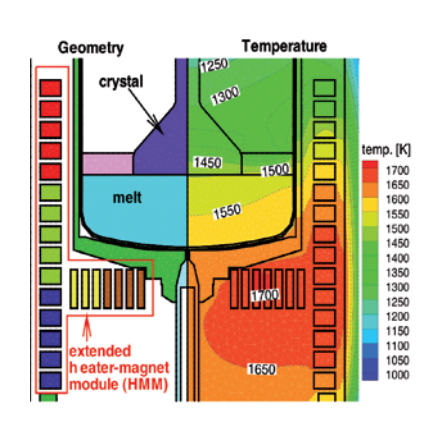
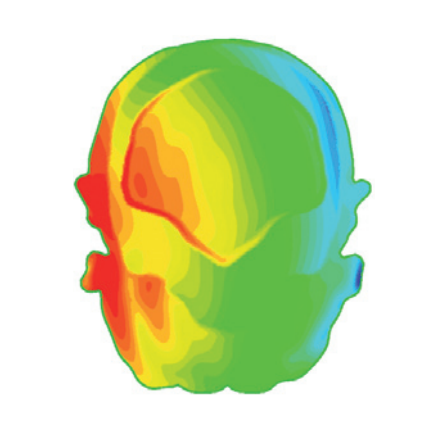
EUROPEAN SCIENCE FOUNDATION

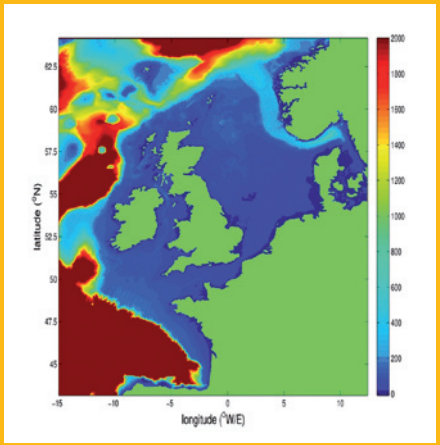
SETTING SCIENCE AGENDAS FOR EUROPE

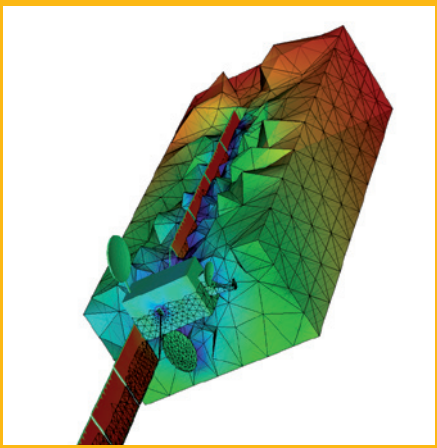
FORWARD LOOK

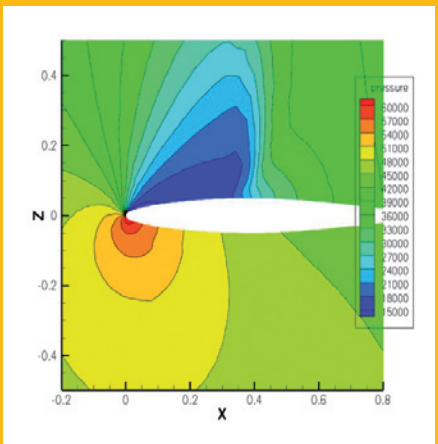
Mathematics and Industry

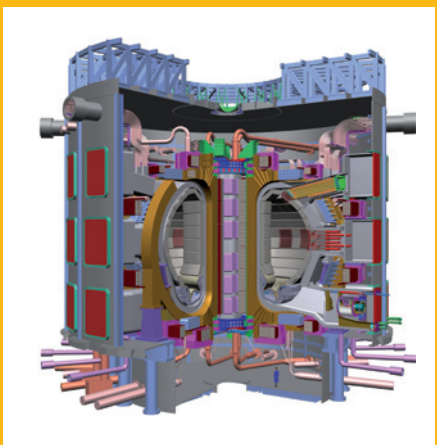


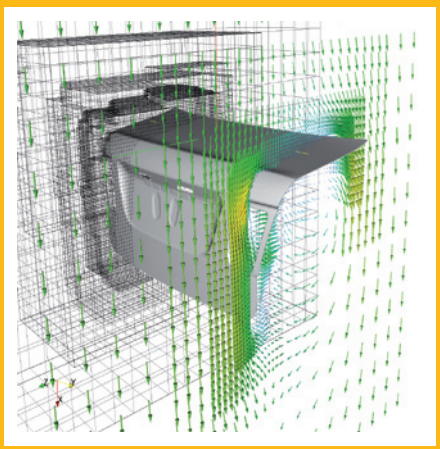












European Mathematical Society

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European Science Foundation

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The Forward Looks are the flagship activity of ESF's strategic arm. Forward Looks enable Europe's scientific community, in interaction with policy makers, to develop medium to long-term views and analyses of future research developments with the aim of defining research agendas at national and European level. Forward Looks are driven by ESF's Member Organisations and, by extension, the European research community.

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Foreword

Many areas in academia and industry depend on mathematical sciences to open up new frontiers and advance discovery. Research in mathematics contributes to advances in areas such as medicine, cyber security, weather prediction, digital data compression and mining, aeronautics and computing, to name a few. Today's challenges faced by academia and industry are so complex that they can only be solved through the help and participation of mathematical scientists.

This ESF Forward Look initiative has involved representatives from all domains of European Mathematics. It has been carried out together with the *European Mathematical Society* (EMS) and in particular with its Applied Mathematics Committee. Moreover, all national mathematical societies in Europe within academia and industry have been directly involved at different stages. In addition, an online survey has been launched among mathematicians in academia and researchers working in the private sector. The results of the survey and the contributions from more than one thousand researchers, engineers and policy makers are reflected in this report.

The Forward Look springs from the strong belief that European Mathematics has the potential to be an important economic resource for European industry, helping its innovation and hence its capacity of competing on the global market. To fulfill its potential, special attention has to be paid to the reduction of geographical and scientific fragmentation in the European Research Area. Overcoming this fragmentation will require the involvement of the entire scientific community and of industry. Europe needs to combine all experiences and synergies at the interface between mathematics and industry and create strong areas of interaction to turn challenges into new opportunities.

The need to consider the issue of synergy is the main engine behind this Forward Look exercise. We identified the groups that carry out significant activities in the field of industrial mathematics and invited mathematicians, through mathematical societies and general communication means, to submit short summaries of their experience and success stories in their cooperation with industry. These "success stories" will be published separately, representing a snapshot of the impact and value created by European groups active in industrial mathematics.

Professor Marja Makarow
Chief Executive
European Science Foundation

Professor Mats Gyllenberg
Chair of the Standing Committee
for Physical and Engineering Sciences
University of Helsinki, Finland

Executive Summary

Where do we want Europe to be in 2020 and beyond?

Three priorities should be the heart of Europe 2020**:

- Smart growth, by developing an economy based on knowledge and innovation.
- Sustainable growth, by promoting a more resource efficient, greener and more competitive economy.
- Inclusive growth, by fostering a high-employment economy delivering economic, social and territorial cohesion.

These three priorities are mutually reinforcing; they offer a vision of Europe's social market economy for the 21st century. Smart growth means strengthening knowledge and innovation as drivers of our future growth. This requires improving the quality of our education, strengthening our research performance, promoting innovation and knowledge transfer throughout the Union, making full use of information and communication technologies and ensuring that innovative ideas can be turned into new products and services that create growth, quality jobs and help address European and global societal challenges. At national level, Member States will need to ensure a sufficient supply of science, mathematics and engineering graduates and to focus school curricula on creativity, innovation, and entrepreneurship.

Purpose of the Forward Look

Knowledge has become the main wealth of nations, companies and people. Hence investing in research, innovation and education is now the key-leverage for competitiveness and prosperity in Europe. At the heart and foundation of this challenge, mathematics plays a crucial role as it provides a logically coherent framework to industry and a universal language for the analysis, simulation, optimisation, and control of industrial processes. Mathematics is an essential factor in the industrial creation of value and a driving force for innovations, but often, its contributions are invisible in the final industrial products. The use of mathematical techniques gives a competitive advantage to industry by suggesting innovative interdisciplinary approaches. In this context, the goal of the present Forward Look was to explore ways of stimulating and intensifying the collaboration between mathematics and industry. European scientific and industrial communities, in close interaction with policy makers, have developed medium to long-term views and analyses of common issues, questions, and "good practices" between mathematics and industry in order to envisage strategies for a stronger interaction of mathematicians with large and medium size companies aimed at technological advancement.

** EUROPE 2020, a strategy for smart, sustainable and inclusive growth, March 2010, European Commission

European context

Although there is a clear need in all areas of industry throughout Europe to use knowledge-based technologies in the development and improvement of products and services, it can be clearly seen that the level of cooperation between academia and industry needs to be increased and that is not equally well established throughout Europe. Strong efforts are necessary to correct this and deliver high-value collaborations across Europe.

Strategic objectives

Europe's competitiveness is to be achieved in a multilateral international environment through a common strategy for European Industrial mathematics by setting up the following key strategic objectives.

Strategic objectives to build the community:

- To foster a European Network in applied mathematics towards a Smart Economy.
- To allow Member States to build up a common strategy for European mathematics.
- To establish mathematics as a necessary component of European innovation.

Strategic objectives to develop the community:

- To overcome geographical and scientific fragmentation.
- To broaden and harmonise educational programs for students and teachers in mathematics at the European level.
- To facilitate the mobility between industry and Academia.

Strategic objectives to empower the community:

- To harmonise the curriculum and educational programmes in industrial mathematics.
- To encourage the exchange of knowledge and information between Academia and industry.
- To promote and improve the career path in industrial mathematics.

Recommendations

Recommendation 1: Policy makers and funding organisations should join their efforts to fund mathematics activities through a European Institute of Mathematics for Innovation.

Roadmap implementation:

- EU and National funding agencies should coordinate *clusters of excellence* in industrial mathematics and create a *European Institute of Mathematics for Innovation* (EIMI) for mathematicians and users of mathematics.
- EU and European governments should set up a Strategy Taskforce for Innovation and Mathematics (STIM) in order to develop a European strategy for mathematics.
- Policy makers should put in place a Small Business Act in Mathematics (SBAM) to encourage spin-off companies explicitly using mathematics.
- EU must identify industrial and applied mathematics as an independent crosscutting priority for the Framework Programme 8.

Recommendation 2: In order to overcome geographical and scientific fragmentation, academic institutions and industry must share and disseminate best practises across Europe and disciplines via networks and digital means.

Roadmap implementation:

- Researchers in academia and industry must adapt their mentalities to the different mathematical and scientific domains they interact with, and disseminate best practises.
- The mathematical community in collaboration with industry should create a journal devoted to industrial mathematics and contribute to a European Digital Mathematics Library.
- Academic institutions and industry must facilitate the employment mobility between academia and companies.
- The mathematics community and industry should work together on real opportunities in application-themed competitions.

Recommendation 3: Mathematical Societies and academic institutions should create common curricula and educational programmes in mathematics at European level taking into account local expertise and specificity.

Roadmap implementation:

- Academia must create a European Curriculum for industrial mathematics and set up a pool of industrial mathematics engineers.
- Academia must develop new criteria to assess and recognise careers in industrial mathematics.

Conclusion

The basic message of this report is that if Europe is to achieve its goal of becoming the leading knowledge-based economy in the world, mathematics has a vital role to play. In many industrial sectors the value of mathematics is already proven, in others its potential contribution to competitiveness is becoming apparent. The benefits resulting from a dynamic mathematics community interacting actively with industry and commerce are considerable and certainly far outweigh the rather modest costs required to support such a community. Nevertheless, such benefits will not be realised unless action is taken to develop mathematics and a coordinated community of industrial and applied mathematicians needed for the future success and global competitiveness of the European economy and prosperity.

1. Mathematics and its relationship with industry

It is very hard or even impossible to give a precise definition of mathematics; however there is consensus that **mathematics is focused on logical and quantitative relations between fundamental abstract objects**. In describing real world objects and processes by means of mathematical tools (**mathematical modelling**), mathematics can provide unique approaches to establish quantitative relations and to describe complex systems, to analyse their behaviour, and finally to control and optimise their output.

Development and progress of mathematics have always been driven both by **internal forces** (to cross the boundary between fields) and by **external forces** (the need of solving problems arising outside the discipline). In the sequel we will often use, for sake of brevity, the term "**industrial mathematics**" to designate all the research that is oriented at the solution of problems posed by industrial applications: It cannot be considered as a field disjoint from the counterpart of "**pure mathematics**", since it uses potentially all fields of mathematics and it is a continuous source of challenges to the fundamental research on structure and methods of mathematics because it has often to deal with completely new mathematical problems. It is our belief that all mathematical fields in the widest sense can of course be important for industry.

Within this report, we use the term "**industry**" to denote business and commerce (including financial services and healthcare), research and development laboratories, and commercial and not-for-profit research, development, and production facilities, i.e., activities outside the realm of education and academic research.

