

Report  
of the ESF-Workshop:

Computational Aspects of  
Stochastic Partial Differential Equations

prepared by

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## 1 Introduction

Participants started to arrive on Sunday, the 17th September, 2006 and most of them left by Thursday, the 21st September. All participants were accommodated at the Billroth Hotel. Since breakfast was not provided at the hotel, breakfast was organised at the cafeteria of the University of Salzburg. Participants had the opportunity to purchase lunch at the cafeteria and the evening dinner was eaten at one of local restaurants. All this provided an additional opportunity for the scientific exchange of opinions, experience and knowledge.

The meeting itself started on Monday, 18th September, with a welcome speech by Prof Schweiger, the Head of the Mathematical Department, the host Institution. It was followed by a presentation by Z. Brzezniak, who stepped in to take the place of an absent representative of ESF. This presentation was followed by a hot discussion, which continued practically through the whole meeting. The first speaker was I Gyöngy from the Edinburgh University. His talk, lasting about 30 minutes, was (as all other talks) followed by an interesting discussion lasting about 10 minutes. It was typical that such formal discussions continued into informal ones for the duration of the breaks. A complete list of speakers, the titles and abstracts of their talks is attached. In one of the following sections we provide brief summary of the talk and the following discussion. The free time after lunch and before and after dinner was usually filled with informal small group discussions between representatives of various research groups represented in the Workshop.

The last event was a discussion on plans for follow-up research activities and/or collaborative actions. It was moderated by Z. Brzezniak. Various ideas were exchanged and it seems that some further developments will take place. For example, a recent application for an EPSRC grant by Z. Brzezniak includes collaboration between him and V. Mezentsev, which would be impossible had they not met at the workshop. Moreover, Gabriel Lord has generated a email list with a monthly newsletter, in which one can post job or conference announcement, and so on.

## 2 Scientific content of the event and discussions

I. Gyöngy's talk began with an introduction to the subject of approximations for Stochastic partial differential equations. He spoke about different methods of approximating the Wiener process (piece-wise linear, convolution etc) and about existing literature. Then he spoke about his new results for a general class of second order SPDES. This class of equations includes the Zakai equation arising in filtering theory. The main emphasis of the talk was on the almost sure convergence of the solutions of the approximating equations. In the discussion, Z. Brzezniak enquired about relationship between the presented results and the rough path theory of T Lyons. In response the speaker expressed his hope that his results would finally lead to a successful bridging of rough paths and SPDES. Various participants asked for clarification about types of SPDES studied.

R. Rautman's talk was about certain system of (deterministic) PDEs arising in the study of the celebrated Navier Stokes equations. The system of equations in question involves certain boundary conditions and these boundary conditions cause the main difficulty in developing a good theory for such equations. The speaker presented the history of the problem and his contributions towards solving it. In particular, showing existence of local (in time) solutions in suitably chosen weighted spaces. The discussion after the talk was centred on the relationship between the equations studied by the speaker and the original Navier-Stokes equations. It was agreed that both are important. The importance of numerical approximations for the system studied by the speaker was emphasised and agreed on.

A. Millet spoke about a topic somehow similar to the topic of I Gyongy but also very different since the class of equations considered by both these speakers are so different. The speaker's main interest were nonlinear monotone stochastic partial differential equations and she explained in detail striking results on the rate of convergence for both implicit and explicit numerical schemes. In the discussion, Z Brzezniak asked if the methods presented could be applied to other types of nonlinear SPDEs. In the response, the speaker speculated on possible extensions and generalisations and it was agreed that any such extension needs substantial work.

L. Zambotti started his talk with a brief introduction to reflected SPDEs and their relationship to applied science. He spoke about his recent results of such problems, in particular on the integration by parts formulae. He finished with description of his recent (not yet written down) results obtained in collaboration with L. Ambrosio, N. Gigli and G. Savare' on the extension of the theory of F Otto of gradient flows to deterministic gradient flows to stochastic ones. The discussion that followed was concentrated on understanding some details of the talk and possible applications to some concrete SPDEs and e.g. the stochastic Ginzburgh-Landau.

