}(\mathbf{x})|^{-1} E\left[[(\dot{\mathcal{M}}_0)^+ \Big| \boldsymbol{\xi}(\mathbf{0}) = \mathbf{x} \right] = E\left[(\boldsymbol{\nu}_{\mathbf{x}} \cdot \dot{\boldsymbol{\xi}}(0))^+ \Big| \boldsymbol{\xi}(0) = \mathbf{x} \right]$$

and $ds(\mathbf{x})$ is the surface element on ∂C.

Back to the Palm distr.

Theorem: Under the above assumptions on $(\boldsymbol{\xi},\boldsymbol{\eta})$, C and $\mathcal{M}$, it follows that, with probability one,

$$\lim_{T\to\infty}\frac{N_T(B_{\boldsymbol{\tau}})}{N_T}=\frac{E\left[N_1(B_{\boldsymbol{\tau}})\right]}{E[N_1]}$$

$$=\quad\frac{f_{\mathcal{M}_0}(0)E\left[I\{\boldsymbol{\eta}(0+\cdot)\in B_{\boldsymbol{\tau}}\}(\dot{\mathcal{M}}_0)^+\Big|\mathcal{M}_0=0\right]}{f_{\mathcal{M}_0}(0)E\left((\dot{\mathcal{M}}_0)^+\Big|\mathcal{M}_0=0\right)}$$

$$=\quad\frac{1}{\gamma_{\boldsymbol{\xi}}}\int_{z=0}^{\infty}zP\left(\boldsymbol{\eta}(0+\cdot)\in B_{\boldsymbol{\tau}}\Big|\mathcal{M}_0=0,\dot{\mathcal{M}}_0=z\right)f_{\mathcal{M}_0,\dot{\mathcal{M}}_0}(0,z)dz$$

$$=\quad\frac{1}{\gamma_{\boldsymbol{\xi}}}\int_{z=0}^{\infty}\int_{\mathbf{y}\in B}zf_{\mathcal{M}_0,\dot{\mathcal{M}}_0,\boldsymbol{\eta}_{\boldsymbol{\tau}}(0)}(0,z,\mathbf{y})dzd\mathbf{y}.$$

where

$$\boldsymbol{\eta}_{\boldsymbol{\tau}}=(\boldsymbol{\eta}(\tau_1),\ldots,\boldsymbol{\eta}(\tau_n))$$

and $B_{\boldsymbol{\tau}}$, $(\tau_1,\ldots,\tau_n)$, B and n are as before.

Palm Distributions, cont.

The above integrals, involve the joint density of $(\mathcal{M}_0, \dot{\mathcal{M}}_0)$ and the conditional density of $\boldsymbol{\eta}_{\boldsymbol{\tau}}(0)\Big|(\mathcal{M}_0 = 0, \dot{\mathcal{M}}_0 = z)$ - which usually do not have simple closed forms - , so f we rewrite them as surface integrals in terms of the distributions of $\boldsymbol{\xi}$ and its derivative:

Theorem: Under the above assumptions on $(\boldsymbol{\xi}, \boldsymbol{\eta})$ and C, and assuming $B_{\boldsymbol{\tau}}$ is open, then

$$\begin{aligned} \frac{E\left[N_1(B_{\boldsymbol{\tau}})\right]}{E[N_1]} &= \frac{1}{\gamma_{\boldsymbol{\xi}}} \int_{\mathbf{x}\in\partial C} \int_{\mathbf{z}\in\mathbf{R}^r} \left(\boldsymbol{\nu}_{\mathbf{x}} \cdot \dot{\boldsymbol{\xi}}(0)\right)^+ \times \\ &\qquad P\left(\boldsymbol{\eta}(0) \in B_{\boldsymbol{\tau}} \Big| \boldsymbol{\xi}(0) = \mathbf{x}, \dot{\boldsymbol{\xi}}(0) = \mathbf{z}\right) f_{\boldsymbol{\xi}(0),\dot{\boldsymbol{\xi}}(0)}(\mathbf{x},\mathbf{z}) d\mathbf{z} ds(\mathbf{x}), \\ &= \frac{1}{\gamma_{\boldsymbol{\xi}}} \int_{\mathbf{x}\in\partial C} \int_{\mathbf{z}\in\mathbf{R}^r} \int_{\mathbf{y}\in B} (\boldsymbol{\nu}_{\mathbf{x}} \cdot \mathbf{z})^+ f_{\boldsymbol{\xi}(0),\dot{\boldsymbol{\xi}}(0),\boldsymbol{\eta}_{\boldsymbol{\tau}}(0)}(\mathbf{x},\mathbf{z},\mathbf{y}) d\mathbf{y} d\mathbf{z} ds(\mathbf{x}), \end{aligned}$$

where $ds(\mathbf{x})$ is the surface element on ∂C.