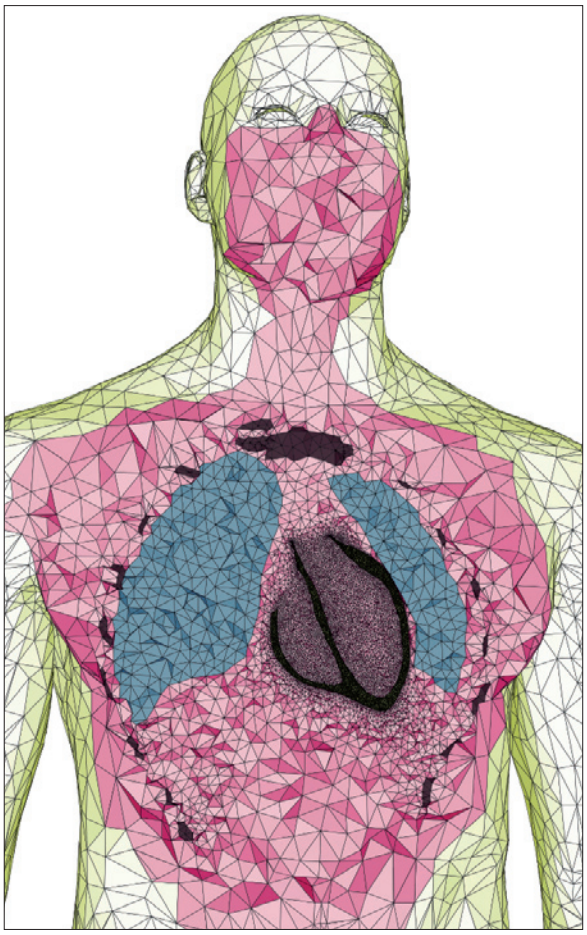
1.1. Mathematics as the language for innovation

> ***Ohne Mathematik tappt man doch immer im Dunkeln.***
> ***(Without mathematics it is like walking in the dark)***
>
> Werner von Siemens to his brother Wilhelm [1]

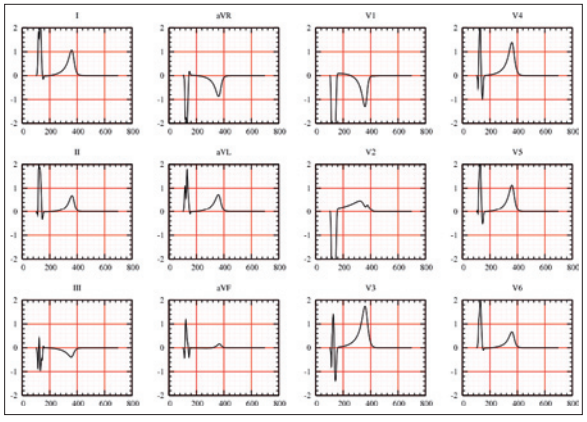
The role of Mathematical Sciences in Civilisation has been of central importance for centuries. The current trend to a global economy and a knowledge society has placed information and innovation technologies, increasingly dependent on scientific research driven by Mathematics, at the forefront. Mathematics also plays an increasing role in the efficient development of novel products and technologies, which are essential for our ageing societies combined with growing worldwide population and limited resources. Mathematics provides the tools, which enable us to understand and reduce the complexity of the mutual interdependencies in economics, and leads the way in predicting, optimising and controlling the respective systems. The study of these mathematical objects and of their relationships and interconnections is a pillar of modern sciences, guiding the design and interpretation of empirical observations and laboratory experiments.

In almost all industries Mathematics opens the way to virtual experiments, the analysis and simulation of multiple scenarios for a given phenomenon and its con-

1. Springer book Mathematik Motor der Wirtschaft, 2008, XII, 125 S. 57 Abb., 54



(Above) The computational domain for the Electrocardiogram
(Below) "Healthy" ECG obtained by simulation
(Courtesy INRIA Paris-Rocquencourt, J.-L. Lions Laboratory and Sorin Group)



trol and optimisation. Besides its role in science and engineering, the domains of application of mathematics include social, environmental, and economic phenomena. This is especially true in areas where innovation is contributing to the well being of society, such as health, security, communications, and environmental stewardship. The search for new life-saving drugs, the development of high-performance materials, the continued miniaturisation in electronics, and the protection of sensitive ecosystems – all of these application-oriented activities, and many others, are strongly dependent on fundamental research, and that research is inextricably linked to mathematics.

The relationship between mathematics, sciences and society has been the subject of study in numerous reports. We have been inspired by these documents that are listed in Annex 1. Here we focus on industrial aspects in the entire European context, aiming at sketching a roadmap for industrial mathematics in Europe. The fragmentation into countries present in the European landscape requires measures which would help combining all experiences and synergies, and creating a strong area of interaction between mathematics, industry and society at large. The need to address this issue is the main driver for this forward look exercise.

1.2. Mathematics as an innovation enabler for industry

> ***The main challenge is to ensure that there is a good supply of people with mathematical skill sets as they are key to the development of our firm. This expertise is endemic within Google and while the firm can never be sure where the next innovation or product is going to come from, it needs a good supply of university graduates with new ideas and concepts.***[2]
>
> Larry Page, Google co-founder

Google is the apotheosis of how innovation in mathematics and technology can transform a few people into a world-leading services company. Google evolved from a research project by Larry Page, a PhD student in the Computer Science department at Stanford University. Page's dissertation explored the mathematical properties of the web, treating its link structure as a huge graph and the number and nature of links to a particular page as an indicator of its importance. From this, Page and Sergey Brin (another Stanford PhD student who joined the project later on) evolved a page ranking algorithm

2. http://www.google.com/intl/en/corporate/execs.html

and a search engine based on the rankings, forming a company based on the technology in 1998, in a garage in Silicon Valley. From these small beginnings, the company has since grown to over 10,000 employees worldwide. This example illustrates that mathematics should be recognised as a true innovation enabler for industry. There is a lack of understanding of this fact in both industry and academia, and it is extremely important for society that this gap is closed. Let us now try to better understand the situation.

1.2.1. Mathematics in the industrial context

The essential program of the applied mathematician when collaborating with industry follows essentially the following paradigm: first, identify the problem of concern; then, build a quantitative mathematical model, analyse and solve it, apply the results, and potentially create appropriate mathematical software that can be commercialised. The emphasis is in pointing out which are the important and relevant variables controlling the problem, which are the constraints and what is the goal. This is done through the understanding of the underlying mechanisms involved in combination with the analysis of the respective observations and data. The next steps concern the analysis of the created mathematical model, its numerical simulation in different scenarios, and the validation the model in comparison with experimental data. In addition it is important to investigate the robustness and sensitivity of the model. Note that this is typically an iterative procedure, since if the results do not explain or fit the observations, one has to modify and adapt the model, and repeat the cycle, until the model describes – as accurately as needed – the situation to be studied or simulated. Typically after the iteration of validating and adapting the model then, when the model is finally accepted, it would be used to improve, optimise or control the process that it describes. Model based control and optimisation is a crucial element of automation in all areas of industry, often reducing the cost and time of product, process and service innovation.

The aforementioned process is often not visible in published research, and requires an active collaboration with the source of the application. Therefore mathematical modellers must have particular skills in communicating with non-mathematical collaborators and the ability to translate in mathematical terms the real world problems, to study them by the use of mathematical techniques and finally to transfer the mathematical results for non-specialists who are typically not interested in the methods of solution. Modellers also have to be able to create models that take into account the main features of the situation or the object to be studied, as well as the overall efficiency of the model when used in a virtual design environment. The complexity can be very high, and the complete system may be too complicated to be described. So, a mathematical modeller often has to make a trade-off: the best model is not the one that takes into account all factors but the one that incorporates the crucial ones. As Einstein said, “Things are to be made as simple as possible… but not simpler”.

It is evident that, in view of the ever-increasing complexity of real life applications, the ability to effectively use mathematical modelling, simulation, control and optimisation will be the foundation for the technological and economic development of Europe and the world.

1.2.2. Lack of recognition of industrial mathematics by the mathematical community

The attitude of the mathematical community with respect to collaboration with industry is far from homogeneous: sometimes one finds a simplistic supercilious attitude that denies industrial mathematics the citizenship in the ethereal world of science, under the unproved postulate that it consists only in the application of standard results and methods without any creativity. The other extreme position would assert that the only justification of mathematics lies in its “usefulness”. Both application-driven research and research motivated by the development of new concept and theories within mathematical sciences are indispensable within the essential framework of scientific research. To quote the OECD report on Mathematics and Industry[3]: “The distinction [*between pure and applied mathematics*] is vague, misleading, and useful at best for classification purposes. Excellent mathematics, however abstract, can lead to the solution of practical problems. In turn, hard problems in nature stimulate the invention of new mathematics”.

It is a common interest of the entire mathematical community to outreach activities to make society and industry aware that mathematics is the common denominator of much that goes on in everyday life, activating the many sectors of society that can benefit from mathematics. Indeed, promoting such awareness will bring resources to all mathematicians. As a result of our questionnaire one can see that the culture of mathematicians in academia to work with partners in industry or to do consulting is not widespread.

An additional delicate matter concerns the evaluation of industrial mathematics, as publication and citation

3. See Annex 1

records mostly guide quality criteria in the academic world. The structure of the academic world is not inclined to appreciate interdisciplinary cooperation and the contribution given by mathematicians to the solution of problems is not always fully evident and thus adequately appreciated.

1.2.3. Lack of recognition of mathematics by industry

An additional contribution to the gap between academia and industry is the lack of recognition of mathematics in industry. This has even culminated in cutting down on mathematics groups in industry, which mainly occurred in the 1990's. The main reason for this is the invisibility of mathematics; one can rightfully say that mathematics is characterised by "**invisible contribution, visible success**". Designers use virtual design environments that rely heavily on mathematics, and produce new products that are well recognised by management. The major effort concerned with the construction of reliable, robust and efficient virtual design environments is, however, not recognised. As a result, **mathematics is not usually considered a key technology in industry**. The workaround for this problem usually consists of leaving the mathematics to specialised small companies that often build on mathematical and software solutions developed in academia. Unfortunately, the level of communication between these commercial vendors and their academic partners with industry is often at a very low level. This, in turn, leads to the observation that yesterday's problems in industry can be solved, but not the problems of today and tomorrow. The latter can only be addressed adequately if an effort is made to drastically improve the communication between industrial designers and mathematicians. Industry should realise that this is the only way their problems can be solved in a rigorous manner.

Example: *A stunning illustration is given in Figure 1. In the electronics industry and far beyond, it is well known that the performance of chips improves by a factor of 2 every 18 months. Moore's law describes this trend. What is not known to the general public, and also not to designers and management in the electronics industry, is that numerical mathematics has achieved a similar speed-up in algorithms underlying the virtual design environments used to develop these new generations of chips! Europe must not miss this challenge that is also an opportunity.*

The lack of recognition also occurs in more direct interactions between industry and Mathematics. When the Stochastic Operations Research Group at Twente University (Netherlands) proposed a new optimal strategy for reducing waiting times in hospitals from 4 weeks to 1 day, there was much scepticism and virtually no acceptance. Only after quite some investment in producing an animation, management was convinced and implemented the strategy with great success. This example shows that **recognition by industry must be earned**, and that it may take additional investment to achieve this. The mathematics community must be prepared to do this investment.

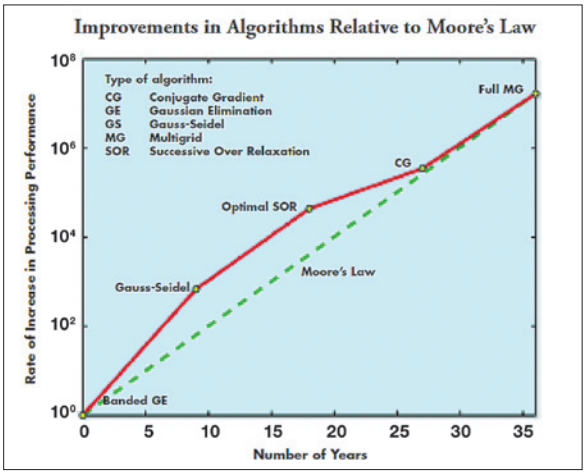


Figure 1. The relative gains in some algorithms for the solution of an electrostatic potential equation on a uniformcubic grid compared to improvements in the hardware (Moore's Law).

1.3. Means of interaction between mathematics and industry

Problems in industry which benefit significantly from mathematics, in addition to the standard workflows, typically are seen on different scales:

1. Specific problems where the current methods or commercial codes fail, where a new feature has to be included, where current models do not meet the requirement. In our survey, quite a number of Small and Medium Enterprises are addressing this kind of situation, but have difficulties to survive on this demand alone. Typically these are small-scale projects up to 12 person-months.
2. Medium scale problems where the transfer of a new technology, algorithmic approach or such has to be performed or where a new mathematical software product has to be brought into the market. This is very often done via companies that are spun out of a research lab or an academic institution. **These mathematical spin-offs show a high survival rate in the market or are bought out by global players thus**

creating novel high-tech jobs with high salaries. This kind of projects typically requires 3-6 person years.

3. Large-scale multi-physics, multi-scale, multi-mathematics problems where different physical models are coupled, where different areas of mathematics are used and where there are essentially no methods available. An example of this is, for instance, the problem of modelling, simulating and optimising the gas transport in large gas networks, where stochastics, partial differential equations, numerics, discrete mathematics and optimisation all have to cooperate and where methods for the simulation are missing and where currently nobody can optimise the whole problem. Typically only a large centre or a consortium can handle such a problem where all the competence is available in-house. Such projects can have an order of 20-30 person years.

1.3.1. Areas of interaction

Probably the central question is the one about the fit or misfit between the competence available in academia and the actual demand in the business sector. The respondents of the ESF questionnaire, both from business and academia, were asked about the main areas of mathematical expertise as well as fields of applications of mathematics that are relevant for activities in their institutions. What follows is an analysis of a stylised supply, understood as a structure of mathematical competence available in academic institutions, versus a stylised demand, understood as the relative importance of particular areas of applied mathematics that are especially of interest to the business sector. In Figure 2, the main areas of competence in academia are plotted versus major business challenges perceived in industry.

The first issue concerns the areas in which available mathematical competence in academia can support the industry, and those which the industry perceive as major business challenges in their companies. The comparison is presented in Figure 1. The location of each area, represented by a bubble, corresponds to relative its importance in academia (horizontal axis) and in industry (vertical axis). Areas above the dotted line are more often pointed out by the business respondents than by respondents from academia. Areas below the dotted line show the other way around: more often by academia than by business.

