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ESF Mathematics Conference in partnership with EMS and ERCOM

Algebraic Methods in Dynamical Systems

The Institute of Mathematics Conference Centre, Bedlewo • Poland
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Chaired by:

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Conference Highlights

Please provide a brief summary of the conference and its highlights in non-specialist terms (especially for highly technical subjects) for communication and publicity purposes. (ca. 400-500 words)

The conference "Algebraic Methods in Dynamical Systems" took place at the Bedlewo Mathematical Conference Center from May 16th to 22nd 2010 and attracted renowned specialists from many countries, representing different schools of mathematics. The Conference was dedicated to the well-known expert in the algebraic theory of differential equations Michael F. Singer on his 60th birthday. Several lectures were delivered by Singer's collaborators and his former students. The first part of the meeting was concentrated on the various subjects of differential and difference Galois theory, especially on algebraic aspects of Galois theory of non linear differencial and q-difference equations. Discrete and continuous interpretations of Galois theory were presented by the experts in the field, among them Lucia di Vizio, Askold Khovanskii and Guy Casale. Further deep applications could be observed during the poster session in the contributions of young researchers. Poster session was not only the occasion for young researchers to present their contributions, but also for the scientists working on foundations of differential Galois theory to meet scientists working on applications in Hamiltonian and quantum mechanics. Many interesting discussions arised and suggested new directions of research. Mathematicians from Japan discussed generalizations of differential and difference Galois theories to the very general context of D-module algebras.

In the second part of the conference, starting with Bernard Malgrange's presentation of non linear Galois theory, the lectures contained more analytic aspects. Andrzej Maciejewski and Maria Przybylska lectured on higher order variational equations and integrable perturbations for homogeneous potentials. Alexander Bruno presented analysis of singularities and ordinary differential equations based on his algorithms of power geometry. The ideas of Bruno were used by Victor Varin in his application of power geometry to the study of stability set of a multiparametric gyroscopic problem. Jean-Pierre Francoise lectured on Abel equations and the algebraic moments problem. Colin Christopher's talk contained new results related to the center-focus problem. Especially interesting were the lectures around the famous Hilbert 16th problem presented by Sergei Yakovenko, Dmitry Novikov and Gal Binyamini. On the other hand several important results in the theory of complex algebraic foliations were presented by Yulij Ilyashenko in his lecture on rigidity properties of polynomial foliations. The ideas of Ilyashenko were included as well in the talks of his students and collaborators e.g. Grisha Kolutsky lectured on the Hilbert-Smale problem and related effective estimates.

The dynamics of billard problems were presented by Alexey Glutsyuk and his student Yury Kudryashov.

Alvaro Bustinduy presented his important results on complete holomorphic vector fields on the complex plane. Finally Henryk Żołądek gave an interesting lecture on his investigation on the famous Jacobian Conjecture.

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Scientific Report

Executive Summary

(2 pages max)

The conference “Algebraic Methods in Dynamical Systems” brought together different aspects of the study of differential equations. The first part of the conference was concentrated on algebraic results and foundations of differential Galois theory. An important contribution to this subject was the lecture delivered by Michael Singer on differential algebraic groups and factorization of partial differential operators. This research which was formerly developed by Phyllis Cassidy and other members of the well known Kolchin’s Seminar was presented by Singer with the more modern Tannakian approach. His ideas were continued in the lecture by Alexey Ovchinnikov on Zariski closures of linear differential algebraic groups. The application of differential algebraic groups to the theory of isomonodromic deformations was presented by one of Singer’s collaborators: Claudine Mitschi from the University of Strasbourg. Besides Singer, other experts from United States lectured on differential Galois theory. Andy Magid presented his results on complete Picard-Vessiot closure which illustrate how complex is the differential setting in comparison with the algebraic one. William Keigher lectured on linear differential equations and Hurwitz series. Thanks to the presence of Bernard Malgrange the subject of non linear differential Galois theory had a strong scientific impact. Just recently Malgrange has finished his great work on non linear differential Galois theory entitled “Pseudogroupes de Lie et théorie de Galois différentielle”, where he gave a very general and extremely clear vision of non linear theory, closing a more than ten years old investigation in this subject. The ideas of Malgrange have had a great influence in several younger mathematicians who participated as well in the conference. Specially interesting were the results presented by Lucia di Vizio in a joint work with Charlotte Hardouin on algebraic and differential generic Galois groups for q-difference equations and by Guy Casale in collaboration with Julien Roques on the discrete version of Morales-Ramis theory. It is worth mentioning that the work by Malgrange contains as well an equivalent formulation of the Morales-Ramis-Simó theory on integrability of Hamiltonian systems. The elegant lecture by Marius van der Put presented non linear aspects of differential Galois theory related with Painlevé equations.
In the early nineties Hiroshi Umemura proposed his own version of non linear differential Galois theory in a more algebraic frame, inspired by former works of Jules Drach and Ernest Vessiot. Umemura’s theory is more general than the theory of Malgrange but not so suitable for applications. In his talk, Umemura reported on the recent results of Heiderich unifying differential and difference theories and proposed a generalization of these theories to the non-commutative case in which algebraic groups should be substituted by quantum groups. The unification of differential and difference theory in the framework of Picard-Vessiot theory was obtained in the PhD thesis of Katsuoshi Amano from Tsukuba University by using the language of D-module algebras. These ideas were presented in Amano’s talk.

