Non-stationary approach to Flood Frequency Modelling and Hydrological Design

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Outline

- Reasons of non-stationarity of flood events
- Direct causes of disastrous floods
- Random variables modelled in FFA
- Uncertainty of upper quantile estimates
- Power of model discrimination procedures
- Asymptotic bias of quantile estimates caused by model misspecification
- Time trend build in estimates of PDF parameters
- Identification of distribution and trend software package
- Implementation of NFFA results
Reasons of non-stationarity of flood events

- Changes in land use and land cover, drainage works
- River regulation and flood control reservoirs
- Global and local climate change or variability
Direct causes of disastrous floods

- Flow discharge exceeds the full bank capacity and water is spilling over crown of levees;
- The flood wave breaks embankment. Prolonged high water levels are softening the river levees finally causing rapidly growing breach. Obviously the exact location of the washout can be hardly predicted but from the observation of past floods one learns that it usually takes place after the flood culmination, i.e. on the falling limb of flood hydrograph. Therefore the break of flood banks does not decrease the magnitude of peak flow discharge.
- Blockage of a river channel by ice jams, sand bar in the river mouth, dumped trees and bushes. (Beyond formal statistical analysis).
Random variables modelled in FFA

One dimensional random variable:

- (i) Annual peak flow \( Q_{\text{max}} \)
- (ii) Annual maximum mean discharge \( \bar{Q}_d \) over a period of duration \( d \) (Javelle, 2001)
- (iii) Annual maximum flow discharge \( Q_d \) continuously exceeded during the period \( d \) (Bogdanowicz et al. 2008)
- (iv) Annual maximum (uninterrupted) duration \([D \ (\text{hours})]\) of flows over the flood alarm threshold \( q_A \)
Random variables modeled in FFA

(i) **Annual peak flow** \( Q_{\text{max}} \)

Water resource management, design of hydraulic structures such as bridges, spillways of dams, embankments, roads and railways, land use management and flood control depend on reliable estimates of annual maximum (AM) flows with various probability of exceedance \( \hat{Q}_{\text{max}}(p) \).

They entail estimation of the upper tail of a probability density function (PDF) of annual maximal flows \( Q_{\text{max}} \):

\[
F(q) = p(Q_{\text{max}} \leq q) = \int_{0}^{q} f(q; \theta) dq
\]
Random variables modeled in FFA

(i) Annual peak flow \( (Q_{\text{max}}) \)

Annual peak flow estimate \( \hat{Q}_{\text{max}}(p) \) is obtained either

(a) directly from the annual maximum instantaneous or mean daily streamflow series \( (Q_{\text{max}}(i); i = 1, 2, \ldots, N) \),

(b) from partial duration series

\[
F(q) = \exp \left[ -\lambda \left( 1 - F^{(\text{POT})}(q) \right) \right] \quad \text{for} \ q > q_0 ,
\]

or

(c) from seasonal approach

\[
F(q) = P(Q_{\text{max}} \leq q) = P(Q_{\text{max}}^{(W)} \leq q, Q_{\text{max}}^{(S)} \leq q) = F^{(W)}(q) \cdot F^{(S)}(q) \quad \text{e.g. TCEV1}
\]
Random variables modeled in FFA

(i) Annual peak flow \( (Q_{\text{max}}) \)

\( Q \quad \text{(m}^3/\text{s}) \)

\( Q_1 = Q^{(w)}_{\text{max}} \)

\( Q_2 = Q_{\text{max}} = Q^{(s)}_{\text{max}} \)

\( Q_3 \)

Peaks used for \( Q_{\text{max}}(p) \) estimation
Random variables modeled in FFA

(ii) Annual maximum mean discharge \( \bar{Q}_d \) over a period of duration \( d \) (Javelle, 2001)

\[ \bar{Q}_d = \Phi(F, \theta_d) \]
Random variables modeled in FFA

(ii)

The duration in ‘discharge–duration–frequency’ model (\(\overline{Q}dF\) model) is considered as a fixed parameter. The product \((\overline{Q}_d \cdot d)\) can be used to determine the volume necessary to reduce the peak to required magnitude.