T. Shardlow began his talk with an exposition of recent works devoted to understanding waves in excitable media, motivated by a wealth of natural systems that exhibit spiral waves. He spoke about a concrete example: the heart, where waves of electrical activity propagate in cardiac muscle to stimulate the heartbeat; irregularities in the cardiac rhythm, such as fibrillations, are related to the self-sustaining activity of spiral and scroll waves. He mentioned other examples, such as the BelousovZhabotinsky chemical reaction and the patterns of slime mould amoebae. Two basic features are common to models of excitable media: spatially localised excitation which diffuses in space and falls into a recovery state after a short time. Cellular automata and PDEs can be used to model excitable media. The main body of the talk was devoted to explaining numerical results obtained for such models. The discussion concentrated on two aspects: (i) how the model is and, (ii) if the model is correct, how good the numerical approximations are. It was agreed that the numerical approximations could help to understand the answer to issue (i).

L. Schimansky-Geier talk was devoted to describing the Brownian Motion from the perspective of a physicist. He began his talk with recalling celebrated works of A. Einstein and M. von Smoluchowski. Then we went through the history, mentioning also fundamental works of R. Feynman on the paths integrals. This was a fascinated talk, especially as the audience was mainly composed of mathematicians. In the discussion that followed it was agreed that much work is needed in order

to improve understanding between those two fields of science.

M. Sanz-Sole spoke about numerical approximation of somehow untypical SPDE, namely an elliptic one with the noise being time-independent. It turns out that this problem has not been much studied and despite its apparent simplicity it is rather difficult. The speaker presented some results on existence and regularity of solutions and then linked those results to the numerical approximations. In particular, as it is the case of ordinary SDE, the speed of convergence is closely related to the regularity of the solution. These results show importance of the theoretical studies. The discussion was about perspectives in extending the results to other time independent equations. It seems that an agreement was reached that these problems should attract more attention.

A. de Bouard began her presentation by recalling that the Nonlinear Schrödinger Equation (NSLE) is one of the simplest models combining both nonlinear and dispersive effects. It is a universal model as it is applicable to Water waves, nonlinear optics, plasma physics, and others. The talk will be about the influence of a random medium on the behaviour of solutions to the Nonlinear Schrödinger equation. The speaker then spoke in more details about different examples of the equations and then she presented a theoretical result of existence of local and global (depending on assumptions) solutions to the NSLE with multiplicative noise. The main part of the talk was about various numerical schemes and the questions of their stability and the rate of convergence. The question of explosion, i.e. non-existence of global solutions and the last part of the talk was devoted to this issue. In particular, she presented some numerical experiments supporting a conjecture that the blow-up indeed happens.

S.S. Sritharan spoke about Control Theoretical issues of the Stochastic Fluid Dynamics. He began his presentation by presenting a typical flow control problem. Then he spoke about the solvability theorem for trajectories. Then he spoke in detail about the feedback analysis and related infinite-dimensional Hamilton-Jacobi-Bellman equation as well as the impulse controls and related infinite-dimensional quasi variational inequalities. He moved then to describing his results on the nonlinear filtering for the Stochastic Fluid Dynamics and finished his lecture with presentation on highlights of computational challenges.

The topic of the talk by P. Kloeden was the pathwise numerical approximation of stationary solutions of semilinear stochastic evolution equations. He started with presentation of a motivating example, a reaction-diffusion equation with an additive noise. He then presented an abstract Hilbert space framework for his results. He spoke about Itô-Galerkin finite dimensional approximations for the solutions and the implicit Euler-Maruyama scheme with constant time step. He then spoke about existence of stationary solutions to the full equation as well as the implicit Euler-Maruyama scheme applied to Itô-Galerkin finite dimensional approximations and about the convergence of the latter to the former when the time step converges to zero. He underlined that his results are applicable to problems that can be solved pathwise. Then he moved on and presented related results for approximating random attractors. Most of the discussion was devoted to the possibility of extending the results to problems that cannot be solved pathwise. It was agreed that the first class of results should be generalisable to SPDEs with multiplicative noise while the second class to only those which can be solved pathwise (as e.g. with one-dimensional Brownian Motion) as the

more fundamental issue of existence of random flows for general SPDEs with multiplicative noise should be studied first.

The topic of the talk by S. Peszat was a certain SPDEs appearing in the theory of mathematical finance. It is called HJM equation and it was introduced by Heath, Jarrow and Morton using Brownian Motion as the driving randomness. The speaker spoke about generalisation of the model to include discontinuous driving processes. mathematically, it involves Levy processes and the so called Musiela parametrisation. The speaker explained the new features of the model caused by the discontinuity of the driving process and formulated the main results on the existence and uniqueness of solutions. The discussion was centred about possible numerical schemes for the problems discussed and their rate of convergence. Various options were discussed and it was agreed that further research, especially as it is directly linked with applications, on this subject should be done.