In general, **the perceptions in both groups are very similar**. This manifests itself in the fact that the areas align themselves quite closely to the diagonal line. The areas that seem to be most important (chosen most often) are those belonging to the areas of Systems Modelling, Computing/Algorithms, and Simulation. These are slightly more often chosen by academia than by industry. Areas chosen least often are Design and

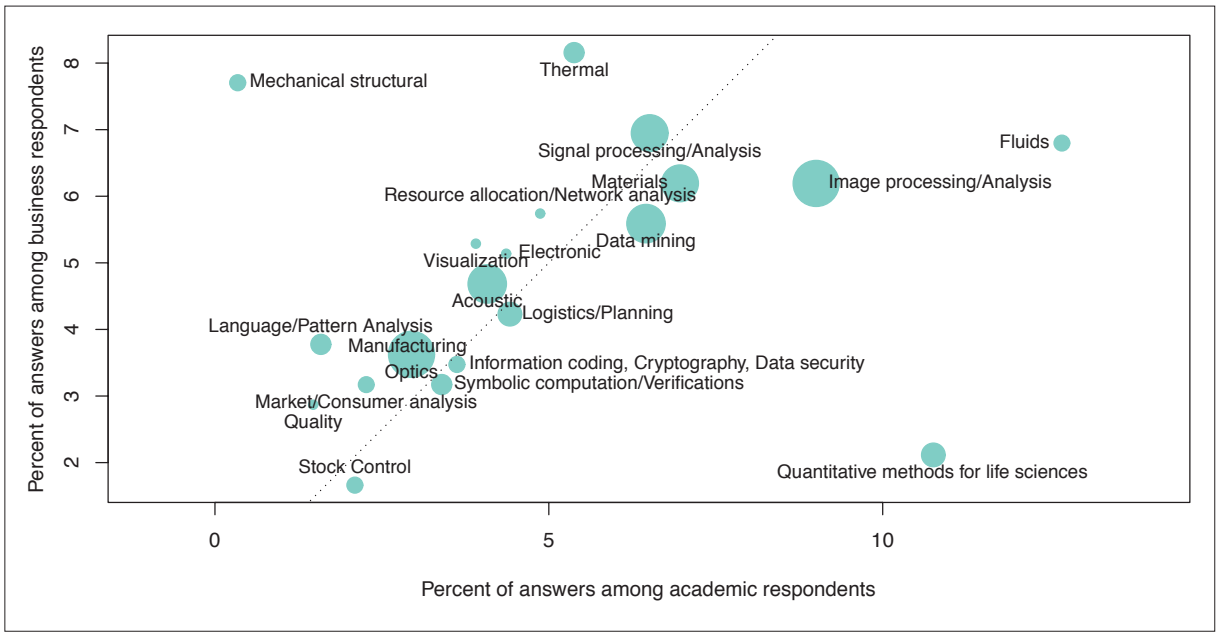


Figure 2. Main areas of competence available in academia versus major business challenges perceived by the industry. Size of the bubbles indicates total number of respondents.

Product optimisation. The largest misfit seems to be related to the areas of Risk management and Simulation. Risk management is perceived as the fourth most important business challenge in industry (12% of answers), however it is chosen by about 8% of respondents from academia (7th most important out of 9 areas).

Based on the declarations of the respondents we conclude that, in general, there exists a fairly good match between the structure of available mathematical competence in academia and the structure of the demand for mathematicians in business (Figure 2). However, there are several notable exceptions. These include mathematics for "structural mechanics" and "thermal engineering", which are perceived as, on the one hand, most needed by the industry, but, on the other hand, not so frequently mentioned by academic respondents. Simultaneously, "quantitative methods for life sciences" and "Fluid mechanics" are popular in academic mathematics departments, but are not mentioned too often as crucial by the business respondents[4].

The deviations in structural mechanics and fluids may be explained by mutual misconception between mathematics and, more engineering-oriented, simulation sciences. However, the result for life sciences has consequences on the entire health care area with a tremendous societal impact. Apparently there exists a significant discrepancy between academics and industry in the perception of the impact of today's mathematics to life science problems. Of course, until today there are only a few examples, such as in pharmacokinetics or biostatistics, where mathematics provided a significant benefit for health care. Moreover, lots of activities in academic mathematics for biology are far away from the needs in health care. On the other side, it is well accepted that the tremendous amount of genomics, proteomics or epigenetic data which can be measured in parallel using the upcoming experimental technologies makes future mathematical involvement essential for the further progress of biomedical sciences. However, apparently both sides have to move significantly in order to realise the benefits from the tremendous investments in data today and in future. Only by novel, efficient information retrieval from these data the efficiency of the health care can be increased, as it is urgently needed in order to prevent the health care systems of the European ageing societies from a financial breakdown.

Example: *let us consider optimisation problems that are frequently encountered amongst the problems proposed by companies. Optimisation is a transverse domain of mathematics, which involves analysis, numerical analysis, combinatorics, and statistics. The problems of interest for industry have some particular features that make them challenging for mathematicians. A particular and frequent feature of the industrial problems is that they often involve a lot of uncertainty, a large number of constraints, and several output quantities to optimise together. Typical problems are related to scheduling and task assignment, physical or chemical processing or optimal design. It turns out that the first challenge in this kind of problems is in their set-up, that is to say, the definition of the optimisation/cost function, which takes into account the budgetary, implementation and regulation constraints. The choice of the cost function is instrumental in the trade-off between the optimisation of several different outputs with several constraints of different natures. Moreover, industrial researchers often wish to keep flexibility, they do not look for "the" solution, but a class of answers or solutions that achieve different trade-offs between the different constraints. Confidence issues and uncertainty quantification are also critical. Industrial researchers of course prefer a reasonable and stable solution than an optimal and unstable one. Mathematicians can incorporate this idea in the optimisation routine, but the challenge for them is to form a group that incorporates experts from the different domains of mathematics needed to achieve this task.*

Another frequent class of problems reported by the industry is related to data processing. Here, two types of problems may appear. The first type of problems arises when few data are available, and the challenge is to overcome data scarcity. Typically the problem is about interpolation estimations (with the minimal amount of data) and the identification of the key parameters of the process. The second type of problems occurs when industrial researchers or engineers have to process very large amount of data, and the challenge is more about data mining and the extraction of useful information. A typical example is the use of geophysical surveys for oil exploration. But it can be said that practically every field of mathematics has or will have its application in the industrial context. In parallel document, we have gathered some recent examples of successful cooperation between mathematics and industry in European research groups that give an idea of the variety of the applications and of the challenges.

1.3.2. Process of the interaction

First, it is relevant to note that mathematicians can contribute in all the different steps necessary to solve an industrial problem as follows.

- **Identification of the problem:** An industrial problem is *almost never* formulated as a mathematical problem; it can be very poorly characterised. One of the main contributions of the mathematicians can be

4. Note that the respondents from industry only represent part of the R&D departments of European companies.

the identification and characterisation of the problem itself and its origins by formulating the problem in an abstract framework, which can distinguish what is relevant and what is not.

- **Modelling of the problem:** The mathematical formulation helps to clarify the problem, identifying the main obstacles, and proposing the tools to tackle the problem. An additional concept that mathematics can bring is the notion of a hierarchy of problems going from easier to solve problems, where one can get analytical insight at a qualitative level, to highly complex ones where the situation to be modelled is better approached, but at a much higher level of detail and cost (for example, coupled electro-thermal or electro-mechanical models)
- **Simulation:** Efficient, robust and reliable mathematical techniques must be developed to solve the problem at hand, and provide realistic simulations. However, academic and industrial people may not agree on the same definition of "solution". Industrial engineers and managers usually look for "a" solution in a given time (say, the best solution given the time allocated to the project, or at least a solution that fulfils the constraints). Academic mathematicians usually work on a different time scale and look for "the" optimal solution. Open discussions are necessary to clarify the different expectations and constraints.
- **Uncertainty:** Due to variability in the production process, uncertainties may occur. This needs to be taken into account by employing stochastic and statistical methods, in combination with the scientific computing techniques already employed to solve the problem.
- **Validation:** This is an extremely important aspect that is, however, often neglected. It is not sufficient to be able to solve the mathematical problem; one always needs to validate the results obtained with measurements. In this sense, there must be strong interactions between modelling and simulation. It is crucial to identify and characterise the range of validity of the model.
- **Optimisation:** due to the increased computational power and the achievements obtained in speed-up of algorithms (see Figure 1), optimisation of products has become into reach. This is of vital importance to industry. Again, however, this calls for open discussions and good communication channels between mathematics and industry.

The question is whether, and to what extent, the above is already taking place in interactions between Mathematics and Industry. In this context, it is interesting to look at the responses given in our questionnaires. Respondents from academia differ in terms of the scientific profile of their host institutions. The majority of respondents come from Mathematics (36%) and Computational Mathematics (23%) departments. A relatively smaller share of respondents come from departments specialising in Control Theory, Financial Mathematics, Computer Science and Bioinformatics, jointly they constitute about 10% of all respondents from academic institutions. Most of the academic institutions of the respondents do consulting for business, on average 71% of respondents answers report that fact. However, consulting for industry varies substantially depending on the scientific profile of the institution. This type of academia-industry collaboration happens most often in departments specialising in Computational Mathematics, it is reported by 85% of respondents from these institutions. Other specialisations reporting a high rate of industry consulting include Engineering Mathematics, Control Theory, Computer Science, and Financial Mathematics. Consulting for industry happens least often in Pure Mathematics departments (61%). A vast majority of people from industry, over 90%, report that their company does run a Research and Development department.

Despite of the outlined limits of our survey, an overall message that can be concluded from the answers of the questionnaire is the high demand for mathematical research and mathematical tools in industry. This is demonstrated, in particular, by the following two features evident from our survey:

- Industry is more and more challenged to accommodate two contradictory requests: the need to optimise all the steps of the industrial process; and the dramatic increase of budgetary and regulatory constraints. **Only mathematics can help industry to optimise more and more complex systems with more and more constraints.** The contributions of mathematicians are also helpful as an internal tool for industry to assess the validity of a new project and also as a proof of rigor and robustness that can be used by supervising and regulatory authorities.
- When new mathematical or computational techniques are introduced and prove themselves to be efficient, the natural trend is to disseminate these techniques and to apply them to other problems. This often makes sense, but may represent a risk. Indeed, mathematical results are always conditioned to a set of hypotheses (a mathematical statement has always the form "if… then") and by extending the domain of application one may exit the domain of validity. It is important that mathematicians be more associated to the development process and that more people receive a basic training in mathematics in order to avoid this risky behaviour. Black boxes can be dangerous, because they can sometimes give answers that are not appro-

priate. Mathematicians or mathematically trained engineers are aware of this and will analyse results before taking them for granted.

1.3.3. Requirements for a fruitful interaction

An important remark is that the use of mathematics in the whole design or production process is most of the time invisible in final industrial products or services. In other words, mathematics is typically an "enabling technology… [whose] economic impact is real, and many companies –old, as well as new– have achieved a competitive advantage though the judicious use of mathematics" [OECD report of Annex 1]. This fact that can be phrased as "invisible contribution, visible success" needs to be acknowledged. **Hence the absolutely most important challenge for mathematicians is to further convince industry that they need more and modern mathematics and mathematicians to develop new competitive products and technologies.**

A means of convincing more people about the presence of mathematics everywhere could be to put stickers "Math Inside" on products where mathematical techniques have proved the reliability and robustness. A real challenge is also to convince also policy makers that it is not just engineering, chemistry or physics that are key technologies, but mathematics as well. A difficulty is the missing profile of the "industrial mathematician". Mathematicians in industry often work in a team and are used as all-rounders or trouble-shooters. It is, therefore, not sufficient to point out that mathematics is everywhere if mathematicians are visible nowhere in their role as mathematicians! Good and relevant mathematics can only be developed and applied by mathematicians, people who know how to model, how to analyse the models and how to make relevant computations, estimate errors and to characterise the range of reliable applicability. These tasks are crucial for high quality products and processes and they are not possible for people who just apply commercial or classical methods that have been available for decades and which might not well adapt to new situations.

As a matter of fact the complexity of most of the problems that are currently encountered and the rate at which mathematics is developing are such that a fruitful collaboration is only possible if the industrial mathematician is familiar with the latest advances of contemporary mathematics; especially when new ideas are needed that can induce real innovation. Indeed, some of the most powerful new ideas in mathematics that are finding fruitful application are complex and sometimes difficult to use, which is why professional mathematicians are required.

Let us note that concerning specific needs from industry, it is very hard to anticipate them, but that clearly model based simulation, optimisation and control will be key issues in the next 10-20 years. Most of the times, the design of a new product requires taking into account opposite constraints (robustness and cost, reliability and speed of conception…). Therefore, in order to achieve a proper balance between opposite needs, mathematical tools are an alternative to the classical means relying on the expertise and lifelong experience of some engineers or on trial and error evaluation of preliminary prototypes that are extremely expensive. Mathematically based tools are already of current use in some industrial fields (aeronautics and aerospace, automotive, energy, etc) but they are not so frequent in other fields (such as pharmacology) and absent in most sectors. Doing this requires some effort for the modelling of the process, the definition of the proper simulation algorithms, the wrapping of these high-level scientific concepts into an expert system that can be used in the conception workshop. The goal of these simulation tools is to provide the same output as a true experience. Thanks to the availability of these in-silico experimental benches, the second achievement is to lead to an in-silico expert tool that is "intelligent" and that out of many numerical simulations is able to provide the optimal product capable to achieve different optimal regimes under industrial constraints.

Another example is related to the time consuming assimilation of over representative data in industrial decision time; mathematical models are able to synthesise the trends in these data and understand which is currently out of reach while automatic acquisition systems are more and more available. On the opposite, there are situations where the data are very scarce and hard or expensive to acquire. Here again mathematical models may lead to get valuable information from the combination of a priori different representations of the same situation.

It should not be hidden that, if most of the problems encountered in the industrial context will certainly benefit from mathematical input, currently most of them are out of reach for at least two reasons: the first one is that industries are not aware that their problem can be cast into a solvable mathematical frame; the second one is that sometimes the necessary mathematical theory is not available, at least under a form that is directly applicable. A one-stop shop where illustration of realisations (success stories) and post list of problems are available would facilitate the meeting of academic research groups and industrial engineers.

Mathematicians can also meet the needs of industry if they are able to exploit all the potential of high performance computing. The proper algorithms for

many problems of interest are available and are the subject of active research, but for the actual problems of industrial needs, with sizes that are not equivalent to academic problems, further investigation needs to be performed.