The Russian school of differential equations has a long and reputed trajectory and had several distinguished representatives in this conference. First of all Yulij Ilyashenko lectured on rigidity properties of polynomial foliations and traced new directions of research in the global theory. The deep ideas presented by Ilyashenko were developed by his students, among them Grisha Kolutsky in his talk on Hilbert-Smale problem and related effective estimates. Secondly Alexander Bruno lectured on applications of his “power geometry” to the analysis of singularities and ordinary differential equations. The ideas of Bruno were used by Victor Varin in his excellent talk on the stability set of a multiparametric gyroscopic problem. Finally Askold Khovanskii presented his ideas of topological Galois theory in his lecture on algebraic functions which can be expressed in terms of radicals. Many of the Khovanskii’s ideas are included in his new book on Galois theory.

The characterization of non-integrability of Hamiltonian systems in terms of first variational equations obtained by Morales and Ramis has been generalized in the recent years by considering higher order variational equations. Several lectures in the conference such as those by Andrzej Maciejewski from Zielona Góra University and Maria Przybylska from Toruń University applied these results to Hamiltonian systems appearing in physical problems.
Other important contributions were made which call attention to long standing problems related to differential equations. Henryk Żołądek from Warsaw University gave a lecture on his recent works on Jacobian Conjecture and Sergei Yakovenko, Dmitry Novikov and Gal Binyamini from Weizmann Institute of Science presented recent important results around Hilbert 16th problem.

Scientific Content of the Conference

(1 page min.)

- *Summary of the conference sessions focusing on the scientific highlights*
- *Assessment of the results and their potential impact on future research or applications*

The conference was held from May 16th to 22nd and according to the final program had 9 sessions, related with the following 5 topics:

1. Analytic theory of differential systems
2. Integrability of dynamical systems
3. Non linear differential Galois theory
4. Algebraic aspects of Differential Galois Theory
5. Algorithmic aspects of Differential Galois Theory

As it usually happens in mathematics, most of the lectures may not be confined to just one of these topics and in fact very interesting results are obtained at the edge between two theories.
With the subject 1 were related (at least partially) sessions 3,4,5,7,8 and 9. Here certainly the most distinguished results were presented by Alexander Bruno, Sergeyi Yakovenko and Yulij Ilyashenko.
Bruno has created his power geometry and applied it to non-linear problems. Its main concepts consist in the local study of these problems not in the original coordinates but in the logarithms of these coordinates. The methods of Bruno allow to simplify equations, resolve their singularities, to isolate their first approximations and to find either their solutions or the asymptotic of the solutions. At this moment it is difficult to evaluate the impact of this theory for future research, but some examples presented by Bruno's collaborators look very promising.
Sergei Yakovienko is working from several years on differential equations related to the Hilbert 16th problem and recently in collaboration with Dmitry Novikov and Gal Binyamini has solved a long standing problem about the zeros of Abelian integrals. It is a new and very deep result on a problem formerly studied by Vladimir Arnold.
Yulij Ilyashenko lectured on his new results on rigidity properties of polynomial foliations. Ilyashenko posed several deep questions on geometric objects related to ODE. Solutions of Ilyashenko's problems may have interesting applications in further study of Hilbert 16th problem.

With the subject 2 were related the following sessions: 2,3,4,5,7 and some of the presented posters. Here the most distinguished results were concentrated around the problems related with integrability of Hamiltonian systems. The theory is still in progress and besides of classical nonintegrability results, quite recently appeared new results related with quantum chaotic Hamiltonian systems presented by Primitivo Acosta-Humanez in collaboration with Juan Morales-Ruiz and with general dynamical systems presented by David Blazquez Sanz in a joint work with K. Yagasaki. It is worth mentioning the research results presented by Andrzej Maciejewski, Maria Przybylska and Andriy Panasyuk. Very interesting applications to molecular physics were presented by Robert Gebarowski from the Department of Physics of Cracow University of

Technology in his poster related with Hamiltonian system for hydrogen atoms and magnetic fields.

With the subject 3 were related the following sessions: 1,2 and 6. Here the most distinguished and influencial lecture was the one given by Bernard Malgrange based on his landmark paper "Pseudogroupes de Lie et théorie de Galois différentielle"(http://www.ihes.fr, Prepublications m/10/11), culminating his long research on nonlinear differential Galois theory. It is difficult to evaluate the future impact of the theory presented by Malgrange, however according to many specialists it is one of the most important works in the Galois theory of differential equations.

With the subject 4 were related the following sessions: 1,2,3 and 4. Here the most distinguished results were the ones presented by Lucia di Vizio in collaboration with Charlotte Hardouin and Guy Casale with Julien Roques. One can expect that very interesting and deep analogies of the classical differential Galois theory in the theory of q-difference equations will have some impact in the arithmetic geometry and number theory.

