ESF Mathematics Conference in partnership with EMS and ERCOM

Third European Set Theory Conference

3 - 8 July 2011

ICMS, Edinburgh, UK

Chaired by:

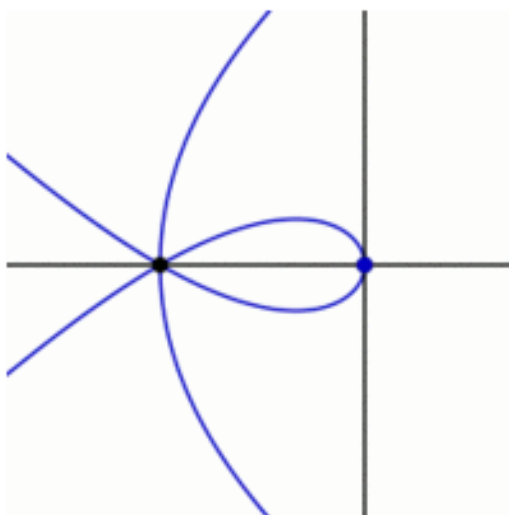Professor Philip Welch, University of Bristol, UK

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Professor Joan Bagaria, University of Barcelona, ES
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Rapporteur:
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Conference Highlights

Please provide a brief summary of the conference and its highlights in non-specialist terms (especially for highly technical subjects) for communication and publicity purposes. (ca. 400-500 words)

Set Theory was quickly recognized by David Hilbert more than 100 years ago as playing a fundamental foundational role in the intellectual underpinning of pure mathematics. Cantor's work on cardinality and wellorderings was seen to establish several basic areas of research whose threads we discern today and indeed will be emphasized in this conference: on the arithmetic of cardinal numbers themselves, and on the `descriptive set theory' that seeks to analyse the logical complexity of sets definable within mathematical
language. Cantor's work derived from his study of trigonometric series, and modern set theory goes back to classical analysis as well as to modern Banach space theory, abstract algebra, ergodic theory, and dynamical systems to find fruitful applications.

Advances in the recent decade in set theory have been rapid. We have seen new techniques developed in
previously impenetrable areas, and have come to an understanding of the interplay between the *prima facie* disparate areas of forcing axioms, infinite cardinal combinatorics and inner model theory. New developments in these areas have been seen to be inextricably connected, and we await resolution of some major problems that this understanding yields. In the last couple of years although we have seen some tremendous progress on the core problems of Set Theory, still some very fundamental problems remain a mystery. It seems that we have some of the tools to tackle them and a conference two years from now can be a great occasion to evaluate the progress and think about new approaches.

The conference focussed on several different major subfields. The first was *inner model and large cardinals,* which was a major theme of our last meeting and would be for any large scale set theory meeting. This theme connected with the second topic of *descriptive set theory.* The research of recent years has uncovered spectacular connections between large cardinals and descriptive set theory. The existence of certain large cardinals in sufficiently iterable inner models of set theory turn out to be equivalent with determinacy hypotheses. The study of Hereditarily Ordinal Definable (HOD) sets of a model of determinacy led Woodin to develop a promising new scenario for settling the continuum problem.

Turning to *descriptive set theory* which studies definable sets and functions (such as Borel, analytic, etc.) in Polish spaces, a recent trend in this field has been the development of the theory of definable actions of Polish groups, the structure and classification of their orbit spaces, and the closely related study of definable equivalence relations. One important motivation comes from basic foundational questions, such as understanding the nature of a complete classification of mathematical objects up to some notion of equivalence by invariants. This theory has a broad scope with natural interactions and applications to many other areas of mathematics, such as model theory, computability theory, the theory of topological groups and Polish group actions.

We had significant discussions on A*pplications of set theory.* Recent years have seen a flurry of applications of set theory to mainstream mathematics on several fronts, both well-established and unexpected. A prominent example is given by recent work of Todorcevic and his collaborators in construction of highly indecomposable Banach spaces; mathematical fields dealing with highly structured objects, such as operator algebras, were traditionally considered to be immune to the independence phenomenon.

Lastly, we planned a minor area of the conference which connected with one theme of the INFTY ESF Network in Set Theory on *Constructive Set Theory* and invited two speakers in this area. This network connects researchers in mathematical, philosophical and computational aspects of infinity.

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Scientific Report

Executive Summary

Scientific Content of the Conference

▪ *Summary of the conference sessions focusing on the scientific highlights*
▪ *Assessment of the results and their potential impact on future research or applications*

A Summary of the Sessions:

Monday 4th July Morning Session   Chair Philip Welch

Alexander Kechris - California Institute of Technology, US, gave a keynote opening lecture on Borel Combinatorics and ergodic theory.

Dima Sinapova - University of California at Irvine, US, gave a lecture on recent work on consistency proofs via forcing for the tree property at small cardinals.