Example: *In the chemical and pharmaceutical industry, the development of novel products and processes has to tackle an increasing complexity. The systems which have to be optimised, e.g. in functional materials or in bioengineering, are highly complex and are not fully understood in detail. Therefore the established mechanistic modelling, which is based on a detailed translation of the underlying mechanisms to mathematics, cannot be applied. Hence hybrid, grey box models have been developed which combine mechanistic modelling with machine learning approaches. Successful applications ranging from process optimisation, materials design up to gene diagnostics show the typical character of high-end mathematics as an enabling technology for a broad range of applications. However, the drawback of the high efficiency is that validation becomes extremely challenging, such that highly skilled mathematicians are the key technology "champions" in industry.*

1.3.4. Forms of interaction

The collaborations of industrial and academic researchers are often the results of personal contacts: typically former students or colleagues working in industries and keeping contact with their former advisors or other members of the academic mathematics departments. A connection can also be established during a workshop or a conference. The early steps of the connections are critical and should benefit from rather simple measures:

- It is not easy for industrial engineers or researchers to know whom to contact in academia when they do not have a priori contacts. There are many mathematics departments throughout Europe but the information about who does what is not easily available. The WebPages of the mathematics departments rarely contain special links to guide the potentially interested people towards a presentation of the recent achievements of the department or specific contacts who could answer the first questions. **This could be improved without too much effort, but there would have to be an initiative from the side of European institutions like EMS in this direction.**
- Academic researchers usually have slides ready to present their most recent work, but usually do not spend time preparing a presentation that could be more accessible to people potentially interested by submitting industrial problems but without the mathematical background expected from a mathematician. It is often much more effective to present practical challenges than 'perfect' solutions.

There is a full spectrum of opportunities for groups who are interested in genuine industrial collaboration. These opportunities can range from setting up a fully-fledged mathematics-in-industry group, with a full-time project facilitator and regular symposia and collaborative meetings with industrial partners to having a group, which occasionally works on industrially relevant problems. In [OECD report April 2009] the forms in which collaboration can start are described on the basis of concrete experiences of many groups and laboratories across the world and the different mechanisms are illustrated. It can be noted that, in cases of complex problems and tasks, typically not all the competence and skills can be found in the same laboratory and research group. For large companies a one-stop-office partner typically is the highest priority, this would require the **formation of large clusters where all the industrially relevant mathematical competence is available.** But sometimes to get the excellence in different areas even trans-regional or transnational groups may be the best answer. As a consequence a set up of databases and a strong coordination activity would be necessary.

Once a connection is established between industrial and academic partners specific programs should be available to develop the collaboration. Several types of such cooperation programs can be identified:

- **Short-term cooperations** (a week up to a few months) that can be implemented at any time and can be realised very quickly. This would allow to make a proof of concept, or to test ideas with a rather tiny budget. As an example, we mention specific internship programs that bring a student and his/her advisor together with the industrial partner, or the cooperations that arise from industry workshops (modelling weeks), where industries present specific problems that are addressed by a group of students and researchers.
- **Medium term cooperations** (approximately one year) where an identified project is carried out jointly by a group of researchers both from industry and academia. Here a clear and very specific project goal needs to be identified and a clear time frame.
- **Long-term projects** (a few years) that include the temporary transfer of academic researchers to the industrial partner of the project as well as engineers and researchers from the industrial partner to the academic one.

In any case a clear estimate of the timescale, the uphold of the deadlines, the focus given to the problem proposed, a careful and well-prepared presentation of

the results (leaving to appendices all technical details that might seem crucial to the mathematician but absolutely obscure and not relevant to the company) are essential ingredients for a successful cooperation. **It is evident from the results of the questionnaire that there is often little experience in academia in setting up such cooperations, in estimating time-frames and in identifying clear and realistic goals.**

Let us end this section by pointing out that, in our opinion, a new type of position is needed in the academic research centres. Indeed, industrial partners usually expect a solution to their problem in a form (a report, an algorithm, a code) that is more finalised than the one that an academic mathematician is prepared to deliver. There are different reasons. First, an academic mathematician may not be an expert in programming. Second, an academic mathematician is evaluated on the number of publications and there is no incentive to spend time and energy on the development of a practical user-guide or a code. The new type of position, which we may call **mathematical engineer**, would be devoted to the management and development of industrial projects. The establishment of such positions should be encouraged.

1.3.5. The particular case of Small and Medium Enterprises (SMEs)

The case of SMEs is very different from that of large companies. Academic mathematicians often have very few contacts with SMEs although, at the same time, a lot of innovation is created in this kind of companies. In fact, SMEs could bridge the gap between Mathematics and Industry. Therefore, it is extremely important that this kind of collaboration is triggered much more than currently is the case.

For instance, it seems very important to keep contact with former students who have not chosen an academic carrier and have started a carrier in industry. These former students should be privileged interlocutors for establishing contacts between industry and academia. An obvious way for that would be to maintain an alumni directory, but this is not so common, and this should be recommended. Another way should be to invite them during special events like the "Modelling Weeks". These invitations are interesting in two different ways. They offer mathematical students the opportunity to discuss with non-academic people and to realise that it is possible to think about a career that is wider than the academic world. They also allow former mathematicians working in industry to feel part of the mathematical community. They can share their experiments, provide advice, and they can present their current problems to the students and researchers, which could initiate collaborations.

The problem of the response time of academic mathematical project partners is at least as critical and probably even more critical for SMEs than for large companies. Here it is important to be able to give at least partial answers very quickly, and it is clear that such answers can only be delivered by a well-organised group with researchers competent in different domains of mathematics and with mathematical engineers capable to deliver answers and products that can be put in a more or less final form. As already noticed, permanent training sessions, internships and modelling weeks proposed by mathematical research departments would certainly be a nice way to start collaborations between SMEs and academic departments.

1.3.6. Improving the mathematics-industry interaction through training

The structure and the contents of curricula in mathematics in most European universities are in most cases excellent with respect to the basic education and to updated information. But they do not contain all the tools necessary to collaboration with other disciplines and even with different branches of mathematics. The current trend to premature overemphasised specialisation is a problem that should deserve a deeper attention within the community. It is important that bachelor programs focus on a broad and fundamental education and that specialisation is restricted to a minimum.

At a later stage, however, a specific training towards the collaboration with other disciplines should be addressed. One possibility that could be offered is a specific curriculum at master degree and/or at graduate level; this is discussed in Section 3.2 below. But also the invigoration of the key area of mathematical modelling in all mathematics degrees is needed. Within this framework (but also having in mind students that will not become industrial mathematicians) some opportunities of encountering non-mathematical problems that can be dealt with mathematical means should be offered. For example:

- Graduate students are usually proposed to follow a variety of courses in mathematics during their training. One could suggest that some of these courses could be taught by industrial researchers or engineers, by taking the form of "**modelling problems**".
- **Study Groups.** The typical format of Study Groups extends over one week as follows. During the first day industrial researchers present different problems to an audience of academic researchers (which may or may not include students). During the next three days the academic researchers split into subgroups to address the different industrial problems. The final day is devoted to the report of the progress made

and exchange between participants. The follow-up may take different forms depending on the progress made during the week.

- In some universities special weeks are proposed during which the students (not only in mathematics) are taken out of the university (in order to focus their attention) and receive different lectures about oral and written presentations, **career opportunities** and the job market. The purpose of this week is to allow them to gain confidence, to learn how to promote themselves and to realise that they have a huge potential.
- It is also common that the students, mostly at graduate levels, organise special seminars, where they present their work. It would be a good opportunity to **encourage invitation of external people** during this seminar so that the students can discover problems presented by industrial researchers. In this non-formal seminar the students would be more willing to ask questions and react to the presentation.
- **Modelling Weeks.** These weeks would have the same form as the Study Groups but would be targeting the students: we note also that this form, as well as the organisation of study groups, can be instruments for promoting more intensive and focused interactions with companies.
- **Internships** where students work on specific problems brought by a company under the supervision of senior mathematicians can also be very interesting for the students and a valuable part of their training. An internship is a research project of short duration (typically 4-6 months) that is undertaken in collaboration with an industry partner and places a graduate student or postdoctoral researcher part-time in the company.
- Finally it may be interesting to build **courses for start-up companies**, with the experience of founders to encourage young students to think to entrepreneurially if they have an idea.

1.3.7. Confidentiality and intellectual property rights

It is clear that academia and industry have completely different attitudes towards research. For scientists, circulation of information is essential and the publication of every interesting results is viewed as the mean of contributing to the advance of knowledge (as well as the means of progressing in the career). On the contrary, an industrial company may be reluctant to publish not only the solutions to its problems, but even the problems themselves, as they may disclose the directions of research and development of the company. It would be very profitable for both partners to smooth this process by avoiding the cumbersome task to write a new agreement for any new collaboration. The idea would be to create an open library that proposes standard agreements [5] that could be the bases of a contract so that everybody can concentrate his time and energy to the problem at hand and not on confidentiality and copyright issues. These agreements would typically describe:

- How the work is carried out (typically the company pays the university for the time spent by the academic researchers or the mathematical engineers on the project).
- Who keeps the intellectual property of the discovery (typically the company keeps the property while the university keeps the free use of the discovery for non-profitable purposes) and how it will be protected under the restrictions of European patent regulations with respect to mathematics and software.
- When the results can be disclosed and published.

1.3.8. Towards a mindset shift for mathematicians

In the previous sections we have shown how companies can benefit from collaborating with mathematicians. But academia also benefits from its interaction with industry. This interaction stimulates research in new directions and also fosters the development of new analytical and computational methods. New tools have to be designed for new needs. Advances in mathematics can and should arise from this interaction. A recent example is the new algebraic theory for structured matrix polynomials, which arose from an industrial project in analysing vibrations for high-speed rail tracks [6].

Mathematicians have played an essential role in the task of making the computer revolution work to the benefit of industry and commerce. Their role and importance in the future is likely to increase not decrease. It follows that there is an urgent need for a new generation of mathematicians to take up the challenges and opportunities presented by industry as Europe seeks to become the leading knowledge-based economy. Despite the wonderful mathematical tradition in Europe in all disciplines, the numbers of students studying mathematics is not increasing in many countries, in stark contrast to the needs of society.

Research that has traditionally been considered as a part of Pure Mathematics and motivated by theoretical development has recently found many practical applications in such areas as tomography, internet searching

5. See for example, the UK's Lambert Agreements at http://www.innovation.gov.uk/lambertagreements
6. D.S. Mackey, N.Mackey, C.Mehl, V.Mehrmann. SIAM Journal Matrix Analysis and Applications, Vol. 28, p 971, 2006

algorithms, cryptology, biology, climate change, insurance, the economy, etc. Application-driven Mathematics has usually been associated with problems appearing in industry, science and engineering. These areas are now understood in a broader sense and also include statistical treatment of large data sets, numerical algorithms and computer packages.

Within modern technological development it does not make sense any longer easy to distinguish between Pure and Applied mathematical problems. The majority of applications require a deep knowledge of fundamental mathematics as well as the development of new tools and algorithms. For many years companies have successfully worked with the classical techniques that were developed years ago. The high complexity of current industrial problems requires the combination of all areas of theoretical and applied mathematics. Furthermore, recent developments in mathematical sciences have reinforced two general tendencies that can be traced back far in the history of mathematics, namely the quest for unity of the discipline and its stimulation through challenges coming from other sciences and engineering. These may appear contradictory but they are not at all. Indeed, when mathematicians speak about unity of their field, they do not have in mind a static architecture of their discipline but rather a very dynamic process that accounts for a permanent rearrangement within the mathematical sciences of its different components and reflects new impacts of one branch on another. A good example for such an application is the development of new optimisation techniques, which immediately move the frontier of solvable problems far beyond current methods. It is essential for mathematicians in academia to be aware of the demand in industry so that they react by developing new techniques and transferring them to industry.

Example: *A typical example of the latter is the growing influence of stochastics in many areas of mathematics, from geometry to the analysis of partial differential equations and combinatorics. On the other hand, challenges coming from outside can be typically taken up through combinations of techniques borrowed from different parts of mathematics and/or the emergence of new concepts. Here also typical examples are new security procedures, critical to data transmission, relying on special codes mixing in new ways combinatorics and algebraic geometry, or medical imaging techniques that rely on sophisticated geometric concepts and discretisation techniques. This shows that it is more important than ever for mathematicians to be on alert for new frontiers of their discipline in both pure and applied areas, and to exchange and share their expertise, tools and knowledge.*

1.4. International context

As mentioned previously, despite some of the deficiencies, the mathematical community in Europe has one of the top positions in international research and development; and in applied and industrial mathematics is clearly in the worldwide leading position. On the other hand keeping this lead position is not for granted, since the number of scientists in some other parts of the world is steadily increasing while stagnating in Europe. The situation can be easily compared to the field of computer science, which started out with many great developments (including the first computer) in Europe and today Europe is lagging behind. In this context, the National Science Foundation (NSF) of the United States (2008) announced Mathematics as an NSF-wide priority area[7] recognising that:

"Today's discoveries in science, engineering and technology are intertwined with advances across the mathematical sciences. New mathematical tools are needed to understand scores of subjects, from the complex processes that drive the climate system to interactions of magnetic fields and fluid flows in the hot plasmas within stars. Mathematical modelling plays a key role in research on micro-scale, nano-scale and optical devices. Furthermore, for the United States to remain competitive among other nations with strong traditions in mathematics education, we must attract more young Americans to careers in the mathematical sciences."

The focus on training and education in mathematics and science in the US was further enhanced after president Obama's launch in November 2009 of the "Educate to Innovate" Campaign for Excellence in Science, Technology, Engineering and Math (STEM) Education. This is a US-wide effort including new investments of over US$ 260M to "move American students to the top of the pack in science and math achievements over the next decade". President Obama[8] furthermore stated that: "Reaffirming and strengthening America's role as the world's engine of scientific discovery and technological innovation is essential to meeting the challenges of this century. That's why I am committed to making the improvement of STEM education over the next decade a national priority."

1.4.1. The European context

Counted together, European mathematics is one of the strongest World leading schools. In many European

7. http://www.nsf.gov/news/priority_areas/mathematics/index.jsp
8. http://www.whitehouse.gov/the-press-office/president-obama-launches-educate-innovate-campaign-excellence-science-technology-en

universities and research centres, mathematicians are frequently organised in communities at national level. In collaboration with industry, they have developed a large expertise at the forefront of research involving most fields of mathematics. There exist scattered networks of centres, exchanging students in common training programs in industrial mathematics, that have a long tradition and experience, forming a scientific community in the making. The diversity of approaches and cultural differences is a key factor for cross-fertilisation.