To avoid inconsistency of the estimates of quantile \(\overline{Q}(d,F)\) for various \(d\), the same distribution function is applied for all duration (Javelle et al., 1999) and the quantile are reduced by decreasing function of \(d\), i.e.

\[
\overline{Q}(d,F) = \varphi(d,v) \cdot Q(0,F) \quad \text{for} \quad d = 0, 1, 2, \ldots ; \varphi(0) = 1
\]

where the parameters \(v\) are estimated from the data. It means that the distributions for various \(d\) values differ in the mean only. Note that \(Q(0,F)\) corresponds to the distribution of annual instantaneous peak discharges.
Random variables modeled in FFA

(iii) Annual maximum flow discharge \( Q_d \) continuously exceeded during the period \( d \)  
(Bogdanowicz et al. 2008)

Definition of annual maximum flow discharge \( Q_d \) continuously exceeded during the period \( d \)
Similarly to the ‘mean discharge–duration–frequency’ model (\( QdF \) model), the duration is in the \( QdF \) analysis considered as a fixed parameter. Taking various \( d \) duration in the \( QdF \) model provides continuous description of flood hydrograph as a function of flow duration.
Random variables modeled in FFA

(iv) Annual maximum (uninterrupted) duration \( [D \text{ (hours)}] \) of flows over the flood alarm threshold \( (q_A) \) is considered as the measure of the risk of flood spilling out of river channel. The flow discharge is a fixed parameter.
Random variables modeled in FFA

(iv)  

The DqF model

The frequency analysis of the data containing several zero values, while zero is the lower limit of the variability, requires using discontinuous PDF

\[ f(d) = \beta \delta(d) + (1 - \beta) f^\circ(d; g) \cdot 1(d) \quad \beta \notin g \]

where \( \beta \) denotes probability of zero event, \( f^\circ(d; g) \) is the conditional probability density function (CPDF), i.e. \( f^\circ(d; g) \equiv f(d|D>0) \), which is continuous in the range \((0, +\infty)\) with a lower bound of zero value, and \( g \) is the vector of parameters while \( 1(d) \) is a unit step function. Note that the estimate of \( \beta \) can be taken from AM cumulative distribution function (CDF):

\[ \hat{\beta} = P(D=0) = P(Q_{\text{max}} \leq q_A) = F(q_A) \]
Random variables modeled in FFA

(iv)

The DqF model

Alternatively, from the likelihood function

\[
L = \beta^{n_1} \cdot (1 - \beta)^{n_2} \prod_{j=1}^{n_2} f^0(d_j; g)
\]

where \(n_1\) and \(n_2\) denote the number of zeros and non-zeros values, respectively, one gets

\[
\hat{\beta} = \frac{n_1}{n_1 + n_2}
\]
Uncertainty of upper quantile estimates

Sampling error and model error both depend on the parameter estimation technique.

**Power of model discrimination procedures.**

The discrimination procedures often favor some functions as shown below for \((IG, LN)\) pair.

Probability of correct selection for the \(LR\) procedure got by sampling experiment:
## Uncertainty of upper quantile estimates

Inverse Gaussian (IG) vs. Lognormal (LN) models – parameters estimated by MLM

Legend: \( N \) – sample size, \( C_v \) – variation coefficient

<table>
<thead>
<tr>
<th>( N )</th>
<th>( C_v = .1 )</th>
<th>( C_v = .25 )</th>
<th>( C_v = .5 )</th>
<th>( C_v = .75 )</th>
<th>( C_v = 1.0 )</th>
<th>( C_v = 1.5 )</th>
</tr>
</thead>
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<td>( IG )</td>
<td>( TRUE= )</td>
<td>( TRUE= )</td>
<td>( TRUE= )</td>
<td>( TRUE= )</td>
<td>( TRUE= )</td>
<td>( TRUE= )</td>
</tr>
<tr>
<td>( LN )</td>
<td>( .59 )</td>
<td>( .19 )</td>
<td>( .62 )</td>
<td>( .20 )</td>
<td>( .69 )</td>
<td>( .22 )</td>
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<tr>
<td>( IG )</td>
<td>( .63 )</td>
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<td>( .65 )</td>
<td>( .32 )</td>
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<tr>
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<td>( .34 )</td>
<td>( .67 )</td>
<td>( .36 )</td>
<td>( .70 )</td>
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</tr>
<tr>
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<td>( .70 )</td>
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<td>( .71 )</td>
<td>( .41 )</td>
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<tr>
<td>( LN )</td>
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<td>( .40 )</td>
<td>( .81 )</td>
<td>( .43 )</td>
<td>( .81 )</td>
<td>( .51 )</td>
</tr>
</tbody>
</table>
Uncertainty of upper quantile estimates

Asymptotic bias of quantile estimates caused by model misspecification

Asymptotic relative model error of quantiles if (True=Log logistic, False=Log Gumbel)
Regardless of the reasons of flood regime changes, when dealing with hydrological non-stationarity in flood frequency modelling and hydrological design, it is necessary to account for trends in upper quantile estimates.