The talk of V. Mezentsev was closely related to the talk of A. De Bouard, as both were about the stochastic Schroedinger equations. However, while the talk by De Bouard was about theoretical research, the talk by Mezentsev was about applied science. It turns out that the stochastic NSLEs are successfully used in many areas of science, in particular in Photonics. V. Mezentsev spoke about photonics and SNSLEs. He began his talk with explaining that Photonics is the science and technology of generating and controlling photons, particularly in the visible and near infra-red spectrum. He spoke about almost infinite spectrum of application, e.g. optical fiber communications in telecommunications. He explained how the equations are derived and what is their physical significance. He spent a lot of time on explaining the origins of the noise in the equations (e.g. "Amplifiers always introduce noise!"). He spoke about important results and challenges related to his research. In particular, he underlined the importance of mathematical underpinning for the numerical and experimental research. The discussion, mainly between the theoreticians working on SNSLEs and V. Mezentsev was concentrated on the questions that according to applied scientists should be investigated by mathematicians and possible methods of collaborations. This was continued in the free time.

The talk by B. Schmalfuss was related to the earlier talk by P. Kloeden as it was about numerical approximations for random attractors but the motivating equation was different, namely the stochastic Navier-Stokes equations. He introduced carefully the model and then formulated a result on the existence of random attractor. Main ideas of the proof were presented and then the question of the numerical approximations was discussed. Again, the explicit Euler scheme was used as it is known that even in finite dimensional cases for the random attractors the implicit scheme leads to divergent results. In the discussion, a question was raised about relationship between the results by Schmalfuss and Kloeden and it was agreed that they are about very different Stochastic PDEs and that each of the models requires different techniques.

E. Buckwar spoke about finite dimensional Stochastic Differential Equations. Her topic was the Stochastic linear multi-step methods and their asymptotic stability problems. She introduced the Stochastic linear multi-step (SLMS) method and explained how the classical methods as the BDF2-Maruyama method and the Milstein variant of the two-step Adams-Bashforth-method fall into her

scheme. She spoke about the aims of her study: understanding the convergence properties, SDEs with small noise and a problem appearing in the simulation of SDAEs for circuit simulation. She then sketched the mean-square convergence results. In the next part of the talk she spoke about the two-step Maruyama scheme and using it she presented both theoretical and simulations for SDEs. She finished by discussing some stability results and their numerical approximations, e.g. the asymptotic mean square stability of zero solutions for SDEs. Discussion:

In his talk on Asymptotic MS-stability of zero solutions J. Voss began with recalling two classical results. Namely that the stationary distribution of a finite dimensional gradient type SDEs with a non-degenerate additive noise has an invariant measure that is absolutely continuous w.r.t. the Lebesgue measure and that the stationary distribution of a gradient type SPDEs with an additive space-time white noise and driven by Dirichlet Laplacian has an invariant measure that is absolutely continuous w.r.t. the Brownian Bridge measure. He went on to generalising the latter result to a wider class of SPDEs and corresponding SDEs. This result then serves as a theoretical underpinning for the problem of sampling (called here "Langevin Sampling") of the solutions of finite dimensional

SDEs with a general drift (not necessarily gradient type). The main question the speaker was interested in was: how can we simulate the corresponding SPDE on a computer? He presented a manifold of fascinating numerical simulations explaining in details which schemes he used.

The primary aim N.J. Cutland's talk was to present some rudimentary background about the Nonstandard Analysis and the results on the stochastic and deterministic Navier-Stokes equations he obtained using this approach. He stated carefully the main results and presented some ideas of the proof. In the discussion, Z. Brzezniak observed that it seems in this framework, the solutions are always strong with respect the Wiener process defined on the Loeb space. N.J. Cutland confirmed but it was agreed that the issue has to be further discussed, especially as it can have far reaching consequences. P. Kloeden became very interested in the talk and has consequently discussed the topic during the free time as well as after the workshop.