Optimal alarm systems, revisited

Theorem: The alarm region given by

$$A_h = \left\{ \mathbf{y} \in \mathbf{R}^m : \int_{\mathbf{x} \in \partial C} \int_{\mathbf{z} \in \mathbf{R}^r} (\boldsymbol{\nu}_{\mathbf{x}} \cdot \mathbf{z})^+ f_{\boldsymbol{\xi}^r(h), \dot{\boldsymbol{\xi}}^r(h) \big| \boldsymbol{\eta}^m(0)}(\mathbf{x}, \mathbf{z} \big| \mathbf{y}) d\mathbf{z} ds(\mathbf{x}) \geq k_h \right\},$$

is optimal of size α_h, where $\dot{\boldsymbol{\xi}}^r$ denotes the derivative of $\boldsymbol{\xi}^r$ with respect to t, $\boldsymbol{\nu}_{\mathbf{x}}$ is the unit vector normal to the surface ∂C at the point $\mathbf{x}$ in the direction of entry into C and the nonnegative constants k_h are such that $P(\boldsymbol{\eta}^m(t) \in A_h) = \alpha_h$.

The integral with respect to $\mathbf{x}$ should really be written as a sum of integrals over those regions for which ∂C is smooth. The areas where $\mathcal{M}(\cdot)$ is not differentiable are of Lesbegue measure less than $r - 1$ and hence of no concern.

Optimal alarm systems - special case

This region A_h is difficult to interpret in the general case, so let assume that C takes the form given by

$$C = \{\mathbf{x} \in \mathbf{R}^r : max(x_1 - u_1, \ldots, x_r - u_r) \geq 0\}.$$

Thus,

$$\partial C = \{\mathbf{x} \in \mathbf{R}^r; \max_i x_i = u_i\} = \cup_{i=1}^r \partial C_i$$

where $\partial C_i = \{\mathbf{x} \in \mathbf{R}^n; x_i = u_i, x_j \leq u_i, j \neq i\}$ for $i = 1, 2, \ldots, r$.

Then the integral with respect to $\mathbf{x}$ can be written as a sum of integrals over the ∂C_i's. As well, the unit vector normal to ∂C_i at $\mathbf{x}$ in the direction of entry into C is then simply the i^{th} unit vector $\boldsymbol{\nu}_{\mathbf{x}} = \delta_i = (0, \ldots, 1, \ldots, 0)$ for all $\mathbf{x} \in \partial C_i$; that is, a zero vector with a one in the i^{th} position. Hence...

Optimal alarm systems, cont.

Corollary The optimal alarm region of size α_h is given by

$$A_h = \left\{ \mathbf{y} : \sum_{i=1}^{r} \int_{\mathbf{x}\in\partial C_i} \int_{\{\mathbf{z}\in\mathbf{R}^r : z_i>0\}} z_i f_{\boldsymbol{\xi}^r(h),\dot{\boldsymbol{\xi}}^r(h)|\boldsymbol{\eta}^m(0)}(\mathbf{x},\mathbf{z}|\mathbf{y}) d\mathbf{z} d\mathbf{x}^i \geq k_h \right\}$$

$$= \left\{ \mathbf{y} : \sum_{i=1}^{r} \left(\prod_{j\neq i} \int_{-\infty}^{u_j} \right) \left[\dot{\sigma}(h) \Psi \left(\frac{\dot{\mu}_i(\mathbf{x}_u^i, \mathbf{y}; h)}{\dot{\sigma}(h)} \right) \right] f_{\boldsymbol{\xi}^r(h)|\boldsymbol{\eta}^m(0)}(\mathbf{x}_u^i|\mathbf{y}) d\mathbf{x}^i \geq k_h \right\},$$

where $x_u^i = (x_1 \ldots, x_{i-1}, u_i, x_{i+1} \ldots x_r)$, $d\mathbf{x}^i = dx_1 \ldots dx_{i-1} dx_{i+1} \ldots dx_r$,

$$\dot{\mu}_i(\mathbf{x}_u^i, \mathbf{y}; h) = \mu_{\dot{\xi}_i(h)|\boldsymbol{\xi}^r(h),\boldsymbol{\eta}^m(0)}(\mathbf{x}_u^i, \mathbf{y}; h) \quad and \quad \dot{\sigma}_i^2(h) = \sigma_{\dot{\xi}_i(h)|\boldsymbol{\xi}^r(h),\boldsymbol{\eta}^m(0)(h)}$$