Despite the work and efforts of the European mathematical organisations, the communities are still not sufficiently connected and remain fragmented. Among European countries large differences remain in the tradition of establishing links between research and industry. Companies have in many cases insufficient awareness of and belief in academic research and particularly in mathematics. This is particularly true for small and medium enterprises, for which innovation should be the main concern, in order to face worldwide competition. Regardless of the vital societal and scientific role of industrial mathematics, its recognition is not high in the European and national political agendas. In addition, the scientific community has often underestimated the value of applied and application driven research. This has led to a difficulty in its evaluation and recognition.

Despite the crisis, the hiring of mathematicians in all business sectors continues, the higher the qualification the better. Even in the present period of financial crisis, more mathematicians are hired. Mathematicians solve many important industrial math problems and mathematical software is provided through academic institutions and small spin off companies, but the demand for good mathematicians is much higher than is currently available.

As is true in many fields but even more in mathematics, if the mathematical schools of the various countries in Europe are considered together, Europe represents the broadest, most vivid and most productive school in the world. There is a strong history on which can be built in all areas of mathematics. The trench between pure and applied mathematics that is still very present in some countries has been bridged in many others. The community of colleagues that are active in developing collaborations with industries is growing and it will certainly benefit from strong promotion of success stories and the success of other people in Europe.

The way pupils, students, engineers, and mathematicians are trained still differs widely throughout Europe and also the job market for mathematicians is very different in Europe. There are also various economic and sociologic models and what is possible in one country may not be feasible in another. Some critical questions as "what falls within the competence of a mathematician" "what falls within the competence of a researcher or an engineer" "how to use the material developed by pure mathematicians in applications" all have elements in various countries that should be compared and publicised. On this firm ground, the exchange of experience is certainly worthy, as some of the approaches for improvement will be possible in some countries.

1.4.2. Strengths and weaknesses of European Industrial Mathematics

Strengths in Europe:

- Mathematics is the language of innovation, which is vital for society and industry.
- Counted together, European mathematics is by far the World leading school.
- In some European universities and research centres, mathematicians are frequently organised in communities at national level. In collaboration with Industry, they have developed a large expertise at the forefront of research involving most fields of mathematics.
- There exist scattered networks of centres, exchanging students in common training programs in industrial mathematics, which have a long tradition and experience, forming a scientific community in the making. The diversity of approaches and cultural differences is a key factor for cross-fertilisation.
- Communication channels between Industry and Academia are already established in many countries. In some cases, mathematicians occupy important positions in industrial R&D, especially within large companies that have in-house research departments.

Weaknesses in Europe:

- Despite the work and efforts of the European mathematical organisations, the communities are still not sufficiently inter-connected and remain fragmented.
- Among the European countries large differences remain in the tradition of establishing links between research and industry. In many cases companies have insufficient awareness of and belief in academic research and particularly in mathematics. This is particularly true for SMEs, for which innovation should be the main concern, in order to face worldwide competition.
- Regardless of the vital societal and scientific role of industrial mathematics, its recognition is not high in the European and national political agendas. In addition, the scientific community has often underestimated the value of applied and application driven research. This has led to a difficulty in its evaluation and recognition.

- For various reasons and for many years, industrial mathematics research was neglected in many academic institutions, and this task has been taken over by engineering groups. While this has changed in several European countries, this attitude still persists in many others.
- Academic careers in the field of industrial mathematics have somehow suffered from a lack of recognition and of a clear identity. Similarly, large companies often have an in-house research department but no specific career development paths for mathematicians.
- In the mathematical community at large, there has been no clear sign of integration of mathematicians working in companies.
- Mathematics as a research topic is highly underrepresented in European research funding despite the fact that it is often well represented in national funding programs.
- Research programs for the direct collaboration between math and industry exist only in very few European countries and such programs are completely missing on the European level.
- The situation to deal with the demands from industry is very unevenly distributed in Europe. In several countries a variety of structures is available that can (in principle) meet this demand, but in several countries such structures are missing completely. Furthermore, in almost all countries, the education systems are not providing the means to generate academically trained staff that can fully meet the challenges. Even if people partially have the appropriate training, typically the know-how has aged after 5-10 years and continuing education is essential. Only very few institutions in Europe provide such (re-education) programs. The fact that the development of methods is so rapid implies that industrial mathematics has to maintain a strict and permanent contact with the University: otherwise, in the long term it will lose its innovation potential and will risk to “sell” obsolete methods.
- The development of new mathematical methods progresses rather rapidly and the software development as well as the relevant analysis is often behind. Very few institutions that have recognised this demand operate on a European level, but they often fail to meet all the demand and only collaborate those partners that are active already.
- There is no infrastructure in Europe to provide the demands and in particular there is no structural support to change the situation in math-industry underdeveloped parts of Europe. The situation is drastically different in the US where mathematics oriented start-ups have quickly become global players (MATLAB) or where the concentration of global mathematical software development is highest (e.g. NASTRAN, ANSYS). European initiatives (even very successful ones like the SLICOT library in control which was developed through the NICONET initiative via EU support) often do not get supported further than over very short periods.

As we have already outlined, mathematics is a universal language in science and in many cases the same strategy can be applied in different contexts. This leads to the conclusion that mathematicians are valuable members of the multidisciplinary teams that are required to carry out modern development projects with industry and companies (and also multidisciplinary projects within Academia). But even more can be said: they are the natural candidates to steer the groups, since they master the language that is the vehicle of trans-disciplinary interaction.

2. Challenges and opportunities

If the boost for doubling the funding of US mathematicians wins the backing of Congress and the next US administration, the whole of science — and society at large — stands to benefit. The initiative will help tackle problems that affect us all.[9]

Rita Colwell, director of the National Science Foundation (NSF) in 2000

NSF budget for the Division of Mathematical Sciences:
US$ 110M in 2000, US$ 241M in 2010[10]

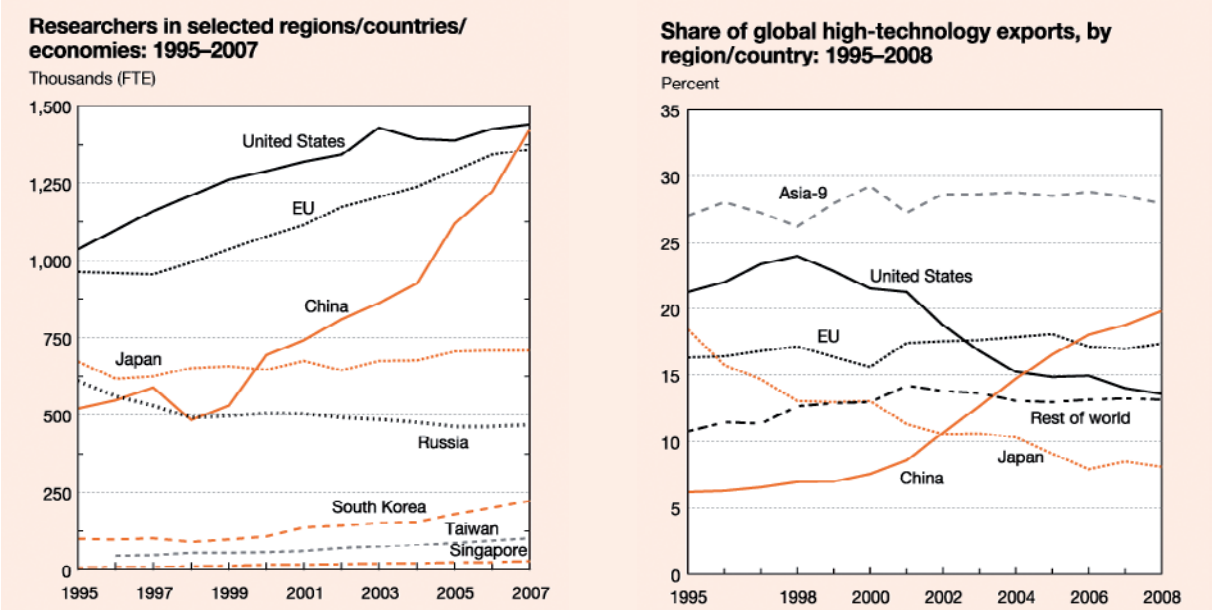
The major players in science and technology, e.g. USA, China, Canada, have made a substantial increase in the funding of research in many areas of Mathematics over the last decade. For example, the total budget dedicated to mathematics in USA from all funding organisations has increased from US$ 380M in 2005 to US$ 540M in 2010. Within this budget, the funding of the National Science Foundation (NSF) Division of Mathematical Sciences has more than doubled from US$ 110M in 2000 to US$ 241M in 2010. It is interesting to note that, in strong contrast to the situation in Europe, the NSF funding for Physics is equivalent to that of Mathematics with US$ 290M for 2010.

9. Nature 407, 929 (26 October 2000)
10. http://www.ams.org/notices/200807/tx080700809p.pdf

Over the past several decades, numerous countries (including some with smaller budgets dedicated to science) have gained international standing in mathematics. More recently, other developing countries have emerged as vibrant centres of mathematical research and training, with faculty and students competing successfully on the world stage.

In each case, it had been understood that a strong national capacity in mathematics is a key to promoting both science and science-based development. Equally importantly, these countries have concluded that domestic competency in mathematics has an important bearing on the overall quality of a nation's educational base, from economics to engineering and from medicine to sociology. India's growing progress in information technology owes much to the mathematical know-how of its knowledge workers. China's contributions to space science and biotechnology are likewise due, in part, to the mathematical skills of its researchers. China now has more than 8,000 graduates with doctorates in mathematics.

The above important developments show that mathematics is a necessary tool for creating innovation (for processes and products) in an interdisciplinary and strategic approach. Hence, innovation and the necessary investment in mathematics are key challenges for European industry and funding organisations in order to face competition in the global market.



From the OECD report entitled *OECD Science, Technology and Industry Scoreboard 2009*.

2.1. Key challenges for industrial mathematics

The start of the twenty-first century is a particularly exciting time, because there is an ever increasing need for new mathematical concepts and tools to solve problems coming from classical fields as well as from new ones, like Biology, Medicine, data mining, security, communications, and other information technologies. Industrial mathematicians can make the difference, allowing these fields to make significant strides, many of them of immediate importance to companies and to the society in general. In order to make that progress, the corresponding actors must have the necessary mathematical background and knowledge, which goes far beyond the common training of engineers or other scientists. Indeed, the latter are essentially taught to implement existing techniques and ideas, but in general they cannot develop new mathematical concepts and tools that are fundamental to provide efficient, robust and reliable methods. Abstraction allows simplifying and going to the essential features of a problem.

The Mathematics Subject Area Group [11] was unanimous in identifying three skills, which it believes every mathematics graduate should acquire:

- The ability to conceive a proof,
- To solve problems using mathematical tools,
- And to model a situation.

It is fair to say that this general principle has been used to design the curricula in many European universities, but has been neglected in a number of countries. For instance, many groups pay lip service to the notion and re-label a differential equations course as being a course in mathematical modelling, which is only a preliminary part of it. In this context, one should differentiate between applications for which the innovation and/or industrial problem comes from including *existing* mathematics within other engineering or scientific disciplines to produce *new* applications, and the ones for which the innovation and/or industrial problem eventually points out the need for *new* mathematics. It is important to note, however, that in this list of skills, the development of mathematical algorithms, the analysis of robustness and reliability of these algorithms and their implementation on modern high performance computers should be added to complete the picture.

The concept of mathematical modelling is central to the practical applications of industrial and applied mathematics. Mathematical modelling and simulation can be identified [12] as the core outlet for applied mathematics in the UK. Furthermore the same document states that: "*mathematics now has the opportunity more than ever before to underpin quantitative understanding of industrial strategy and processes across all sectors of business. Companies that take best advantage of this opportunity will gain a significant competitive advantage: mathematics truly gives industry the edge.*" This sentiment can be extended to Europe (See MACSI-Net roadmap in Annex 1 for example), and the coordination of our mathematical resources in this way is desirable, essential even, for the regeneration of economic success.

Mathematical modelling alone, however, is not enough; it has to go hand in hand with mathematical methods for the numerical simulation, control and optimisation. Mathematical methods have to design and analyse, and they also have to be implemented and maintained as production software that incorporates error and sensitivity analysis as well as failure control. In this respect another item has to be added to the list of important skills that a mathematician has to master, i.e. understanding, analysing and implementation of algorithms for numerical simulation, control and optimisation methods in finite precision arithmetic.

The computer revolution initiated during the 20th century is an essential part of the scientific revolution and has transformed science and engineering as a whole and its importance cannot be overestimated. The computer has created needs of mathematical expertise in almost all fields of human scientific endeavour. Close interaction between mathematicians, computer scientists and application field experts is necessary to correctly and fully exploit the enormous potential that the computer has brought. This interaction is not optimal today.

The data revolution is connected with the computer revolution. In many applications, coming from e.g. finance, astronomy, meteorology or life sciences, both the number of measured variables and samples can be very large (10^4-10^{10}) resulting in enormous datasets. Large amounts of money are invested worldwide in obtaining these data and much could be gained if we used just a small additional fraction of this money to analyse the data more thoroughly. In this endeavour mathematicians are necessary. A point connected with all the above and worth singling out is "software development".

It must be emphasised that the philosophy of industrial mathematics is to operate in a problem driven environment based on the core research areas of expertise available to the group. While this approach is multi-dis-

11. EU commissioned project: "Tuning educational structures in Europe" in "Towards a Common Framework for Mathematics Degrees in Europe"; Newsletter of the European Mathematical Society September 2002, pp. 26-28

12. Mathematics: giving industry the edge – http://www.industrialmath.net/web/?url=News/Roadmap/index_html/view

ciplinary by its very nature, it is a model that is known to work and to produce both scientific advancement and significant mathematical innovation as illustrated by the unquestionable success of many research groups. One of the consequences of all this is that in industrial mathematics the aim is not to write the most possible elegant and general proof in the infinite world of mathematics, but to solve a real problem in an efficient manner and with the constraints imposed by a finite world (finite time and finite resources). Many mathematicians have accepted this, but it is far from being accepted by the community at large, which generates difficulties of recognition and valorisation.

