



**RESEARCH CONFERENCES**

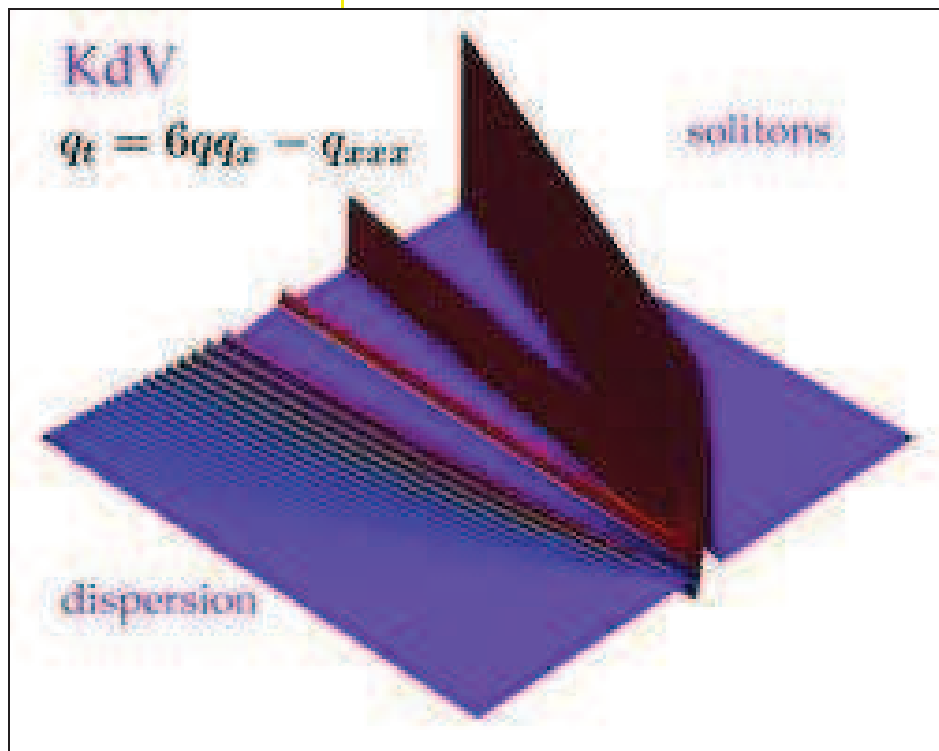
ESF Mathematics Conference in Partnership with  
EMS and ERCOM

# Completely Integrable Systems and Applications

The ESI, Vienna • Austria  
3-8 July 2011

Chair: **Gerald Teschl**, University of Vienna, AT  
Co-chair: **Spyros Kamvissis**, University of Crete, GR  
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## Conference Highlights

*Please provide a brief summary of the conference and its highlights in non-specialist terms (especially for highly technical subjects) for communication and publicity purposes. (ca. 400-500 words)*

The conference was focused on a few very active areas in the theory of integrable systems, including PDEs, ODEs and discrete (in space and time) systems.

One such was the area of initial boundary value problems for integrable equations and the application and rigorous justification of the generalized inverse scattering method initiated by Fokas and his collaborators (Talks by Bona, Boutet de Monvel, Chen, Fokas, Hitzazis Lennels, Mantzavinos, Pelloni).

A second focus was the further development of the asymptotic method of Deift and Zhou which relies on the Riemann-Hilbert formulation of initial and initial boundary value problems for integrable systems (Talks by Baik, Cafasso, Do, Grava, Kamvissis, Kuijlaars, Mikikits-Leitner, Miller, Moro, Shepelsky).

A third focus was on random matrices and related problems arising in probability (Talks by Baik, Kuijlaars, Soshnikov, van Moerbeke, Wang). The talk of Kuijlaars was a very interesting tying up of recent work on orthogonal and biorthogonal polynomials, nonintersecting Brownian motions, random matrix models and the Hele-Shaw problem, Riemann-Hilbert problems and equilibrium measures for logarithmic potentials. Also a very interesting development was presented by Srinivasan concerning the appearance of integrable models in the study of Burgers turbulence.

Other points of focus included the work of the Trieste school on the study of non-integrable Hamiltonian PDEs before and during break via integrable techniques (Grava, Masoero, Moro, Wu), peakons and the Camassa-Holm and Degasperis-Procesi equations (Constantin, Eckhart, Gesztesy, Grunert, Lundmark, Szmigielski), some recent results on the Ablowitz-Ladik lattice (Michor, Nenciu) and Weyl functions (Rybkin, Sakhnovich) as well as connections with theta functions and algebraic geometry (Kalla, Kamvissis, Konopelchenko) and state of the art numerical work (Kalla). We should also mention the presentation of work by the Kharkiv team and collaborators on step-like initial value problems (Egorova, Minakov), the presentation of recent work on semiclassical limits (Masoero, Miller, Sergyeyev) and the singular presentations of Holden on the nonlinear variational wave equation and Quispel of recent results in discrete (in space and time) integrable systems.