are the conditional mean and variance respectively of $\dot{\xi}_i(h)$ given $\boldsymbol{\xi}^r(h) = \mathbf{x}_u^i$ and $\boldsymbol{\eta}^m(0) = \mathbf{y}$, $\int_0^\infty x f_X(x) dx = \sigma\Psi(\mu/\sigma)$ for $X \sim N(\mu, \sigma^2)$ so $\Psi(z) = \phi(z) + x\Phi(z)$ with $\phi(z)$ and $\Phi(z)$ the standard normal probability density function and distribution function respectively, and the nonnegative constants k_h are such that $P(\boldsymbol{\eta}^m(t) \in A_h) = \alpha_h$.

The conditional mean and variance can be obtained using multi-variate normal theory.

The integrals in are non-trivial to evaluate and so this is done numerically.

Examples

We consider as alarm and catastrophe process a centered Gaussian with covariance function

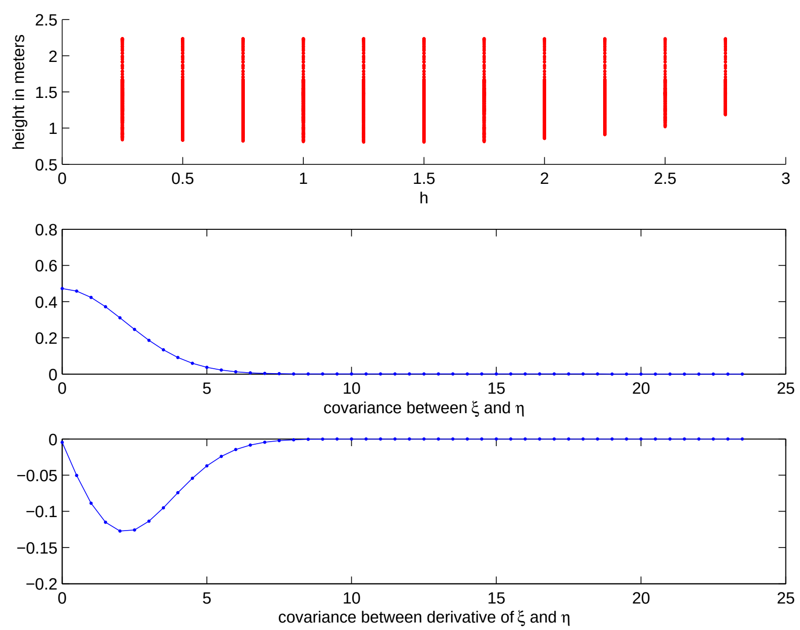
$$r(x,y,t) = \lambda_0 \exp\left(-\frac{1}{2\lambda_0}(x,y,t)\Lambda(x,y,t)'\right),$$

with

$$\Lambda = \begin{pmatrix} 0.7 & 0 & -0.09 \\ 0 & 0.2 & 0.04 \\ -0.09 & 0.04 & 0.2 \end{pmatrix}$$

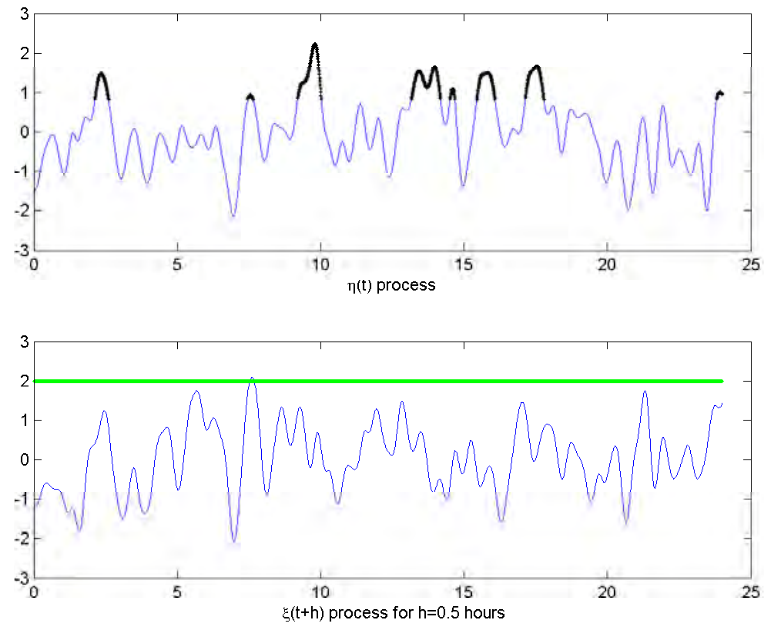
$\lambda_0 = 1.$

Examples, Cont.