Benjamin Miller - University of Muenster, DE gave a lecture on the G_1 dichotomy, thereby beautifully generalising a number of known arguments in seemingly disparate areas.

Afternoon Session Chair Jouko Vaananen

Grzegorz Plebanek - University of Wroclaw, PL, gave a lecture on Baire measurability in some C(K) spaces.

Christian Rosendal - University of Illinois at Chicago, US spoke on Bounded representations and maximal symmetry

Ilias Farah - York University, CA gave a lecture on new developments in Set theory and C*-algebras,where he has shown new independence results in such algebras.

Alain Louveau - CNRS and University of Paris VI, FR gave a lecture entitled: A dual Ramsey result for finite co-structures. These structures generalize the more well known Ramsey arguments.

Tuesday July 5th Morning Session Chair Istvan Juhasz

Ralf-Dieter Schindler - University of Muenster, DE, introduced his recent work on the Bounded Proper Forcing Axiom (BPFA) entitled ``Vagaries of Bounded Forcing Axioms''.

Piotr Koszmider - Polish Academy of Sciences, PL gave a lecture on some cardinal invariants on Banach spaces.

Afternoon Session Chair Alessandro Andretta

Andrew Brooke-Taylor - University of Bristol, UK gave a lecture on new methods as to how certain large cardinal properties could be rendered indestructible in further forcing extensions**.**

Riccardo Camerlo - Polytechnic University of Turin, IT joint work with **Alessandro Andretta** – University of Turin, IT gave a lecture on a result in descriptive set theory concerning the of the Lebesgue density theorem

Vladimir Kanovei - Institute for Information Transmission Problems IITP, RU gave a lecture entitled: ``On effective compactness and sigma-compactness.''

Wednesday July 6th Morning Session Chair Carlos Di Prisco

Peter Aczel - University of Manchester, UK spoke on recent work re-interpreting Martin-Lof's type theory in the theory of homotopies, rather than set theory. This work is provoking a lot of interest amongst category theorists.

Jana Flaskova - University of West Bohemia Pilzen, CZ gave a lecture on ideals and ultrafilters on the natural numbers.

Lajos Soukup - Hungarian Academy of Sciences, HR gave a lecture on the properties of families of sets.

Thursday July 7th Morning Session Chair William Mitchell

Natasha Dobrinen - University of Denver, US spoke on Canonical Ramsey Theorems and applications in the Tukey theory of ultrafilters.

Ernest Schimmerling - Carnegie Mellon University, US gave a lecture on the extent to which certain combinatorial properties of ordinals called `square sequences' can hold in various inner models. Such sequences are important since they give a litmus test of the construction of such a model on the one hand, and the degree to which certain forcing axioms which may hold on the other.

Heike Mildenberger - University of Freiburg, DE gave a lecture on preservation properties for non-elementary proper forcings.

Afternoon Session Chair Philip Welch

Boban Velickovic - University of Paris VII, FR spoke on the model theory of Transfinite Ehrenfeucht Fraissé games.

Matteo Viale - University of Torino, IT spoke on the current state of his work on a long standing problem concerning the relation between the Proper Forcing Axiom and the large cardinal property of supercompactness.

Poster Sessions

There were 23 posters that were successfully displayed over the course of the conference with three particular Poster Sessions scheduled. These were very well attended and provoked a good discourse back and forth between their authors and readers. The authors and poster titles were:

Adolf, D., Generalized Namba forcing on successors of regular cardinals.

Bablola, K.O., Early coefficients of certain close-to-star functions of type alpha.

Balcerzak, M., On monotone hull operations.
Bartosova, D., Universal minimal flows for closed subgroups of permutation groups.
Blaszczyk, A.., Transversal and T_1-independent topologies.
Chodounsky, D., Games with Towers.
Cox, S., Martin's Maximum, diagonal stationary set reflection, and well-determined generic embeddings.
Farkas, B., Idealized MADness.
Glab, S., Ideals with bases of unbounded Borel hierarchy.
Ikegami, D., Omega-logic and 2nd-order Boolean-valued logic.
Juhasz, I., G_{δ}-resolvability of compact spaces.
Kwiatkowska, A., Ample generics in the group of homeomorphisms of the Cantor set.
Malicki, M., The descriptive complexity of the family of all Polish groups admitting a compatible complete left-invariant metric, and the existence of certain universal objects.
Marks, A., Some combinatorial questions related to the universality of recursive.
Pawlikowski, J., On the concept of Sigma12 completeness.
Sabok, M., Forcings embeddable into a sigma-closed*ccc iteration and Baumgartner's Axiom A.
Schlicht, P., & Thompson, K., Embedding structure of kappa-trees.
Servais, D., Ambiguous Cardinals.
Soukup, D., Guessing clubs for aD, non D-spaces.
Tomescu, A., Simple Statements of Infinity in Set/Hyperset Theory.
Villaveces, A., Model Theory around lower infinite cardinals.
Weinert, T., Partition relations between countable ordinals.