With the subject 5 were related the following sessions: 2,3,5,7 and 9. Here a very interesting lecture on coding with the use of skew polynomial rings were presented by Felix Ulmer. The lack of unique factorization in the rings of skew polynomials is the source of a big number of codes. In the classical context of Hamiltonian mechanics algorithmic aspects of differential Galois theory were presented by Andrzej Maciejewski and Maria Przybylska and in the context of general dynamical systems by Jacques-Arthur Weil. In the near future one can expect valuable implementations of the presented algorithms together with applications in the study of hard computational problems.

Forward Look

(1 page min.)

- *Assessment of the results*
- *Contribution to the future direction of the field – identification of issues in the 5-10 years & timeframe*
- *Identification of emerging topics*

Many important scientific problems for future work and collaboration may be mentioned. A number of them were indicated by the participants during the forward look discussion which took place on Friday. The scientific discussions made during the conference will certainly be a germ for future collaborations. An important fact of this conference was that it assembled specialists working in close subjects but with different methods and points of view. Synergy will then arise in their future collaboration which will produce a large number of important results.
We list the most important problems for future work and collaboration.

1. *Differential algebra and Galois theories*. In this topic, extensions and unifications of various approaches to Galois theory were postulated, especially in connection with model theory, which allowed recently to elaborate very general and powerful methods for difference and differential theories as well as for algorithms. It is worth mentioning that these theories have important applications to integrability of differential systems appearing in different physical problems as well as problems arising in other disciplines; discrete dynamics, bifurcations; isomonodromic deformations. Some work in progress by Casale is intending to express solvability in the context of Malgrange's non-linear differential Galois theory. Further progress along this line will allow for new interesting applications.
   Another open problem which deserves further research is building a Picard-Vessiot theory in the case in which the constant field is not algebraically closed. Some work in progress in the case of formally real fields was presented during the poster session. Extending differential Galois theory to real fields is certainly important for physical applications. The case of number fields will have interesting arithmetic applications.
2. *Ordinary and partial differential equations and foliations*. In this topic, a further study of the

oscillatory properties of surfaces in space as well as a search for new functions measuring zeros of solutions of connections were suggested. Ilyashenko suggested to study global extension of geometric objects related to ODE's as well as considering other geometric objects such as limit cycles, saddle connections.

3. *Power geometry*. This is a very powerful tool developed by Bruno who proposed a more intensive use of it in the study of higher variational equations and integrability of differential systems.
4. *Integrability of dynamical systems.* In this item, Morales-Ramis theory appeared around ten years ago and had a big impact. It has been generalized recently to higher variational equations in collaboration with Carles Simó. It gives a necessary condition for integrability of Hamiltonian systems hence provides a method to prove that a given Hamiltonian system is not integrable. The need was pointed out to find sufficient conditions for integrability allowing to determine that a given Hamiltonian system is integrable. Maciejewski and Przybylska have started research in this direction but much should still be done.
   Michael Singer pointed out that there are different notions of integrability arising from different physical contexts which should be unified or related between them. Hence there are also mathematical theoretical problems related to integrability. It was also suggested to establish a classification of integrable differential equations following Lie and working on the research results of Malgrange and Casale. This is certainly an interesting aim which will take several years to be reached.
5. *Algorithmic and explicit aspects.* The implementation of Kovacic's algorithm has allowed to apply effectively the criterion given in Morales-Ramis-Simó theory to obtain non-integrability of Hamiltonian systems. In connection with point 4. Maciejewski indicated the necessity of disposing of an efficient way to compute the solutions of a differential system once we know that it is integrable. Explicit aspects appear as well in the direct and inverse problem in Picard-Vessiot theory, namely computing the differential Galois group of a linear differential equation or realizing a given linear algebraic group as differential Galois group. Still open problems survive in this context, especially in the case of non algebraically closed field of constants.

We are planning to publish the proceedings of the conference as a Banach Center publication. This will allow to spread out the ideas discussed during the conference among a larger audience so that the benefits of it will reach to more mathematicians and facilitate a more intense collaboration.

- Is there a need for a foresight-type initiative?

The Conference held in Bedlewo had a precedent in the conference with the same title celebrated in Barcelona in February 2008. The intensive research activity around algebraic methods in dynamical systems and related subjects makes it interesting to try and celebrate such an event every two years in order to take account of the new obtained results. The next one will most probably take place in Valladolid (Spain) in 2012.

Atmosphere and Infrastructure

- *The reaction of the participants to the location and the organization, including networking, and any other relevant comments*

All participants in the conference appreciated the good conditions offered by the Bedlewo Mathematical Conference Center. The housing has a comfort level much higher than other mathematical conference centers and the staff is always ready to help. The fact that the meals were taken in the palace and the possibility of walking in the near forest contributed to the good-feeling of the participants. The isolation of the center makes it a very good place for scientific activity. The atmosphere of the conference was really enjoyable both from the scientific and human points of view. The center disposes of several rooms suitable for scientific discussions which were

used all along the conference. The only weak point is the flatness of the lecture room which does not allow a good visibility of all assistants to the lectures.

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