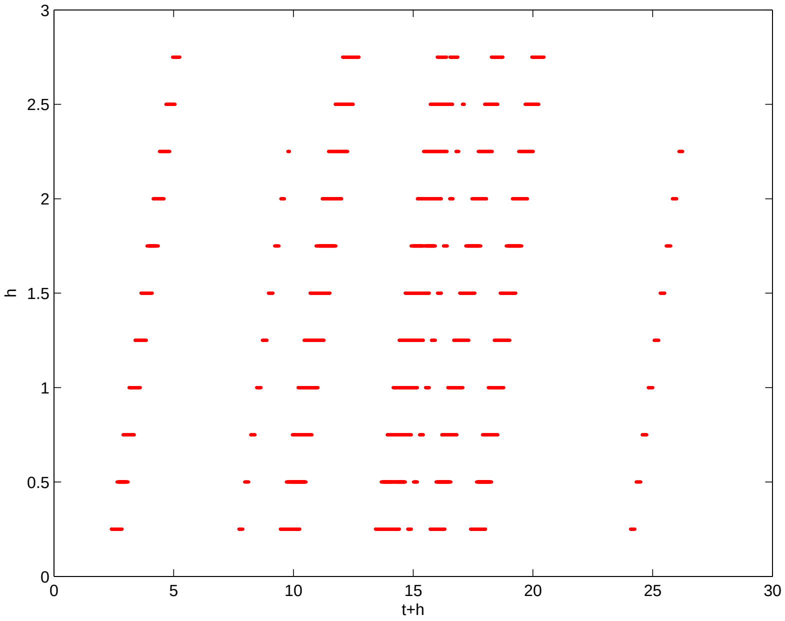
Top: Plot of the optimal alarm region for different values of h with alarm size $\alpha \in [0.1, 0.2]$. *Middle:* Covariance function between $\xi(\mathbf{p}, t+h), \eta(\mathbf{s}, t)$ for $\mathbf{p} = (0,0)$ and $\mathbf{s} = (1,2)$. *Bottom:* Covariance function between $\dot{\xi}(\mathbf{p}, t+h), \eta(\mathbf{s}, t)$ for $\mathbf{p} = (0,0)$ and $\mathbf{s} = (1,2)$.

Examples, Cont.



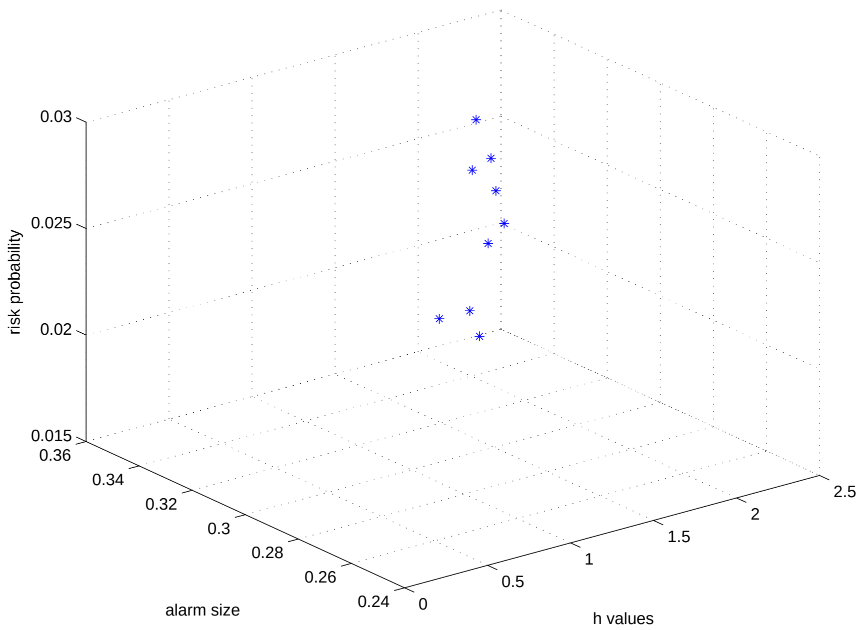
Top: Simulation of process $\eta(\mathbf{s}, t)$ for $\mathbf{s} = (1, 2)$. The red line indicates the corresponding optimal alarm region of size $\alpha =$. *Bottom:* Simulation of the process $\xi(\mathbf{p}, t + h)$ for $\mathbf{p} = (0, 0)$ and $t = 0.5h$. The black line indicates the catastrophe region. We have used $u = 2$.

Examples, Cont.



Sets $\{(t+h,h);\eta(\mathbf{s},t)\in A_h\}$

Examples, Cont.