C. Canuto spoke about somehow different kind of stochastic problems, in a sense similar to those from the talk of M. Sanz-Sole: deterministic equations in random domains. His main interest lied in the numerical approximations of such problems. The discussion was centred around the problem of modelling, i.e. whether the crude models the speaker spoke about are sufficiently precise to make useful engineering predictions. The answer to such worries seems to be that although more precise models are desirable, they are so far too complicated numerically to be useful and the engineers, e.g. in aerodynamics, are happy to use the crude models as they agree to large extend with experiments.

The talk on M. Hairer was about his recent result with J. Mattingly. It consisted of an introduction in which all necessary machinery was introduced. Then he recalled recent related results, especially the one on uniqueness of invariant measures for stochastic 2-D Navier-Stokes equations driven by degenerate additive noise. He spoke about a general method that allows to show the existence of spectral gaps for Markov semigroups on Banach spaces. Unlike most previous work, the type of norm we consider for this analysis is neither a weighted supremum norm nor an  $L_p$ -type norm, but

involves the derivative of the observable as well and hence can be seen as a type of Wasserstein distance. This turns out to be a suitable approach for infinite-dimensional spaces where the usual Harris or Doeblin conditions, which are geared to total variation convergence, regularly fail to hold. In the first part of this paper, we consider semigroups that have uniform behaviour which one can view as an extension of Doeblin's condition. He then proceeded to explaining situations where the behaviour is not so uniform, but the system has a suitable Lyapunov structure, leading to a type of Harris condition. He finally explained why the latter condition is satisfied by the two-dimensional stochastic Navier-Stokes equations, even in situations where the forcing is extremely degenerate. The talk on J Printems was related to the talk of I Gyongy as both talks were about numerical approximations of the Zakai equation. However, as the talk of Gyongy was about approximations of the driving random processes, the talk of Printems was centred on approximations of the solution via time and space discretizations. He spoke about an important tool used to numerical integration called optimal quantisation of random variables. He showed how this idea can be used in order to obtain sharp error estimates. The talk was concluded with two concrete examples, the Burgers equation and the McKean-Vasov equation. The talk was illustrated by many fascinating computer simulations.

The talk by M. Tretyakov was based on the joint research with G. Milstein. M. Tretyakov spoke about Monte Carlo algorithms for backward equations in nonlinear filtering. He began with presenting basic preliminary material, in particular he explained the fundamental Kallianpur-Striebel formula. Then he introduced the backward pathwise filtering equations and spoke about the numerical algorithms for these equations. In the next part of the talk, the speaker introduced the backward SPDEs and spoke about the numerical algorithms for these equations. He finished his talk by discussing an important issue of the variance reduction and by presenting very interesting numerical examples.

M. Romito spoke about joint works with F. Flandoli and D. Blömker on Markov solutions for the stochastic Navier-Stokes equations and other dissipative equations. He began his talk with recalling what a Markov process is. He spoke about existence of Markov solutions by e.g. disintegration & reconstruction and the abstract selection principle for Markov families. He then presented two models: The 3-D stochastic Navier-Stokes equations and the surface growth equations. For both these problems there is no satisfactory theory related to the uniqueness of solutions. He then spoke about Markov solutions for each of these equations separately. He mentioned about existence of the martingale solutions. Then he explained how one can prove existence of the Markov solutions for the NSEs (!). He concluded with a brief discussion of the strong Feller property, the invariant measures and ergodicity.

G. Lord gave a fascinating presentation on Neural dendrites & wave propagation. He presented some background material, especially about the Baer-Rinzel model and reason for adding noise to that model. He discussed Discretization of the equations in time: explicit and nonstiff. He spoke about the computation of the travelling wave solutions. Then he spoke about the Spike-Diffuse-Spike and an almost analytic solution to that model. He spoke then about the noise induced propagation and the temporal filtering. The conclusion of the discussion was that this very impor-



tant and fascinating subject should attract more attention from researchers, especially joint research collaboration involving people of different background should push forward the understanding of models.

C. Carstensen gave an introductory course on AFEM (Adaptive Finite Element Methods), especially aimed at people who do not have experience in this field. He introduced a method which is quite successful in practise, but is verified only by computer experiments. A statement which is unusual within researcher coming from theory, but common by practitioners.

