Scientific report on the project: Internal Diffusion Limited Aggregation for non-centered random walks (Program RDSES, reference 554)

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1 Purpose of the visit

The Internal Diffusion Limited Aggregation was first introduced by Diaconis and Fulton in 1991. It is a discrete growth process defined on an infinite set associated to a Markov chain on this set. On $\mathbb{Z}^d$, for a centered random walk (with moments conditions) the shape of the process tends (up to a suitable renormalisation) to the unit ball of $\mathbb{R}^d$ with a norm depending on the transition law of the random walk.

Up to now, almost nothing is known when the random walk has a drift (except on $\mathbb{Z}$). The idea of the project is to study this process for non-centered random walks on $\mathbb{Z}^d$. Computer simulations show that the evolution of the shape of the process could be related to the the asymptotic behavior of the generalized Green function. Our goal is to understand this link and to derive precise results on that evolution.

2 Work carried out during the visit

- Improvement and completion of the work, with Sara Brofferio, on Internal DLA on groups with exponential growth.

- Begining of the project about Internal DLA associated to random walks with drift, with a preliminary study in the case of homogeneous trees.

3 Description of the main results obtained

Internal DLA on groups with exponential growth, without drift:

- A limiting shape theorem by comparison with balls for a new distance (that we called "hitting distance") related to the Green function;

- Upper bounds for the fluctuations around that limiting shape (logarithmic in the radius for the inner fluctuations and like the square root of the radius for the outer fluctuations);

- The sharpness of these upper bounds in the case of the simple random walk on the homogeneous trees.

Internal DLA with drift:

- A first tecnical result linking the generalized Green function and the shape of the internal DLA on an homogeneous tree associated to the nearest neighbors random walk with a drift in one direction;

- the order of the size of the cluster in the direction of the drift (number of particles divided by its logarithm).

4 Future collaboration with host institution

Continuation of the project about Internal DLA on graphs with a drift for the associated random walk.

5 Projected publications

- The article "Internal DLA on discrete groups with exponential growth" (with Sara Brofferio) is closed to finish and will be submitted within one month.

- A future article should contain shape results about Internal DLA for random walk with drift, if the tecnics we used throught the generalized Green function happens to be successful beyond the homogeneous trees.