Alarm sizes and risk probabilities for the values of h where a catastrophe occurs exactly h time after an alarm was given.

Preliminaries for Slepian models

As we have seen the distributions for $\boldsymbol{\eta}$ and ξ conditionally on $A(t)$ and $C(t)$, were defined by means of Palm distributions and under the assumption $(\boldsymbol{\eta}, \xi)$ is ergodic. The conditional distributions thereby obtained can be interpreted as the distributions:

- for $\boldsymbol{\eta}$ around points in time at which the catastrophe process $\boldsymbol{\xi}^r$ enters the set C
- for ξ around points in time at which the alarm process η^m enters the set A.

Slepian Models

We can obtain the multivariate version of the long-run Rayleigh distribution of the derivative of the field at an upcrossing. Assuming $(\boldsymbol{\xi}, \boldsymbol{\eta})$ are jointly Gaussian and satisfy the conditions we mentioned, we obtain the Slepian model for $\boldsymbol{\eta}|C(0)$.
Let $\boldsymbol{\kappa}(t)$ denote a non-stationary m-dimensional normal process with mean zero and the covariance matrix

$$r^{\boldsymbol{\eta}}(t_1,t_2) = r_{\boldsymbol{\eta}}(t_1-t_2) - \left[r_{\boldsymbol{\xi},\boldsymbol{\eta}}(t_1)^T \quad -\dot{r}_{\boldsymbol{\xi},\boldsymbol{\eta}}(t_1)^T \right] r_{\boldsymbol{\xi},\dot{\boldsymbol{\xi}}}(0)^{-1} \begin{bmatrix} r_{\boldsymbol{\xi},\boldsymbol{\eta}}(t_2) \\ -\dot{r}_{\boldsymbol{\xi},\boldsymbol{\eta}}(t_2) \end{bmatrix}.$$

Also $(\boldsymbol{\chi}, \boldsymbol{\zeta})$ is a $2r$-dim r.v. independent of $\boldsymbol{\kappa}$, taking values on

$$\{(\mathbf{x}, \mathbf{z}) \in \partial C^- \times \mathbf{R}^r : \boldsymbol{\nu}_{\mathbf{x}} \cdot \mathbf{z} > 0 \text{ for each } \mathbf{x} \in \partial C^-\}$$

and with distribution given by

$$f_{\boldsymbol{\chi},\boldsymbol{\zeta}}(\mathbf{x}, d\mathbf{z})ds(\mathbf{x}) = \frac{\boldsymbol{\nu}_{\mathbf{x}} \cdot \mathbf{z}}{\gamma_{\boldsymbol{\xi}}} f_{\dot{\boldsymbol{\xi}}(0)|\boldsymbol{\xi}(0)}(\mathbf{z}|\mathbf{x}) f_{\boldsymbol{\xi}(0)}(\mathbf{x}) d\mathbf{z} ds(\mathbf{x}),$$

where $\partial C^- = \{\mathbf{x} \in \partial C \text{ with first order der.}\}$

Slepian Models

The Slepian model is given by:

$$\boldsymbol{\eta}_{\partial C}(t) = \mathbf{m}^{\boldsymbol{\eta}}_{\boldsymbol{\chi},\boldsymbol{\zeta}}(t) + \boldsymbol{\kappa}(t)$$

where

$$\mathbf{m}^{\boldsymbol{\eta}}_{\boldsymbol{\chi},\boldsymbol{\zeta}}(t) = \left[r_{\boldsymbol{\xi},\boldsymbol{\eta}}(t)^T \;\; -\dot{r}_{\boldsymbol{\xi},\boldsymbol{\eta}}(t)^T \right] r_{\boldsymbol{\xi},\dot{\boldsymbol{\xi}}}(0)^{-1} \begin{bmatrix} \boldsymbol{\chi} \\ \boldsymbol{\zeta} \end{bmatrix}.$$

Using the characteristic functions for the finite dimensional distributions we can show:

Theorem: Under the conditions on $(\boldsymbol{\xi}, \boldsymbol{\eta})$ and C and ∂C , the finite dimensional distributions for the conditional process $\boldsymbol{\eta}|C(0)$ are the same as those for the process $\boldsymbol{\eta}_{\partial C}$.

Future Work

This is work on progress. So there are a lot of things to be done! Still, we are currently working on

- special cases of covariance functions and for different numbers of alarm and catastrophe locations
- on developing the theory for the case of catastrophe occurring at random locations and along a curve,

and we intend to

- test the theory using satellite and buoy data of significant wave height in order to predict floods and other related extreme events.

References

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