M. Ondřejat's talk on stochastic wave equations was began with a short overview of recent results, with an emphasis on various various approaches to the subject. He presented various models of equations being studied and hinted some techniques used in both deterministic and stochastic cases. He then moved to discuss his own results on the properties of martingale solutions. He spoke about the finite speed propagation and how it is used to prove existence and uniqueness of solutions. The talk was finished with a brief presentation of his most recent results on stochastic geometric wave equations, i.e. wave equations with values in some finite dimensional manifold. The discussion centred on two topics. Firstly, on the differences and similarities between various approaches to stochastic wave equations. In this case, the conclusion reached that the approach should naturally follow the problem to be studied. The questions about motivation for the study of the geometric wave equations were convincingly answered by the speaker. Discussion:

C. Odasso spoke how the asymptotic strong Feller property introduced by Hairer and Mattingly can be used in numerical approximations of solutions of certain class of SPDEs. He presented complete results for a toy model and speculated about the extensions to the concrete SPDEs in question.

The last speaker was L. Quer-Sardanyons who in his talk on space semi-discretizations for a stochastic wave equation spoke about results obtained in collaboration with M. Sanz-Sole and E. Hausenblas. The first part was about the linear equations and in the second about nonlinear ones. In each part he spoke about existence and uniqueness, and of the convergence of the numerical schemes. He spoke about the rate of convergence and gave some hints of the proof. The discussion was mainly about possible generalization of the convergence results to the stochastic geometric wave

equations discussed in the talk by M. Ondřejat. The speaker and participants have speculated how can this be done and agreed that the problem seems to be rather challenging.

The workshop was finished with a discussion (moderated by Z. Brzezniak) about the future.

### **3 Some Highlights of Discussions**

Since it would be boring to read all discussion, which were sometimes quite technical, we portray in this section some interesting highlights.

We invited three researchers working in the numerical aspects of purely deterministic partial differential equations. The discussions after their talks can be ranked within the most interesting.

Prof Rautmann gave a talk about the well posedness of the Navier Stokes for  $t \rightarrow 0$ . Within the techniques the time regularity of the perturbation was essential. In the discussions hereafter, it turns out that it never comes up to his mind relaxing the time regularity on the perturbation. On the other side, dealing with stochastic perturbation, this time regularity is not given and obliged to deal with process of quite pure time regularity. This was interesting, since problems which arises when one deals with Stochastic PDEs often not only not treated by researcher from deterministic PDEs, but also not coming up to there mind.

The next discussion to mention on this place was initiated by Prof Casterensen. He is a typical representative of a person coming from deterministic Numerics. Thus, he presented a method in finite element to refine the space, a method which leads to very good results in practise, but where no theoretical proof of order of convergence exists. Something unthinkable for a person coming from theory. In the discussions hereafter, he asked for existing benchmarks. Here, it turns out that first no benchmarks exists for SPDEs or even for filtering problems and secondly, none of the community of SPDEs ever thought about benchmarks.

Claudio Canuto gave an excellent talk. His originally research topic are wavelets. He had known SPDEs before, nevertheless, some interesting discussion about peculiarities of the wavelets and their self similar properties, followed the talk. The self similarity is a property which the wavelets are sharing with the Wiener process. Hence, one may assume that they are approximate the solution of SPDEs in a better way than finite elements. The drawback is, that e.g. in case of Navier Stokes no divergence free wavelets exists until now.

The talk of Ann de Bouard arises in a nice discussions. Her talk was about the nonlinear Schrödinger equation, an equation, which is quite important in physics. If one add stochastic noise, the behaviour changes essential. To be more precise, there are certain parameters, where in the deterministic case blow up happens, however in the stochastic case nothing appears. Unfortunately, there exists no theoretical evidence, the difference can only shown by experiments gained by simulation on computers. It turns out that the group around Mezentsev have observed the same behaviour, but also have now problems how to deal with this fact. It is not know if this fact is true in theory or if it is a artefact's due to computer simulations. Therefore, either theoretical results or other methods which converge has to be founded. Both groups want to keep in touch.

Martin Hairer's talk resulted in a long discussion, which centred around the important issue whether the results presented in the talk are valid for other systems, e.g. stochastic NSEs in general domains and about possibility of finding suitable numerical simulations.