Historically, the investigation of real problems has been the mechanism for some of the most significant mathematical developments. The exposure to industrial problems naturally leads to associate theoretical problems that, de facto, involve the areas that are necessary to give industry and science an edge. Industrial mathematics groups thus include in their mission the wider aim of building up expertise in the core mathematical areas of most relevance in industry and science e.g. modelling, optimisation, statistics, differential equations, multi-scale phenomena, numerical simulations. There is a need for mathematics to be considered as an enabling technology in its own right and for mathematicians to be regarded as scientists with a particular "technological" expertise that goes to the heart of all scientific disciplines. This has not necessarily been the case for all mathematicians in the past and will involve industrial mathematicians of this type obtaining a broader fundamental education with at least introductory courses in physics, chemistry, biology, finance, economics etc. being part of the undergraduate program. Industry needs flexible access to a more connected and well-informed academic community.

New priority areas in industrial mathematics for the next 5 years are:

- **Optimisation** discrete, continuous with all types of constraints, differential equations, stochastic in particular with all these combined.
- **Control and dynamics** of real world processes, Simulation and control of whole process chains e.g. in production, including development of software tools.
- **Mathematical modelling**, in particular directly for industrial projects with high complexity, creation and extension of modelling software, model reduction and validation.
- **Visualisation** and geometry processing.
- **Data mining** handling for large data sets in sciences and engineering as well as the respective model identification tools.
- **Software and algorithm development** in collaboration with computer scientists in order to be able to fully exploit the potentiality of new computer hardware.

General challenges for Europe:

- Industrial mathematics is at the basis of the economical pyramid and is instrumental in the innovation process and in governing complexity. In the context of globalisation, **a lack of innovation will make Europe less competitive and will have a dramatic influence on the job market.** This will reduce our reaction time to adapt to new challenges.
- A lack of political, societal and financial support will also prevent young researchers from choosing a mathematical career. As a consequence, **Europe may not keep its leading role in mathematics**.
- International competition is very tough, not only from the traditional competitors for Europe, but also, and more importantly, from emerging countries, like China or India.
- **Fragmentation needs to be overcome!** Confinement to diverse and independent national priorities would indeed be counter-productive without a strategy at the European level. The launching of a Joint Programming Initiative in the domain of European mathematics and industry could overcome fragmentation.
- **Engineering systems and manufacturing processes are becoming increasingly complex;** design optimisation, time-to-market, and cost effectiveness have become major concerns.
- The ubiquity of powerful microprocessors and the advent of inexpensive data storage devices have led to an ever-expanding capability to collect data, but the useful integration of such data in an industrial context requires that they be processed, preferably in real time, and transformed into information and knowledge.
- **Societal concerns have led to regulatory actions that reflect more stringent requirements for the safety and reliability of products;** they demand new methods for validation, verification, and the quantification of uncertainties.
- **Globalisation, awareness of resource limitations, increasing sensitivity to anthropogenic effects on the environment, and general concerns about sustainability impose constraints on industry, as well as on society as a whole. They force industry to continually analyse and evaluate its activities in a broader social context, beyond the bottom line.**
- The traditional methods of the physical sciences and engineering are no longer adequate to answer many of the questions raised in an industrial environment.

Today's problems are complex and nonlinear, they involve phenomena on multiple length and time scales, and their analysis can extend well beyond the realm of textbook mathematics. **Industry requires more than ever access to qualified mathematicians** who appreciate and understand its needs, who have been trained to capture the essence of an industrial problem in mathematical terms, who can apply methods of contemporary mathematics, and who are familiar with recent advances in scientific computing and numerical algorithms. Only such people can produce the transformative new ideas that drive future innovations.

- **The educational challenge appears at the curriculum level and in the way applied mathematics is taught**. Also, examples of success stories based on mathematical concepts and tools may be fascinating for students and attract good people to the field.

Therefore, mathematicians are the natural candidates for the coordination in industrial problems solving groups, where in addition to the modelling, simulation, control and optimisation, the implementation of robust and reliable production software is also necessary. Therefore, there must also be a basic education in modern computer science methods and modern computer architectures.

2.2. Opportunities for Europe

Recognition for mathematics is overdue

Nature 407, 929 (26 October 2000)

We recall again that the scientific revolution is happening now. There have never been so many scientists as now [13]. At the same time, all over Europe, there is a general phenomenon of lack of attraction towards scientific studies. But luckily mathematics is not the most affected field in this respect.

Most mathematicians in positions in Europe have been trained in Europe. Moreover, the European scientific policy has helped in starting the creation of a European mathematical community through networking and mobility. Due to this, among scientists the feeling of belonging to a European community is stronger than the average, and this can be used to build initiatives at the European level allowing overcoming fragmentation.

The European mathematical school is being structured through the recent initiatives of the European Mathematical Society but it does not exist as such yet. If all the national indicators in mathematics were summed up at the European level, the European mathematical school would be the most diverse, innovative and powerful one in the world. It is an opportunity to direct at least a part of this potential towards applications to industrial and societal problems.

On the other hand, we are not aware of any model of collaboration between mathematics and industry that has not been implemented in some European country. The existing expertise can be used to spread the best-suited models all over Europe, taking into account the diversity and the richness of the different European countries.

General opportunities:

- **There is a general increasing awareness about the necessity of using mathematics to improve the competitiveness of the world economy** (see for instance the various reports that are listed in Annex 1).
- **The arising awareness of the needs of mathematical modelling, illustrated recently for instance in the financial crisis or the global environmental changes, together with the willingness of the mathematicians, makes the timing right to create the necessary synergy.**
- Even if complexity is a problem addressed by mathematicians only recently, results already exist and allow to deal with some real life problems in a pertinent way. The latter is amplified by the exponential increase in computing power.
- **The expertise in mathematics is extremely high in Europe, both in variety and quality.** More and more mathematicians are ready to participate in industrial projects. The potential is there.
- **Existing experiences and knowledge can be shared in order to increase the level of industrial collaboration in all countries.** Indeed, the existing and reproducible examples of collaboration with industry can and must be spread all over Europe.
- **It is timely to start developing common networks and databases of industrial problems, mathematical experts and examples of collaboration.** An effort has to be made, but many of the necessary ingredients are already available and will be put in place if our recommendations are followed. Indeed, Europe has already greatly contributed to the building networks across the European boundaries at the academic level. At the same time, companies are more and more international.
- **The European industrial landscape is characterised by the strong presence of SMEs. The**

13. http://www.its.caltech.edu/~dg/crunch_art.html

development of stronger interactions and networking in the mathematical community would be an opportunity for both industry and Academia.

- **The present increase of the amount of data in many fields will require the development of new mathematical and statistical approaches.** Moreover, the necessary level of mathematics is nowadays often too sophisticated for a single researcher or research group, so building a strong community is more crucial than ever in order to be able to tackle those problems. Reciprocally, available tools of data mining will be able to treat the huge amounts of data that were impossible to deal with until recently. This gives the opportunity to structure data in such a way that models can be built upon it.
- **Knowledge is the key source of competitive advantage in today's advanced economies. European Member States need to boost productivity and performance through better use of knowledge, especially for innovation. Productivity is about making the most of available resources, combining them in new ways and creating new sources of value. It is the key driver of smart economic performance and sustainability enabled by industrial and applied mathematics.** That is why Europe needs to adapt to a smarter economy that is also a more sustainable and innovative economy – characterised not least by the capacity to adapt quickly to changing circumstances.
- Finally, concerning the very delicate question of confidentiality in industrial research, the mathematicians already involved in industrial have learned to deal with it and can share this knowledge with colleagues in other European countries.

3. Mathematics and industry: the missing strategic link

Europe invests billions of euros in knowledge-based industries, but there is concern that links between industry and research are weak, and that market fragmentation is causing investment to leak out of the research infrastructure.[14]

Kostas Glinos, European Commission,
DG Information Society and Media
Head of GEANT and e-Infrastructure Unit

One way to look at or model the interaction between society as a whole and the university is the *Triple Helix approach* developed by Henry Etzkowitz and Loet Leydesdorff. It is based on the viewpoint of the University as a leader of the interaction with Industry and Government to generate new knowledge and innovation. In the words of Etzkowitz:

"**The increased importance of knowledge and the role of the university in incubation of technology-based firms has given it a more prominent place in the institutional firmament.** The entrepreneurial university takes a proactive stance in putting knowledge to use and in broadening the input into the creation of academic knowledge. Thus it operates according to an interactive rather than a linear model of innovation. As firms raise their technological level, they move closer to an academic model, engaging in higher levels of training and in sharing of knowledge. Government acts as a public entrepreneur and venture capitalist in addition to its traditional regulatory role in setting the rules of the game."[15]

As just one small manifestation of this new global innovation landscape the *Journal of Knowledge-based innovation in China* was created in China in 2009. The objective of the journal is to publish research that addresses emerging or developed innovative practices in modern China, and illustrate how innovation is feeding into the country's rapidly growing knowledge economy. Furthermore, the journal will explore the nature of the Triple Helix Model in China and the creation of the journal is propelled by the fact that China ***"is undergoing a fast transformation from the world's manufacturer to a leading player in the field of knowledge-based innovation."*** [16] As is acknowledged by the already quoted "Educate to Innovate" campaign in the US in STEM-education, mathematics plays a key role in this emerging innovation landscape, being the unifying theme and common language of all science, technology and engineering. That education in mathematics already at the basic school levels is crucial for a competitive knowledge-based society is stated in the report "Building a Science, Technology, Engineering and Math Agenda" [17]:

"*Studies of results from TIMSS 2003 suggest that the top-achieving countries have coherent, focused, and demanding mathematics curricula*"

Naturally, coherent, focused and demanding mathematics curricula at the university level are also mandatory to create and maintain a successful knowledge driven economy. It is clear that **Mathematics plays a key role as the backbone of the Triple Helix.** In this context,

14. ERCIM News, Simulation and modelling for Research and Industry, Number 81, April 2010

15. Social Science Information, Vol. 42, No. 3, 293-337 (2003)
16. Editorial Objective of the Journal of Knowledge-based innovation in China
17. http://www.nga.org/Files/pdf/0702INNOVATIONStem.pdf

we remind that, counted together, Europe already has the potential of being the world leader in Mathematics when it comes to either fundamental or applied and industrial research. However, the European Mathematical community is fragmented. **We have the opportunity to focus the strength of European mathematics and use it as a catalyst for European innovation.** Major pillars for success in such a venture are education, training and career possibilities in industrial mathematics. Key to making such a venture a truly European effort are student and staff mobility and the spread of best practices. Mobility, joint ventures and the spread of best practices have the potential to positively influence and strengthen every aspect of education, training and research. This includes the unfortunate gender imbalance in the mathematical community and the differences in gender imbalance across Europe as reported in the work "Statistics on Women in Mathematics" by Catherine Hobbs and Esmyr Koomen [18]. This work shows considerable differences between regions in Europe where southern countries such as Italy, Portugal and Spain have a much higher proportion of women in mathematics than northern countries such as Germany or Sweden. The work shows that there exists an unused potential pool of female talent and expertise particularly in the northern countries. In the same work one can see that the trend is positive, but that development goes slowly. This progress would certainly go much faster with increased mobility and truly joint ventures in European Mathematics.

3.1. Building a community

Strategic objectives to build the community:

- **To foster a European Network in applied mathematics towards a Smart Economy.**
- **To allow Member States to build up a common strategy for European mathematics.**
- **To establish mathematics as a necessary component of European innovation.**

There is a need to coordinate groups interested in mathematics in industry at the European level. An obvious way of doing this is a bottom-up approach where each country is given responsibility for coordinating its own network and there is a European coordinating organism that takes care of common activities (from a web portal for the organisation of events, sharing of expertise databases, etc), promotes transnational research projects whenever the complexity of the problems requires this approach and develops and coordinates opportunities of training. This can be done in the framework of the European Mathematical Society and could benefit from the experience of ECMI, of its Educational Committee and Special Interest groups.

The coordination should not be too difficult to organise at the academic level. But that will be much more difficult at the level of companies. One of the main problems is that people working in industry and dealing with mathematics, such as engineers, physicists, technicians, MDs or CEOs, do not consider themselves as mathematicians any longer. One of the reasons for this is the lack of recognition, and the lack of evaluation of what they are doing. Moreover, many of them become often in charge of tasks, which are more administrative than scientific. This should not be avoided necessarily, but there should be a way of a better evolution of those mathematicians and also it should be possible for them to remain close to problem resolution activities. Structuring this part of the community is thus not so obvious.

On the other side also, the activity of mathematicians that get confronted with collaborative efforts with industries is barely recognised when it comes to promotion on faculty positions. The output of these activities may give rise to a journal publication but this reflects only a little part of the work done, and from the promotion point of view it often pays off much more to concentrate on academic problems than solving real world ones.

So, it is high time to actively promote ways of highlighting the scientific and intellectual value of industrial mathematics. The creation of a journal of industrial mathematics has been evoked, together with the associated with pros and cons. On the "pros" side: such a journal could become a platform for learning how to evaluate these activities, and also to promote by the example. On the "cons" side: most of the work done in the interaction with industry is of a bespoke nature, and it is not research oriented and the evaluation should not be done with standard tools… others should be invented.

It is fair to say that many mathematicians do not really wish to solve real industrial problems. This fact should be acknowledged from the outset. But it is enough that those who want to do it, can do it and are properly evaluated and recognised for that work. Another situation that should be avoided is where a group of mathematicians, with no real modelling experience, claims to solve industrial problems, when actually they only solve mathematical problems of interest to themselves rather than to industry. This sort of activity undoubtedly damages the case for mathematics in industry. After experiences like these, industrial and company partners will invariably

18. http://womenandmath.files.wordpress.com/2007/09/statisticswomen.pdf

disengage from the collaboration and express the opinion that mathematicians do not solve real problems. In the controversial words of Bernard Beauzamy, one time professor of pure mathematics at the University of Lyon, who later set up his own mathematical consultancy [19]: "It (mathematics) brings solutions nobody understands to questions nobody asked." It is not sufficient to *claim* an interest in industrial applications to be an industrial mathematician. In this context, a good example is the so-called "industry-motivated papers" where the industrial problem just appears in the title or in the introduction to justify a bunch of theorems (that in any case could have an interest in themselves); this is a real danger that also journals devoted to industrial mathematics should avoid. For instance, Springer is planning a new journal on Industrial mathematics together with ECMI and it should be worth following its evolution and applying appropriate evaluation measures to deem its success and impact.