I hereby authorize ESF – and the conference partners to use the information contained in the above section on 'Conference Highlights' in their communication on the scheme.

# Scientific Report

## Executive Summary

(2 pages max)

There were 14 sessions: 8 sessions of 2 invited talks of 40 minutes each and 6 sessions of shorter talks: 3 sessions of 4 25-minute talks and 3 sessions of 5 20-minute talks.

At the end of the conference, there was also a presentation of open problems by 5 delegates.

## Scientific Content of the Conference

(1 page min.)

- *Summary of the conference sessions focusing on the scientific highlights*
- *Assessment of the results and their potential impact on future research or applications*

In the first session we had a general presentation of the inverse scattering method for boundary initial value problems for integrable equations of Fokas and a presentation of some applications of the method to the Ernst equation by Lenells.

The second session included a presentation by Bona of different aspects of nonlinear dispersive, not necessarily integrable equations, including questions of well-posedness and stability. It was followed by a talk of Pelloni on the application of the Fokas method to the elliptic Sine-Gordon equation.

The third session started with Konopelchenko who discussed local properties of the families  $W$  of algebraic varieties in Birkhoff strata of the Sato Grassmannian containing hyperelliptic curves. This was followed by Cafasso who showed how one can approximate a Pearcey process by Airy processes using a suitable variable rescaling. Hongqiu Chen spoke about long wave limits of periodic solutions of nonlinear wave equations in Sobolev spaces. Finally, Yen Do presented his work on how to extend the Deift-Zhou method for Riemann-Hilbert problems with non-analytic phase, based on earlier work of Varzugin.

The fourth session started with Grunert who presented work with Holden and Raynaud on a Lipschitz metric enabling them to study the stability of some conservative solutions to the Camassa-Holm equation. Nenciu talked about work with L.-C. Li on the Liouville integrability of the Ablowitz-Ladik by use of an appropriate Poisson-Lie group. Hitzazis presented his work on the application of the Fokas method on KdV in an interval and also some higher dimensional equations. Eckhart presented work with Teschl on the spectral theory for Sturm-Liouville problems appearing in the dispersionless Camassa-Holm equation. Sergyeyev presented work with Marvan on a general method for constructing recursion operators for multidimensional integrable dispersionless systems.

In the fifth session Holden presented a proof of existence of a global semigroup of conservative solutions of the nonlinear variational wave equation. Constantin described how the Camassa-Holm and Degasperis-Procesi equations arise in the modeling of the propagation of shallow water waves over a flat bed.

In the sixth session Grava gave a progress report of the Trieste school on the Dubrovin conjecture concerning behaviour of solutions of Hamiltonian PDEs near critical points and their relation to solutions of Painlevé equations. Baik presented recent work on maximal crossings and nestings of random complete matchings and their asymptotic analysis via Riemann-Hilbert methods.

In the seventh session Rybkin made a link of the theory of Hankel operators to completely integrable systems and as a consequence yielded new well-posedness results for KdV. Peter Miller talked on recent results about the semiclassical Sine-Gordon equation. Minakov analysed a shock problem for mKdV with step-like initial data via Riemann-Hilbert techniques. Moro showed how the balance equation for a gas in thermodynamic equilibrium is equivalent to an integrable nonlinear system of equations of hydrodynamic type and discussed the critical point of gradient catastrophe.

In the eighth session, Soshnikov discussed results about the fluctuation of the outliers in the spectrum of finite rank deformations of Wigner random matrices and results about the fluctuation of the matrix entries of regular functions of Wigner and sample covariance random matrices. Sakhnovich presented several examples and new results on the Weyl theory and inverse spectral transform for the Dirac system with rectangular potential. J. Michor showed that for decaying solutions of the Ablowitz-Ladik system the leading

asymptotics are time independent and that bounded solutions that are asymptotically close at time zero remain so forever. Szmigielski presented a construction of peakon solutions to an integrable system first introduced by Geng and Xue. His construction uses Hermite-Pade approximants and Cauchy biorthogonal polynomials. Kavitha Louis talked about the integrability and the magnetization dynamics of ferro- and antiferromagnets.

In the ninth session the talk of Anne Boutet de Monvel presented work with Kotlyarov and Shepelsky concerning long time asymptotics of the focusing NLS equation with step-like data via Riemann-Hilbert techniques. Shepelsky's talk presented long time asymptotics for the short-wave model for the Camassa-Holm equation, also via Riemann-Hilbert techniques.

