Purpose of the visit

During my short visit of the Theoretical Physics Department at Loughborough University I had numerous discussions with Prof. Kusmartsev and prepared a very first draft about a project entitled

PHASE TRANSITIONS AND HYSTERESIS IN A SYSTEM OF TWO COUPLED MAGNETIC NANOPARTICLES

Short description about the project work

We study dynamical and static properties of two interacting magnetic particles subjected to exchange interaction, anisotropy and an external magnetic field. We present a complete magnetic phase diagram as a function of the field, we classify all possible magnetic hysteresis loops and show the dependence of the magnetic moments on the external field.

In principle we can start with the Hamiltonian of the classic anisotropic Dirac-Heisenberg model

$$H = -J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j + K \sum_{i=1}^{N} (\mathbf{S_i} \cdot \mathbf{e}_y)^2 - \mathbf{H} \cdot \sum_{i=1}^{N} \mathbf{S}_i$$

and assume that the particles are elementary mono-domain particles in the shape of very flat ellipsoids. We assume further that the external field points into the direction of the easy axis. Under these assumptions we can reduce the problem to the study of one-dimensional chains of small magnetic particles. Specifying the magnetization direction of particle i with respect to an arbitrary reference axis by the variable x_i the total energy of such a system can be given by the two-particle Hamiltonian

$$\mathbf{H} = -2J \cos(x_1 - x_2) + \frac{K}{2} \left(\sin^2(x_1) + \sin^2(x_2)\right) - H \left(\cos(x_1) + \cos(x_2)\right)$$

The first term defines the exchange energy, specified by nearest-neighbour interactions, while the next two terms are due to the anisotropy. The last two terms specify the Zeeman energy. Since H and K can be scaled by J we choose $|J| = 1$. Since for ferromagnetic coupling the system will always remain in a ferromagnetic phase, we only consider antiferromagnetic interlayer coupling.

We find two ferromagnetic phases, $\mathbf{F}^{\uparrow\uparrow}$ and $\mathbf{F}^{\downarrow\downarrow}$ specified by the angles $x_1 = x_2 = 0$ and $x_1 = x_2 = \pi$, respectively. The corresponding energies are $E_F = 2 - 2H$ and $E_F = 2 + 2H$. The two ferromagnetic phases $\mathbf{F}^{\uparrow\uparrow}$ and $\mathbf{F}^{\downarrow\downarrow}$ are stable for $H > 4 - K$ and $H < K - 4$, respectively. We further find an anti-ferromagnetic phase **AF** characterized by the angles $x_1 = 0$ $x_2 = \pi$ with the corresponding constant energy $E_{AF} = -2$. The antiferromagnetic phase is stable for $|H| < \sqrt{K(4+K)}$. Eventually there exists a socalled scissored phase specified by the angle relation $x_1 = -x_2$ with

$x_1 = \text{Arccos}\frac{H}{4-K}$. The corresponding energy takes the value $E_{SC} = (K-2) - \frac{H^2}{4-K}$. This phase is stable for $|H| < 4-K$ and $|H| > (4-K)\sqrt{\frac{K}{4+K}}$. Note that the scissored phase exists only for $K < 4$. The phase diagram is depicted in Fig. 1.

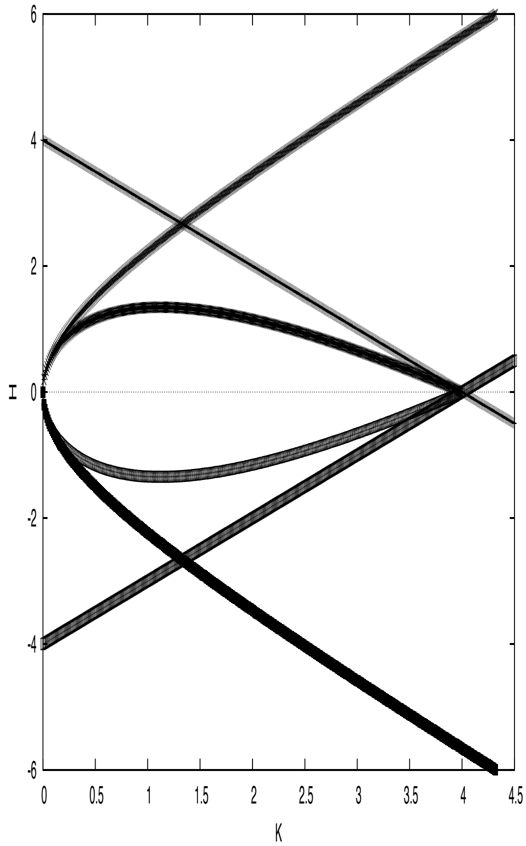

Figure 1: Phase diagram as function of the magnetic field H

Note that we can have coexistence of two and three phases, respectively. Moreover, we have the triple points $P_3 = (\frac{4}{3}, \pm\frac{8}{3})$, while at $P_4 = (4,0)$ we have coexistence of all four phases. These critical points will be decisive in order to give a proper classification of all possible hysteresis loops.