Assuming a distribution function to be time-invariant, the trends in quantiles result from time-variability of distribution parameters:

\[ f(x; \theta_t) \Rightarrow f(x; m(t), \sigma(t), \text{shape parameter}) \]

Why the heteroscedasticity option is of interest?

For upper quantile \( x_F \) one gets \( R > 1 \),

\[ R = \left( \frac{\partial x_F}{\partial \sigma} \right) / \left( \frac{\partial x_F}{\partial m} \right) \]

Denote \( R = \left( \frac{\partial x_F}{\partial \sigma} \right) / \left( \frac{\partial x_F}{\partial m} \right) \)

For upper quantile \( x_F \) one gets \( R > 1 \),

\( R \) equals 2.33 and 3.14 for the normal and Gumbel distributions, respectively.
Identification of distribution and trend software package

Model = type of PDF + class and form of time trend in distribution parameters.

Set of alternative distributions:
(2000):
  2-parameters: N, Ga(2), Gu, LN(2),
  3-parameters: Pe(3), LN(3)
(2007 supplement):
  2-parameters: LG, LL, We(2), CD(2), IGa(2)
  3-parameters: GEV, GLL, We(3), CD(3), IPe(3).
Time trend build in estimates of PDF parameters

Identification of distribution and trend software package

Classes of trend:

A: in the mean value;
B: in the standard deviation;
C: both in the mean and the standard deviation related by a constant value of the variation coefficient ($C_V$);
D: unrelated trend in the mean value and the standard deviation

Forms of trend:
(2000) Linear and square trinomial (parabolic);
(2007) Linear only.
Identification of distribution and trend software package

ML estimation of covariate-dependent PDF parameters. Model discrimination by the Akaike information criterion
Standard error of time-dependent quantile produced by
- Fisher information matrix (2000)
- “Resampling” (Katz et al. 2002).

Hydrologic design

Denoting service life as $T$ years and the beginning of operation in $t_1$ year, the probability of exceedance of peak discharge $x_d$ during this period is

$$P_T (X > x_d) = 1 - \prod_{t=t_1}^{t_1+T-1} \int_{-\infty}^{x_d} f(x; \hat{\theta}_t) \, dx$$

For given probability of exceedance $P_T$ one can find by iterative technique the design flow discharge $x_d$. 

*Time trend build in estimates of PDF parameters* (4 of 4)
The deficiency of the ML estimation in the presence of covariates results from the fact that various models may show similar fit to the time-series while time dependent estimates of moments and upper quantiles strongly depend on the model.

Moreover, the L-ratio (hence AIC) discrimination procedure favours some distributions and its capacity is low for hydrological series.

Computational difficulty of the ML method depends on a model and it increases fast with the number of parameters to be estimated.

All these tend to incline towards an estimation of time-dependent moments by distribution-free techniques.

While some authors consider the GEV distribution option only, we suggest stationarisation of time series by WLS method (Strupczewski and Kaczmarek, 1998, 2001) and then selection of a proper distribution function using the L-moment technique for parameter estimation.
Implementation of NFFA results

- Hydrologic design under non-stationary conditions is a direct consequence of accepting the idea of environmental changes.
- It requires a two-dimensional extrapolation of usually short time series, namely, in probability and in time, to cover the design life of a flood control structure, which can be over 100 years in the case of a major structure.
- Is the statistical prediction for such a long period reliable??
- Prevailing tendency of Polish river flood regime (1920-2005) is a decreasing trend in both the mean and standard deviation while keeping $C_V$ fairly constant.
- Making allowance for non-stationarity brings in this case decrease of water structure dimensions.
Implementation of NFFA results

If time-series of summer and winter peak flows exhibit different nonstationary properties, such analysis should be based on the Seasonal Maxima approach.

It is easy to accept the idea of the impact of environmental changes on flood regime but its implementation is still in the stage of infancy.

A physical explanation of the observed trend can make the prediction more meaningful.
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Thank you !!