In the tenth session Kuijlaars talked about the normal matrix model and accumulation of eigenvalues in two-dimensional regions in the complex plane when the dimension of the matrices is large. He presented a way to redefine orthogonality in terms of a well-defined Hermitian form and formulated a Riemann-Hilbert problem for the resulting multi-orthogonal polynomials. He presented an asymptotic analysis (work with P. Bleher) in the case of a cubic potential. Srinivasan presented work with G. Menon on a stochastic coalescence model arising when considering shock statistics in scalar conservation laws with Markov initial data. The evolution is a completely integrable equation, analogue of the N-wave model in nonlinear optics.

In the eleventh session Egorova presented a rigorous inverse scattering theory for steplike problems for KdV and Toda, and also some long time asymptotics. Kamvissis presented work with Teschl on long time asymptotics for a perturbed periodic Toda lattice, using the nonlinear stationary phase analysis of a Riemann-Hilbert posed on a Riemann surface.

In the twelfth session van Moerbeke presented work on the tacnode process, which appears when non-intersecting random walks or Brownian motions meet momentarily. Gesztesy presented work with Weikard on a generalisation of the spectral problem underlying the Camassa-Holm hierarchy using a Birman-Schwinger type operator approach.

In the thirteenth session, Dong Wang discussed worked with Baik on the limiting distribution of the top eigenvalues of Hermitian matrices with general potential when the external source is of finite rank (spiked). Quispel presented recent results on discrete integrable systems. Chaozhong Wu presented a quasi-triviality result for all orders, in the spirit of Dubrovin's work on Hamiltonian perturbations of evolutionary PDEs. Enolskii's talk was about SU(2) monopoles of charge 3 in the Bogomolny-Prasad-Sommerfeld limit.

In the fourteenth session, Mantzavinos talked about applications of the Fokas methods to 2+1 dimensional integrable models. Masoero presented a result with Raimondo on the convergence of solutions of the small dispersion KdV to solutions of the Burgers equation, before the break. Lundmark talked about constructing n-peakon solutions of Camassa-Holm and Degasperis-Procesi via orthogonal and biorthogonal polynomials. Mikikits-Leitner presented work with Teschl on the long time behavior of the perturbed finite gap KdV, using the Riemann-Hilbert method on Riemann surfaces initiated by Kamvissis and Teschl. Kalla presented a theoretic and numerical work with C. Klein on the multicomponent NLS system.

Overall the results presented were of a very high caliber, at the heart of mainstream research, and we expect them to be of seminal importance in the development of the field of integrable systems.

## Forward Look

(1 page min.)

- *Assessment of the results*
- *Contribution to the future direction of the field – identification of issues in the 5-10 years & timeframe*
- *Identification of emerging topics*

Several distinguished speakers offered to give a short presentation of open problems and further directions in the field.

Pierre van Moerbeke spoke of the intriguing connection of the Tracy-Widom distribution (for beta equal to 2 or 4) and a simple linear PDE and asked whether there is a similar connection for any values of beta. This could give insight into a possible interpretation of non-standard beta-ensembles appearing in random matrix theory.

Kuijlaars asked whether some interesting S-curves could appear in the Riemann-Hilbert analysis for the multiorthogonal polynomials mentioned in his main talk. He speculated that there could be an underlying max-min type equilibrium measure problem like the one studied by Kamvissis and Rakhmanov appearing in the theory of semiclassical NLS.

Peter Miller compared the effectiveness of two alternative methods in the theory of semiclassical/zero-dispersion limits of integrable systems: namely the Lax-Levermore theory and the Deift-Venakides-Zhou theory involving Riemann-Hilbert problems.

Dimitry Shepelsky presented some important problems in the theory of initial boundary value problems for integrable PDEs. He pointed out that for the Fokas method to be useful asymptotically for long times, the Dirichlet-to-Neumann map has to be better understood. For example, if the Dirichlet data are periodic (or exponentially oscillating) in time, are the Neumann data asymptotically periodic (or exponentially oscillating) for long times?

Jerry Bona suggested some answers to Shepelsky's problems via PDE methods.

The forward look section identified what we believe could be some main directions of research in the next 5-10 years: beta-ensembles in RMT, problems appearing in the implementation of the Riemann-Hilbert method and the associated equilibrium measures, initial-boundary value problems, interaction of PDE theory and integrable theory.

▪ Is there a need for a foresight-type initiative?

At the conclusion of the conference, delegates agreed that further meetings would be beneficial for closer looks at the future of the problems addressed in the forward look section.

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▪ *The reaction of the participants to the location and the organization, including networking, and any other relevant comments*

We have heard very positive comments about the location (Vienna and the great ESI facilities) and the organization, including the dinner and the beautiful trip to Wachau.

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