The evolution of the two magnetization angles for the ansitropy strength $K = 1$ is illustrated in Fig. 2. We start with a sufficiently large positive field and drive the field slowly down. The first transition occurs, when the $F^{\uparrow\uparrow}$ phase becomes unstable. ($P_{F^{\uparrow\uparrow}\rightarrow SC} = (1,3)$) This second order transition is a pitchfork bifurcation. Then the SC phase becomes unstable and there is a discontinuous transition at $P_{SC\rightarrow AF} = (1, \frac{3}{5}\sqrt{5})$. Then the AF phase becomes unstable and we reenter the SC phase at $P_{AF\rightarrow SC} = (1, -\sqrt{5})$. Eventually the SC phase becomes unstable at $P_{SC\rightarrow F^{\downarrow\downarrow}} = (1, -3)$) and we enter continuously the complementary ferromagnetic phase $F^{\downarrow\downarrow}$.

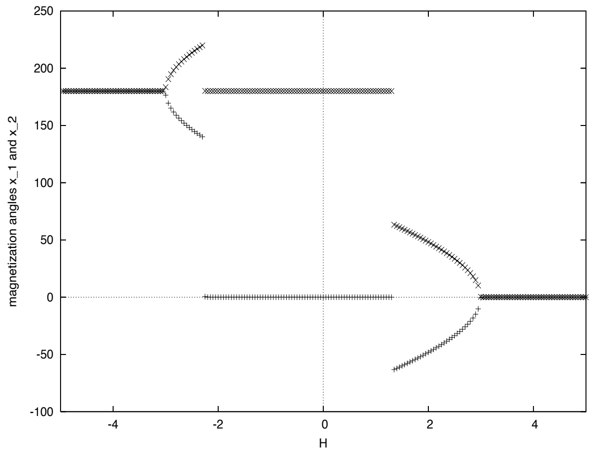

Figure 2: Magnetization angles for $K = 1$ as a function of the magnetic field H

Figure 3 shows the corresponding evolution of the hysteresis loop, which can be extracted completely from our phase diagram depicted in Fig. 1. According to experimental studies we start with a positive large saturation field and drive it sufficiently slowly down to the opposite extreme. In contrast to the evolution of the magnetization angles in Fig. 2 we restart with a negative large saturation and drive the field slowly up to the positive saturation again.

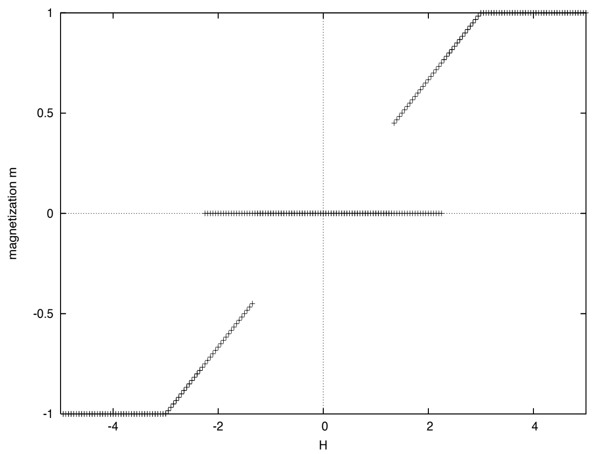

Figure 3: Hysteresis loop for $K = 1$

The evolution of the two magnetization angles for the ansitropy strength $K = 3$ is illustrated in Fig. 4. We start with a sufficiently large positive field and drive the field slowly down. The first transition occurs, when the $F^{\uparrow\uparrow}$ phase becomes unstable ($\mathbf{F}^{\uparrow\uparrow} \to$ **SC**). This second order transition is is also a pitchfork bifurcation. Then the **SC** phase becomes unstable and there is a first order transition (**SC** $\to$ **AF**). Then the **AF** phase becomes unstable and we reenter directly the complementary ferromagnetic state $\mathbf{F}^{\downarrow\downarrow}$ without passing through the **SC** phase. In fact, this behavior is well known from experiments with antiferromagnetic materials.