## 4 Assessment of the results

The workshop was inspiring for several people. This can be seen in the follow-up research activities:

- Z. Brzezniak and V. Mezentsev met for the first time during the workshop. As a result of their conversations V. Mezentsev is now collaborator in the new project, which has been recently submitted to the ESPRC (a research council in the UK) by Z. Brzezniak. Also, Mezentsev has been invited to give a talk York.
- G. Lord has generated the email list sde-net, which provides a free service to those working in stochastic (partial) differential equations and is hosted by the Heriot-Watt University. Subscribers receive a plain text digest of announcements of meetings, new books or journals, jobs, software, etc related to stochastic (partial) differential equations and its applications. The third newsletter appeared already.
- A group consisting of A. De Bouard, A. Millet, A. Debusche and F. Flandoli are preparing a proposal for a network that will be submitted to the ESF in October 2007. At the moment, they are contacting people and building up the structure.

E. Hausenblas, as a result of the workshop, plans to strengthen her contacts with people researchers working in the theory of deterministic PDEs. Not only because of collaboration but also to call their attention to problems arising in the stochastic PDEs. Moreover, she has started to formulate a benchmark for SPDEs driven by Poisson random measure.

## 5 List of the Participants:

In the table below we give a lists of the participants and their affiliation:

Name	Affiliation	Country
Zdzislaw Brzezniak	Department of Mathematics University of York	UK
Evelyn Buckwar Anne de Bouard Claudio Canuto	Institut für Mathematik Humboldt- Universität Berlin Laboratoire de Mathématiques Université Paris-Sud Dipartimento di Matematica Politecnico di Torino	Germany France Italy
Carsten Carstensen	Institut für Mathematik Humboldt- Universität Berlin	Germany
Nigel Cutland Istvan Gyöngy	Department of Mathematics University of York School of Mathematics University of Edinburgh	UK UK
Martin Hairer	Mathematics Department The University of Warwick, Coventry	UK

Erika Hausenblas Peter Kloeden	Department of Mathematics Faculty of Natural Science Faculty of Computer Science and Mathematics, Frankfurt	Austria Germany
Gabriel Lord	Department of Mathematics, Heriot-Watt University Edinburgh	UK
Vladimir Mezentsev	Aston University, Aston	UK
Annie Millet Cyril Odasso Martin Ondrejat Szymon Peszat	Samos-Matisse, CES, Paris Universit'e Paris 12 - UFR des Sciences et Technologie Academy of Sciences of the Czech Republic, Prague Polish Academy of Sciences, Institute of Mathematics, Warsaw	France France Czech Rep Poland
Roger Pettersson	Matematiska och systemtekniska institutionen Vaxjo universitet	Sweden
Jaques Printems	Faculte de Sciences et Technologie Universit Paris XII	France
Lluís Quer- Sardanyons	Departament de Matemàtiques Universitat Autònoma de Barcelona	Spain
Reimund Rautmann Marco Romito Marta Sanz-Sole	Universitt Paderborn, Paderborn Dipartimento di Matematica Universita di Firenze Facultat de Matemàtiques Universitat de Barcelona	Germany Italy Spain
Lutz Schimansky- Geier	Institut für Physik Humboldt Universitt zu Berlin	Germany
Tony Shardlow	School of Mathematics University of Manchester, Manchester	UK
Björn Schmalfuß	Institut für Mathematik EIM Universitt Paderborn, Paderborn	Germany
Sivaguru S. Sritharan	Department of Mathematics University of Wyoming Laramie	USA
Michael Tretyakov Lorenzo Zambotti	Department of Mathematics University of Leicester, Leicester Politecnico di Milano Dipartimento di Matematica	UK Italy
Jochen Voss	Warwick Mathematics Institute University of Warwick Coventry	UK

## 6 Final programme

### Sunday 17 September 2006

Evening *Arrival*

### Monday 18 September 2006

09.00-09.15 **Opening: Presentation of the European Science Foundation (ESF)** because the

representative of ESF was unable to come, the presentation was given by **Z.**

**Brzezniak**

09.15-09.55 **I. Gyongy** (Edinburgh): *Rate of Convergence of Wong—Zakai Approximations for Stochastic Partial Differential Equations*

10.05-10.45 **R. Rautman** (Paderborn): *A direct approach to the vorticity transport & diffusion equation*

10.45-11.10 *Coffee/Tea break*

11.10-11.50 **A. Millet** (Paris): Rate of convergence for discretization schemes of stochastic

evolution equations

11.55-12.35 **L. Zambotti** (Milano): *Reversible processes with log-concave invariant measures*