3.2. Developing the community

Strategic objectives to develop the community:

- **To overcome geographical and scientific fragmentation.**
- **To broaden and harmonise educational programs for students and teachers in mathematics at the European level.**
- **To facilitate the mobility between industry and Academia.**

Education for Innovation

Since higher education plays a key role in the transformation of the society from industry to knowledge based, many actors are entering the scene: not only professors and students, but also public authorities and social partners. All actors have to adjust to this new reality: a reality with a greater number of legitimate actors representing the interests of society, wanting to give their views on and indications to the direction of the development of higher education in the future. Research and development must be developed as a whole covering universities, polytechnics, research institutions and industry. Public investments are crucial when the role and quality of higher education institutions in innovation need to be improved. The role of public funding will remain crucial, and indicative of the determination of the public authorities. Furthermore, one way of developing research resources of universities is promoting the use of private investments from industry together with public funding. In order to establish "one European Mathematical Community" that is ready to act as a catalyst for innovation in Europe is the establishment of common points of reference at the Bachelor and Master Degree levels. The formation of European Model Master Curricula in Mathematics and Industrial Mathematics are necessary (See Annex 2 for an example).

19. Irish Math Soc. Bull., 48 (2002), 43-46

3.3. Empowering the community

Strategic objectives to empower the community:

- **To harmonise the curriculum and educational programmes in industrial mathematics.**
- **To encourage the exchange of knowledge and information between Academia and industry.**
- **To promote and improve the career path in industrial mathematics.**

A European Model Master in Industrial Mathematics

It is certainly true that Mathematical sciences always have developed in symbiosis with real world applications. This was clear already from the time of Archimedes but has been even clearer since the days of Newton. It is also clear that Mathematics, together with Science as a whole, has taken a giant leap during the 19th and 20th century and today present many new potentially revolutionising opportunities for Society. It is clear that these opportunities for innovation can only be realised with a focused investment on education and training in mathematics and science across Europe. That a new curriculum development for Industrial Mathematics, taking all the above points into account, can be achieved on a European scale has been proved by the European Curriculum Development program called ECMIMIM [20] that took part during 2007-2009. The outcome was a "European Model Master in Industrial Mathematics" developed by the nine participating European Universities. The project work has already been acknowledged by the fact that at the Dissemination Conference of the project in Dresden in September 2009 the nine participating universities from eight different European countries signed a statement that the Model Master program will be implemented at the participating nodes [21]. In conclusion, Europe should ensure the production of mathematical students suffi-

20. http://www.uc3m.es/portal/page/portal/postgrado_mast_doct/masters/Masters_in_industrial_mathematics/Project
21. tu-dresden.de/die_tu_dresden/fakultaeten/fakultaet_mathematik_und_naturwissenschaften/fachrichtung_mathematik/emp-im-2009

cient in number, quality and breadth to meet the needs in teaching, in research in the industrial mathematics and in other disciplines. **Europe should consequently increase its funding for the mathematical sciences to a level comparable to those in the physical and biological and engineering. It should also encourage activities that connect mathematics to other areas of science, technology, business, finance and government, together with strengthening the connections between pure and applied mathematics.**

Opening up higher education in a lifelong learning perspective

Higher competence and skills are not only a question of higher education but also vocational training. Different levels of formal education must not be seen as opposite, but as complementary elements in lifelong learning. Europe needs to take two leaps simultaneously to raise the qualifications and skills of its researchers and workers in industry, academia and companies. On average the number of graduates from higher education is lower than in competing economies, but this is not the case in all member states. At the same time the number of non- or low-skilled workers is too high in Europe. Statistics shown in the Commission's communication indicate a clear link between the educational level of the population and employment. There is also a correlation between the educational level of the population and the GDP in western countries. Lifelong learning is a key to achieving the EU objectives of full employment, enhanced competences, high qualifications and worker mobility, as well as a fairer distribution of income and the balance between professional and family life. Access to training in mathematics should be opened up to motivated and talented adults who did not have the chance to study in their youth and for those who come from socially and financially disadvantaged backgrounds. Moreover, higher education institutions need to increase their role in continuous training of their graduates as part of improving their attractiveness among industry and working life. The task of higher education could be divided into initial and continuous training where continuous training or adult education does not only mean postgraduate education. **Lifelong learning and the modernisation of the national educational and vocational training systems, especially in industrial mathematics will be the cornerstones in preventing long-term unemployment and increasing the labour force mobility in Europe.**

4. Recommendations: giving European mathematics and industry the edge

> ***Business-as-usual in science will leave an ever-growing gap between 'have' and 'have-not' nations – in financial as well as human-development terms.***
>
> US Inter Academy Council Report
> January 2004

The essential role of the mathematical sciences in almost all aspects of the scientific, engineering, and educational enterprise has become increasingly apparent. Together with this greatly expanded capacity for discovery and its subsequent applications to meet societal needs has come a dramatic demand for new mathematical techniques and capabilities that will ensure the continued growth of European scientific and technological capacity. To respond to this demand, substantial progress in the development of new fields of fundamental mathematics is required. Further, in light of our increasing reliance on science, engineering and technology to sustain economic growth and to improve the national quality of life, there is a growing need for improved education and training in mathematics and statistics, both for the scientific and technical workforce and for the population at large. **Europe's competitiveness is to be achieved in a multilateral international environment through a common strategy for European Industrial mathematics.**

To answer the strategic objectives presented in section 3 for **a strong European future in industrial mathematics**, and to enable maximal socio-economic benefits, the Forward Look Steering Committee proposes a number of concrete, near-term actions. These recommendations are listed below in order of impact and urgency, starting with the highest priority:

4.1. Recommendations to policy makers and funders

Recommendation 1: Policy makers and funding organisations should join their efforts to fund mathematics activities through a European Institute of Mathematics for Innovation.

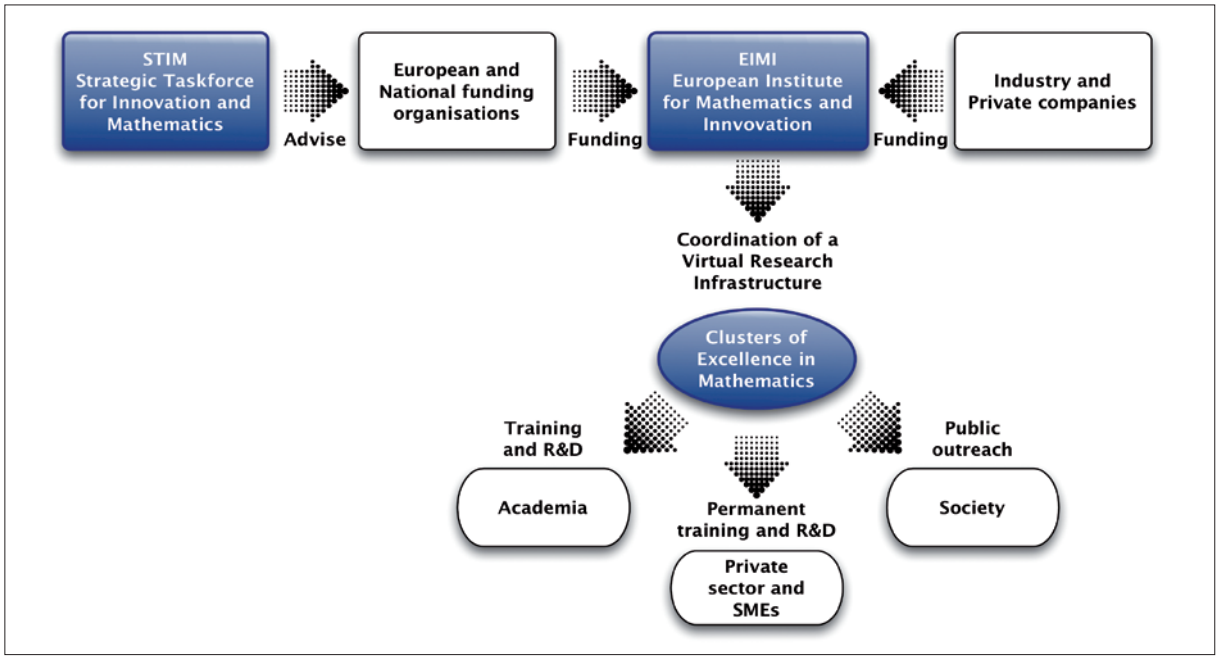
Roadmap implementation 1.1:

EU and National funding agencies should coordinate *clusters of excellence* in industrial mathematics and create a *European Institute of Mathematics for Innovation* (EIMI) for mathematicians and users of mathematics

Global companies often need to collaborate with more than one specialised area in mathematics and so, capitalising on existing leading-edge mathematical expertise, large clusters or competence centres should be formed as for example the *Excellence Clusters*[22] in Germany. They should be highly visible, collaborative and provide competences in all areas of applied mathematics including computational mathematics. These should be closely linked to universities and research institutes so that young students get involved directly. The clusters should have clear points of contact to access experts and industrial mathematicians, and create appropriate engagement mechanisms where they are not already available. Clusters must be based on knowledge hubs of international excellence and market foresight. Indeed, the development of groups of experts should not only be based on a greater allocation of funds but also through a clearer recognition that an intensification of economic globalisation and the rise of new global players require strong international partnerships.

The European funding organisations should provide financial support for the creation of a dedicated and sustainable **Institute of Mathematics for Innovation** through a world-class **European virtual infrastructure** to act as the basis of sciences and innovation. It would involve Academia, Researchers and Industry across Europe and beyond. Additional funding could be provided by national governments and agencies. **Such an Institute would be built from the knowledge and expertise of industrial mathematicians to access the best experts, databases, libraries and facilities for mathematicians and users of mathematics. It would provide mechanisms for exchange and dissemination of knowledge and be a unified channel for international funding in the area of industrial**

22. http://www.excellence-initiative.com/



and applied mathematics. The EIMI should include fundamental, industrial, and computational mathematics in order to support a sufficient number of researchers and activities, and should offer companies a one-stop-shop for collaborations.

Indeed, the strength of European higher education and research does not lay only in excellence of a few institutions but also in the high quality of the majority of institutions. There should be a closer coordination between the EU and Member States, for mutual benefit, with a more coordinated use of all available instruments, through better strategic planning and the developing of new working methods and procedures. The creation of a coordinated and coherent virtual structure in mathematics and its applications is therefore crucial for Europe's competitiveness and impact as it is done for example in Canada through MITACS[23], Australia with MASCOS24, or in the UK with the Industrial Mathematics Knowledge Transfer Network25. In addition to universities and research centres, private partners should constitute the core of the partnership within the European virtual infrastructure. The European Institute of Mathematics for Innovation would be particularly instrumental for SMEs that are seen as a major source of future job creation in Europe and will be central to how member states emerge from the ongoing financial crisis. Moreover, small companies face major challenges when attempting to tap into European funding programmes, given the time and financial resources required to apply. A European Institute of Mathematics for Innovation would provide a fertile environment in which SMEs can access research infrastructure available within universities. In addition, it would also allow research institutes to tap into the dynamism offered by small firms and start-ups.

Roadmap implementation 1.2:

EU and European governments should set up a Strategy Taskforce for Innovation and Mathematics (STIM) in order to develop a European strategy for mathematics

In order to support the creation of a political, technological and administrative framework for a coordinated and efficient use of distributed resources in industrial mathematics, we recommend the creation of a strategy task force to define and recommend best practices and major initiatives across Europe. It would include official government delegates and key representatives from the private sector from all the EU countries. The Strategy Taskforce for Innovation and Mathematics (STIM) would produce white papers, roadmaps and recommendations, and analyses the future foundations of the European Knowledge Society and the added value of industrial mathematics to address Global Grand Challenges. The STIM would also coordinate activities with similar international initiatives. Europe's competitiveness is to

23. www.mitacs.math.ca
24. www.complex.org.au
25. www.innovateuk.org.mathsktn

be achieved in a multilateral international environment through a common strategy for European Industrial mathematics.

Roadmap implementation 1.3:

Policy makers should put in place a Small Business Act in Mathematics (SBAM) to encourage spin-off companies explicitly using mathematics

Europe should establish an industrial policy creating the best environment to maintain and develop a strong, competitive and diversified industrial base in Europe. National and International supports for the creation of spin-off companies using mathematics, which have shown to have very high success rates in Germany and Sweden for example, should be provided, taking into account the timescale of the market dynamics for Small and Medium Enterprises (SMEs). Standardisation, confidentiality and Intellectual Property regulations should be disseminated in the Community through the Small Business Act in Mathematics.

The main objectives of the Small Business Act in Mathematics should include:

- Simplifying the conditions and providing incentives to involve mathematics and mathematicians in innovative businesses, improving the entrepreneurial environment and reducing the administrative burden,
- Simplifying the access of SMEs to the EU' s pool of industrial mathematicians,
- Simplifying the access of SMEs using mathematics to investment funding,
- Openly fostering industrial mathematics engineers at the interface between SMEs and academia.
- Giving specific guidelines for possible forms of user-friendly interaction.
- Suggesting some indicators for the evaluation of the success of the collaboration

Roadmap implementation 1.4:

EU must identify industrial and applied mathematics as an independent crosscutting priority for the Framework Programme 8

Research is an area where Member States have the main responsibility, hence the important differences of research policies across Europe. The European Union's role in this area is essentially that of establishing a framework encouraging more public and private investment in research activities, as well as the direct funding of research activities through the Framework Programme. Mathematics should be an essential priority of the future Framework Programme 8 from the European Commission. If the Lisbon strategy is to create a knowledge-based economy and to raise the educational level of society, then Europe has to invest more in the key and fundamental area of mathematics. A dedicated call from the European Commission should also be created for research and development activities in the domain of industrial and applied mathematics.

4.2. Recommendations to both academia and industry

Recommendation 2: In order to overcome geographical and scientific fragmentation, academic institutions and industry must share and disseminate best practises across Europe and disciplines via networks and digital means.