Zelenyuk, Y., Metrizable refinements of group topologies and the pseudo-intersection number.

Assessment of results:
The conference covered most areas of modern set theory; there was a good spread across the field, with almost all talks covering some new aspect of their respective subject matter. Constructive set theory was not covered as had been hoped as one of the two speakers there had to cancel. The other (P. Aczel, Manchester) explained Voevodsky's recent work on interpretations of type-theory in homotopy theory which will be the basis of a research year at IAS Princeton next year. Besides this however the fields of forcing, descriptive set theory, and applications of set theory to mathematics – in particular analysis - were very well represented, and inner model theory well represented. Fully eight lectures of the 21 lectures could be said to be on applications of set theory to more traditional mathematics. A highlight was the keynote lecture of A. Kechris (CalTech) on ``*Borel Combinatorics and ergodic theory.''*

Emerging topics for the next 5-10 years were discussed as follows:
   a) Woodin's work on suitable extender sequences and his attempts to define and construct an "Ultimate L" inner model of set theory are very likely to play a central role in inner model theory, and in the foundational aspects of the subject as a whole. This process is still slowly unfolding.
   b) At the last meeting we reported that the classification of countable structures by Greg Hjorth would feature prominently: we are saddened by the very early untimely death of Greg, so we have lost a major force behind this project. It is expected that others will pick up this torch however.

Forward Look

(1 page min.)

▪ *Assessment of the results*
▪ *Contribution to the future direction of the field – identification of issues in the 5-10 years & timeframe*
▪ *Identification of emerging topics*

State of the Art:
(A) Emerging topics
1)The continuance of likely advances in Ramsey spaces and neighbouring fields was identified.
2) Woodin's work on strong extender sequences and the concomitant inner models is still emerging and will continue to influence research directions in the near future.

(B) Scientific Challenges: Knowledge Gaps were identified as follows::
1) Combinatorics and forcing axioms for the interval [aleph_2,aleph_omega+1); eg, what are natural forcing axioms for aleph_3?
2) The consistency strength of PFA
3) Sigma^2_2-absoluteness relative to Diamond.
4) Inner models for a supercompact.
5) The value of the continuum itself.
6) Forcing axioms for CH
7) The Omega Conjecture
8) Shelah's bound of aleph_omega_2 and the pcf conjecture.

Vision for the future:
1) Solutions to some of the above are of a visionary nature as some (such as 5) appear nowhere in sight.
2) There was some discussion of the flow of ideas in set theory in the decades since Cohen's result on the independence of the CH and AC: many theories have been tried out, found consistent or otherwise and have been assessed by the set theoretical community. Our knowledge of the interactions between such theories has deepened and matured. Interest is converging on a smaller grouping of these, as the ideas crystallize out. There is a lot of interest focused on some form of "super inner model" or "super Forcing Axiom". Candidate axioms for such offer reasonably complete theories extending ZFC,
   There was some further discussion on whether there might even conceivably be some form of problem involving mathematical physics, quantum mechanics say in the eg, the field of Hilbert spaces that tip considerations to one form of set theory rather than another, if the axiomatisations they delivered differed on the solutions for the physically

modelled problem. This is however speculative.

Another viewpoint expressed was that now might be the time for an increased self-reflection in set theory, such as spending more effort in attempting to discover what large cardinals or strong axioms of infinity "really are."   Lastly it was suggested that more time should be spent looking at connections to model theory, as this is a neighbouring theory.

<u>3)</u> Exposition: More effort could be made for exposition of set theory and set theorists' work to a broader mathematical and philosophical communities; this would help advance the field and others' perceptions of it.

Plans:
1) It is intended to hold in Barcelona during 2013 the Fourth European Set Theory Meeting. As mentioned by the Rapporteur: there are currently no ESF calls for research conferences in mathematics, so this will have to funded from elsewhere.
   a) There should be a problem session at future conferences.
   b) Consideration should be given to having 5 rather than 4 day conferences..
2) Continuance of the INFTY Network. This will help to fund future meetings, as well as support for younger researchers.

▪ Is there a need for a foresight-type initiative?

Given the success and interest of the Forward Look discussion, this would probably be a feature of future conferences. In particular it was thought that aspects concerning future directions for younger researchers needed to be emphasised.

Atmosphere and Infrastructure
▪ The reaction of the participants to the location and the organization, including networking, and any other relevant comments
There is a questionnaire on this, but the overall impression appeared favourable, and the conference venue organisation etc were fully satisfactory.

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Date & Author:

August 3rd 2011.   Philip Welch