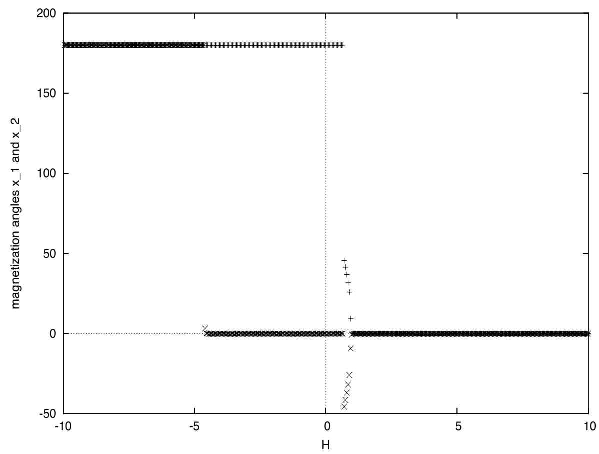

Figure 4: Magnetization angles for $K = 3$ as a function of the magnetic field H

Figure 4 shows the corresponding evolution of the hysteresis loop, which can also be extracted completely from our phase diagram depicted in Fig. 1.

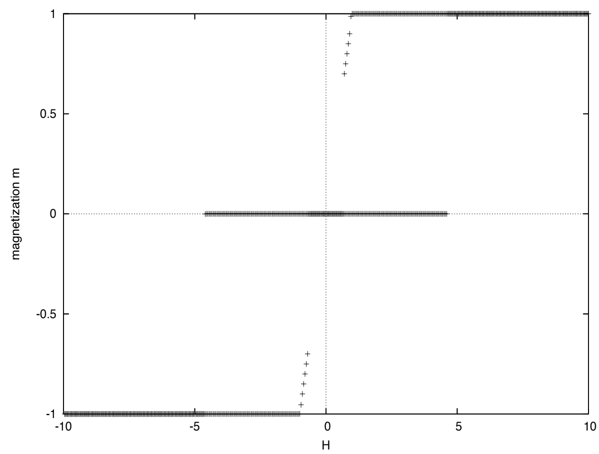

Figure 5: Hysteresis loop for $K = 3$

Fig. 6 depicts the evolution of the magnetization angles for a larger value of K, where we only have transitions from the ferromagnetic phase $F^{\uparrow\uparrow}$ to the complementary ferromagnetic $F^{\downarrow\downarrow}$ phase and vice versa.

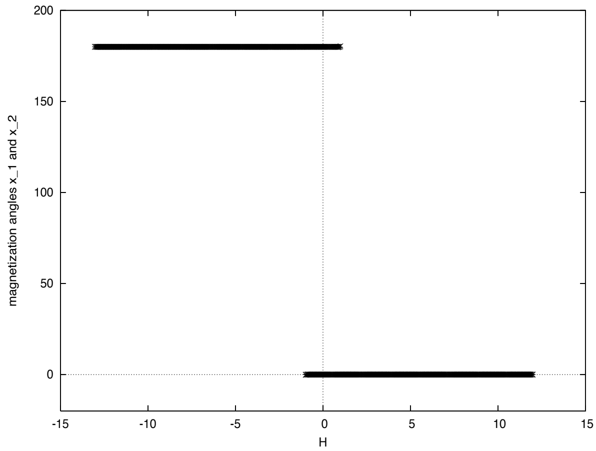

Figure 6: Magnetization angles for $K = 5$ as a function of the magnetic field H

Figure 4 shows the corresponding evolution of the hysteresis loop.

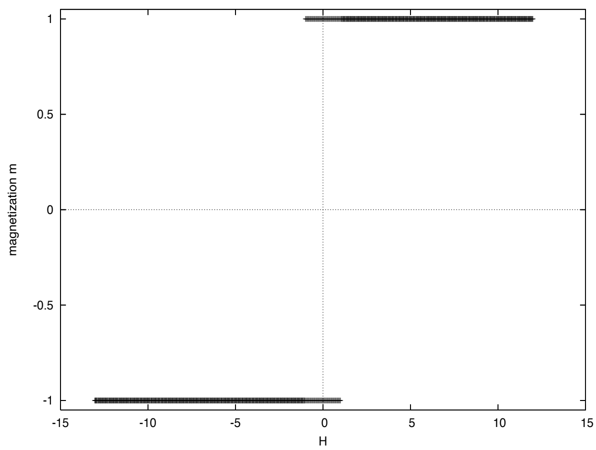

Figure 7: Hysteresis loop for $K = 5$

Note that a complete classification of all possible hysteresis curves is of fundamental importance in any kind of technical application. Moreover, in the same manner also all possible magnetoresistance curves can directly be extracted from the phase diagram depicted in Fig.1 .

Future collaboration with the host institution

The long lasting collaboration Vienna-Loughborough with Prof. Kusmartsev will be continued in the future. Considering that the subject is of high actuality, especially in the range of technical applications, we plan to do further joint work in the field of magnetic nanoparticles and thin magnetic layers.