Roadmap implementation 2.1:

Researchers in academia and industry must adapt their mentalities to the different mathematical and scientific domains they interact with, and disseminate best practises

It is crucial for mathematicians in industry and even more in academia to change and adapt their view regarding other fields of mathematics and other scientific domains. Indeed, the mathematical community needs a stronger cohesion and unity. In this context, a wider and more frequent dialogue between fields is essential and can be achieved through a federated structure such as the European Institute of Mathematics for Innovation (EIMI) suggested previously. Mathematicians could also make more regular use of existing research infrastructure such as High Performance Computing centres in which they could play a role. Moreover, sharing best practices, and also unsuccessful investigations, across Europe will help to level out geographical and economical differences between Member States. Web access to information about problems, methods, solutions, centres of excellence, and available expertise especially for multi-disciplinary approaches, will be particularly useful for companies. Moreover, mathematics has an important role in education and permanent training for companies and society. The community should develop educational documents and electronic tools to disseminate knowledge about success and failure, and assess the quality of information in the field of industrial and applied mathematics. The use of open databases of experts, expertise, and problems and of the corresponding search tools should be created to allow users to identify the

best available partners and methods for their specific problems. It would allow publicly funded research to be made available to society at large.

Roadmap implementation 2.2:

The mathematical community in collaboration with industry should create a journal devoted to industrial mathematics and contribute to the European Digital Mathematics Library

A Journal on mathematics in industry would raise the visibility of the community. It would bring together high level research works on the developments of applications of mathematics to industrial problems, including both methods and computational challenges they encompass. It would show the actual use of mathematics in industry, through the improvement of industrial processes and major challenges deriving from cost and ecological issues. By publishing peer reviewed papers of high quality and originality, it would serve as an essential resource for academic researchers and practitioners alike, and would provide a common platform for scholars interested in the mathematics required and specifically motivated by concrete industrial applications, thus favouring the interaction of academia and industry. The journal would be set-up with a worldwide editorial board consisting of scientists in industry, academia and contract research organisations.

In the light of mathematicians' reliance on their discipline's rich published heritage and the key role of mathematics in enabling other scientific disciplines, the European Digital Mathematics Library has strived to make the significant corpus of mathematics scholarship published in Europe available online, in the form of an authoritative and enduring digital collection, developed and curated by a network of institutions. Mathematicians from academia and industry should contribute actively to this collective effort in constructing the Europe-wide interconnections between their collections to create a document network as integrated and trans-national as the discipline of mathematics itself, through secure and open access to their texts.

Roadmap implementation 2.3:

Academic institutions and industry must facilitate the employment mobility between academia and companies

Industry can make its environment more attractive for qualified researchers if an acceptable compromise can be reached between the long timescale typical for mathematics research and the much shorter timescale typically faced by industry. Industry can enhance its presence in academia by actively supporting activities that potentially increase the impact of mathematics in industry. The creation of joint positions, or special chairs or faculty positions for researchers from industry should be given careful consideration and would provide real role models for students. Academia should also promote successful implementations of such flexibility and provide mechanisms to answer industrial short-term needs. Incentives should be put in place to facilitate a stronger mobility.

Connections, formal and informal, between non-academic organisations and academic mathematicians can build pathways for a two-way flow of both concepts and results. A better transfer of mathematical methods and technology into industry is needed. Societies and academic institutions should offer opportunities for mutual exchange of knowledge and workshops, where industry can come with their specific problems and where these can be addressed. Academia should adapt training to long term and future research fields in addition to the short timeframe adaptation of training for short-term needs of companies. Moreover, the challenges for mathematics arising from future industrial needs must be defined and addressed. Societies and academic institutions should organise specific "grand challenge workshops", focussed on industrial application areas with high relevance for future economic needs in Europe. Their multidisciplinary teams of experts from academia and industry should jointly identify the mathematical gaps which have to be solved in order to enable quantum leap innovations.

Short courses in continuing education as proposed by academic mathematics departments should be a privileged way to promote interaction between mathematics and industry. It would allow the academic and industrial researchers to meet and allow engineers to acquire or to upgrade a mathematical culture. It would allow the academic researchers to have immediate feedback from the industrial researchers about their needs through the questions asked during the training sessions, and would be the opportunity to establish direct contacts between academic and industrial researchers, creating opportunities for research participation at all levels. Early exposure whets the appetite, and nothing beats hands-on experience. The opportunities may include industrial internships, modelling camps, and summer schools.

Roadmap implementation 2.4:

The mathematics community and industry should work together on real opportunities in application-themed competitions

The mathematics community and industry have shown some reluctance to bid for projects within application-

based funding competitions, largely because of previous experiences where mathematics has not been considered core to the application. If funders can include mathematics explicitly then the mathematics community should respond by moving into this new territory. The mathematicians from academia and industry should use all opportunities and expertise in from other scientific fields outside mathematics to participate in dedicated proposals in application-themed competitions. The participation in dedicated calls could be done through the EIMI (See recommendation 1).

4.3. Recommendations to academia

Recommendation 3: Mathematical Societies and academic institutions should create common curricula and educational programmes in mathematics at European level taking into account local expertise and specificity.

Roadmap implementation 3.1:

Academia must create a European Curriculum for industrial mathematics and set up a pool of industrial mathematics engineers

For this crucial recommendation to overcome fragmentation, the European Master and PhD programs for mathematics and industry should be created, where students learn the necessary mathematics and software skills but where they also learn the language of practitioners e.g. engineers, medical doctors etc. Education in Master programs should provide the option of specific problem solving on industrial internships[26] with support from interdisciplinary scientists if necessary. They would give students the flexibility to explore career options outside academia. To develop curriculum options that prepare the students for a career at the interface of mathematics and industry, curricula should reflect the reality that such a career requires both a solid background in mathematics and the intellectual curiosity to go beyond mathematics. The curriculum should be flexible but subject to rigorous quality control. It should stress innovative applications of mathematics, highlight problems that are industry-driven, and encourage students to broaden their scientific interests. The curriculum should also take into account the training of schoolteachers in order to create a continuum in the dissemination of knowledge in mathematics. Above all, it should be designed to demonstrate that the interaction of mathematics and industry leads to exciting research opportunities and benefits.

26. See for example www.ktn-internships.net

Considering the need for Academia to adapt to the short timescales of companies, a pool of specialists who can quickly and adequately answer requests from industry in terms of mathematical modelling and solutions would be a valuable resource for companies to draw upon. As a consequence, new long-term professional careers of mathematical engineers should be offered by scientific organisations at national and European levels mainly funded by industry. Permanent positions should be the norm. The communication between industry and the world of mathematics should be facilitated through the creation of techno-mathematician translators such as the Technology Translators[27] at the UK's Knowledge Transfer Network, since the problems in industry are not expressed in the language of mathematics in an obvious way, and since this language itself has developed its own idioms and syntax.

Roadmap implementation 3.2:

Academia must develop new criteria to assess and recognise careers in industrial mathematics

The community should set standards for the evaluation of the dual career of industrial mathematicians between academia and industry. There should be teaching and research positions devoted to industrial and applied mathematics following educational programs built with industry. Such programs offer a first-hand experience of working in an industrial environment and thus a realistic perspective on a career in industry, while preserving the option of an academic career. The younger generation of mathematicians should also be encouraged to start spin-off companies and to move between countries and disciplines.

27. www.industrialmath.net/web/?url=About/mathsKTN/KTNManagement

Annexes

Annex 1

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Annex 2

Example of Model Master in industrial mathematics

The aim of the master programme in Industrial Mathematics is to develop the student's mathematical and computational skills to solve industrial problems and development tasks in innovative ways in collaboration with other science departments, e.g. engineering. Fundamental capabilities to be trained are:

- Modelling and analytical skills, knowledge of numerical methods, skills in programming, simulation, experience with mathematical models in industry,
- Ability to handle huge amounts of data by integrating mathematical, numerical and statistical methods,
- Team working, cooperation, communication with and presentations for mathematicians and engineers.

The master programme is international. The education is given in English when possible. The duration of the master programme is two years (120 **E**uropean **C**redit **T**ransfer and Accumulation **S**ystem or ECTS). There are three course blocks (A, B and C) which are related to both the requirements for admission to the master study and compulsory courses of the master programme.

Requirements for admission to the master programme

(A) 180 ECTS of undergraduate study at university level (bachelor degree), (B) Prerequisite courses covering the topics in course block A, (C) Prerequisite courses covering topics of an amount of at least ½ of the total ECTS in block B, (D) Since the education is given in English, good communication skills in English are required both orally and in writing.

Requirements for the master degree

The following is required in the master study to receive the master degree. (A) At least 120 ECTS in total, corresponding to two years study, (B) Courses covering the remaining topics in block B (see requirements for admission), (C) At least 60 ECTS of elective courses in course block C. Among these courses there are required mathematical courses summing up to at least 42 ECTS as well as courses in a minor field of study related to mathematics of at least 12 ECTS, (D) Modelling activities of at least 9 ECTS, and a master thesis of 30 ECTS, (E) Further Courses up to 120 ECTS in total.

Additional requirements

In addition to the above requirements for the master degree in industrial mathematics the following is required (1) at least one semester of the study or the final master thesis project at one of the other universities of the network, (2) practical training in mathematical modelling, organised in a regular modelling seminar, Workload: 6 – 15 ECTS, (3) Participation in the International Modelling Week (3 ECTS), (4) Mathematical courses in Scientific Computing, Optimisation and Mathematical Statistics. Each student's qualification is finally assessed on the basis of two review reports of the thesis.

Study blocks

- **Block A:** Prerequisites
  (for admission to the Master programme)

  Required topics:

Basic knowledge in calculus	18 ECTS
Linear Algebra	6 ECTS
Some basics in numerical analysis	6 ECTS
Programming skills	6 ECTS
Basic knowledge in physics or mechanics	6 ECTS
Basics in statistics	6 ECTS
Total amount of courses	48 ECTS

- **Block B:** At least ½ of the listed topics are required for admission.
  The remaining topics are required during the Master study.

  Required Topics:

Transforms, linear systems, basics in ODE	6 ECTS
Basics in PDE	6 ECTS
Numerics for differential equations	6 ECTS
Minor field of study related to mathematics	6 ECTS
Total amount of courses	at least 24 ECTS

- **Block C:** Elective courses in the field of industrial mathematics.
  Every student can choose among offered specialisation courses at her/his home university or at an ECMI partner university.

  Requirements:

Total amount of courses in block C, among these courses:	at least 60 ECTS
Mathematical courses	at least 42 ECTS
Courses in a minor field of study related to mathematics	at least 12 ECTS

- **Modelling activities**

Modelling activities (e.g. modelling seminar, summer school, study groups, industrial project, internships etc.)	at least 9 ECTS

Annex 2

Example of Model Master in industrial mathematics.

- **Master thesis:** The thesis should be related to a real industrial problem. It could preferably be carried out in an interdisciplinary environment involving participants from industry.

  30 ECTS

Total ECTS for Master degree:
12 – x (Block B) + 60 (Block C) + 9 (Modelling) + 30 (Thesis) + 9 + x (free) = 120 ($0 \leq x \leq 12$)

Details on the topics in block B

Transforms, linear systems, and basics in ODE: (examples of topics)
- Fourier series, Fourier and Laplace transforms
- Systems of linear differential equations (state space theory, diagonalisation, stability)
- Quadratic forms and Input-output relations (transfer function, impulse response)

Basics in PDE: (examples of topics)
- First order PDE, Linear second order PDE
- Series expansions, Fourier's method, Green function
- Wave propagation, Functions spaces and norms, Distributions

Numerics for differential equations: (examples of topics)
- Methods for time integration, Finite difference methods
- Explicit and implicit Runge-Kutta, Multistep methods
- Error analysis, stability and convergence
- Object-oriented programming

Annex 3

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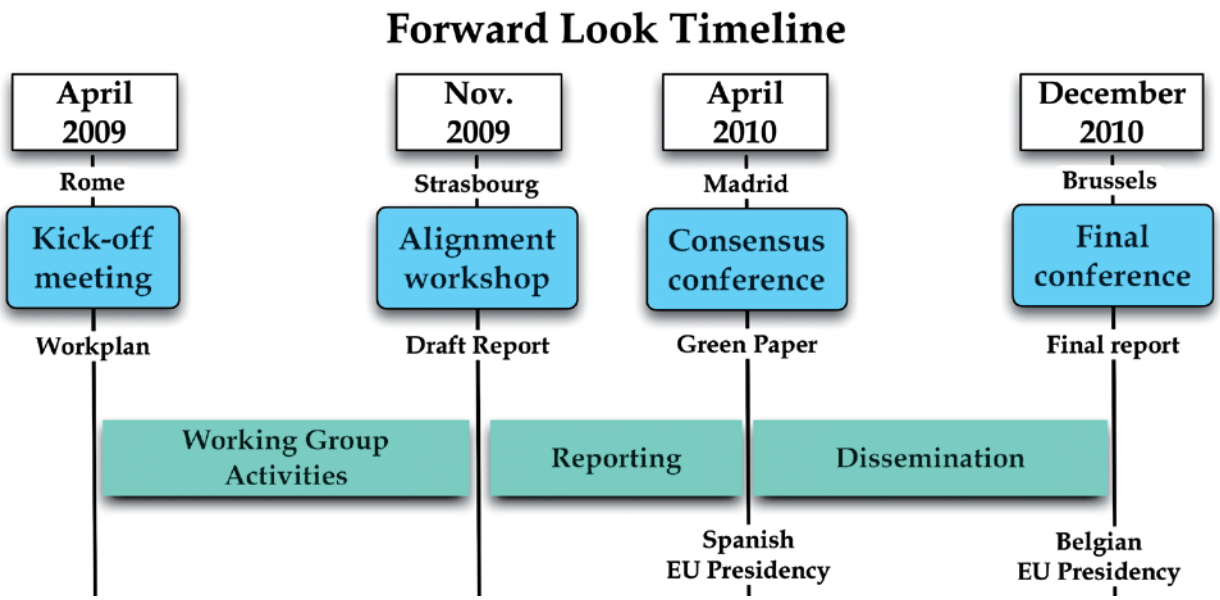
Activities of the Forward Look



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