12.30-14.30 *Lunch break*

14.30-15.10 **T. Shardlow** (Manchester): *Stochastic PDEs and excitable media*

15.20-16.00 **L. Schimansky-Geier** (Berlin): *100 years of Brownian Motion*

16.00-16:30 *Coffee/Tea break*

16.30:17.10 **M. Sanz-Sole** (Barcelona): *A discretisation scheme for a stochastic Poisson equation in dimension  $d>3$*

17.20-18.00 **A. De Bouard** (Paris): On the numerical analysis of stochastic nonlinear Schrodinger

equations

### Tuesday 19 September 2006

09.00-09.40 **S.S. Sritharan** (Laramie): *Control Theoretical Issues of Stochastic Fluid Dynamics*

09.50-10.30 **P. Kloeden** (Frankfurt am Main): *The pathwise numerical approximation of stationary solutions of semilinear stochastic evolution equations*

10.30-11.00 *Coffee/Tea break*

- 11.00-11.40 **S. Peszat** (Krakow): *Solutions to the Heath-Jarrow-Morton equation*
- 11.50-12.30 **V. Mezentsev** (Aston): *Stochastic PDEs in Photonics*
- 12.30-14.30 *Lunch break*
- 14.30-15.10 **R. Pettersson** (Växjö): Numerical solution of a reflecting stochastic heat equation
- 15.20-16.00 **B. Schmalfuss** (Paderborn): Dynamics of numerical schemes for random and stochastic pde's
- 16.00-16:30 *Coffee/Tea break*
- 16.30-17.10 **E. Buckwar** (Berlin): Stochastic linear multi-step methods and their asymptotic stability properties
- 17.20-18.00 **J. Voss** (Warwick): An MCMC Method for Sampling Diffusion Bridges

## Wednesday 20 September 2006

- 09.00-09.40 **N. Cutland** (York): *Loeb space methods for stochastic Navier-Stokes equations*
- 09.50-10.30 **C. Canuto** (Torino): *Numerical methods for solving PDEs in stochastic domains*
- 10.30-11.00 *Coffee/Tea break*
- 11.00-11.40 **M. Hairer** (Warwick): *Spectral gap results for infinite-dimensional evolutions*
- 11.50-12.30 **J. Printems** (Paris): Discretization and simulation of Zakai equation
- 12.30-14.30 *Lunch break*
- 14.30-15.10 **M. Tretyakov** (Leicester): Monte Carlo algorithms for backward equations in nonlinear filtering
- 15.20-16.00 **M. Romito** (Firenze): Markov solutions for the stochastic Navier-Stokes equations and other dissipative equations
- 16.00-16:30 *Coffee/Tea break*
- 16.30-17.10 **G. Lord** (Edinburgh): Neural dendrites, noise and wave propagation
- 17.20-18.00 **C. Carstensen** (Berlin): AFEM (Adaptive Finite Element Methods)

## Thursday 21 September 2006

- 09.00-09.40 **M. Ondrejat** (Prague): Stochastic wave equations
- 09.50-10.30 **C. Odasso** (Paris): Conservation of qualitative properties for numerical scheme
- 10.30-11.00 *Coffee/Tea break*
- 11.00-11.40 **L. Quer-Sardanyons** (Barcelona): Space semi-discretisations for a stochastic wave equation
- 11.50-12.30 **Discussion on plans for follow-up research activities and/or collaborative actions** (moderated by **Z. Brzezniak**)
- 13.00 *End of the workshop*
- 13.00-14.30 *Lunch break and Departure*

## 7 Statistical Information

Since we had not recorded the age of the participants, we cannot provide any statistical information about it. However, we can say that the number of participants who have received the PhD's in the last 6 years is 7 (seven).

- Male: 24. Female: 5 (It seem from our experience that this is about an average value in Mathematics.)
- Ordered by Countries. (From the 9 participants coming from the UK, 7 are originally from other countries (with 5 of them from other EU countries).
  - Austria: 1
  - French 4
  - Germany 6
  - Italy 3
  - UK 9
  - Poland 1
  - Czech 1
  - Sweden 1
  - Spain 2
  - USA 1
- Ordered by Scientific Aspects:
  - Numerical aspects of Deterministic Partial Differential Equations: 3
  - Numerical aspects of Stochastic Partial Differential Equations: 5
  - Numerical aspects of Stochastic Differential Equations: 4
  - Applications of Stochastic Partial Differential Equations: 5
  - Theory of Stochastic Partial Differential Equations: 9