Phase Transitions and Fluctuation Phenomena for Random Dynamics in Spatially Extended Systems (RDSES)

An ESF scientific programme

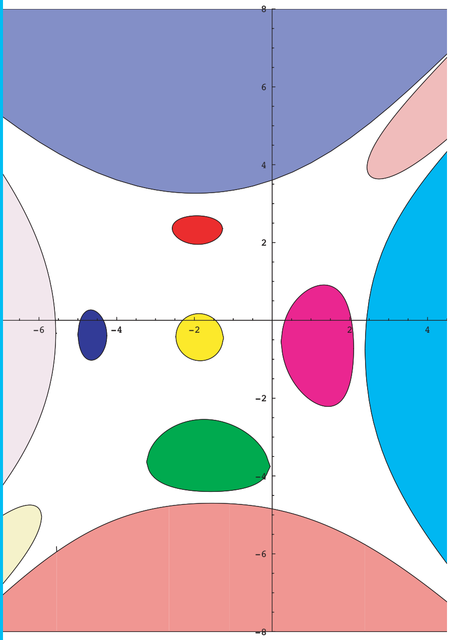
Mathematical statistical physics is going through a phase of rapid and exciting development. Spatially extended systems, consisting of a large number of mutually interacting components, exhibit intriguing phenomena, many of which are currently beginning to be understood at the mathematical level through the application of powerful probabilistic techniques.

Points of departure for the mathematician are model systems subject to a random dynamics acting on the components of the system. This microscopic dynamics defines a transformation on the phase space of the system, which typically is highly complex. The important challenge is to give a precise mathematical treatment of the interesting physics that arises from this complexity. The macroscopic behaviour of the system is the result of an appropriate space-time rescaling in combination with probabilistic limit theorems.

The main goal of this programme is to bring together the core of the researchers in Europe that are active in this area of mathematical statistical physics. Researchers are widely scattered and work in small groups, but European contributions have been extensive and of high quality. The network activities will help to strengthen the ties between the various national groups and to make the developments more accessible for young researchers.

The programme is unique in bringing together mathematical physicists and probabilists in Europe working in statistical physics and stochastic modelling of natural phenomena. Mathematical statistical physics is widely known to foster inter-disciplinary approaches and to provide mathematical expertise and training in analysing and modelling complex dynamical processes. Mathematical statistical physics continues to have a major impact on large parts of mathematics and physics. Ideas from the subject areas covered by the programme are beginning to play a crucial role in the mathematisation of computer science, biology, economics and communication technology as well.

The running period of the ESF scientific programme RDSES is January 2002 – December 2006.



The European Science Foundation acts as a catalyst for the development of science by bringing together leading scientists and funding agencies to debate, plan and implement pan-European initiatives.

Scientific background

In spatially extended systems the components interact with each other and with their environment locally, but this can lead to a global dependence, resulting in anomalous fluctuation phenomena and phase transitions. The RDSES programme is concerned with aspects of this behaviour both in and out of equilibrium. The concept of entropy is essential in connecting the microscopic dynamics with the macroscopic phenomena. This needs a mathematical understanding of the interplay between different space-time scales, through renormalisation methods in combination with multi-scale perturbation techniques.

The programme centres around the following research topics:

- Gibbsian versus non-Gibbsian spin systems
- Polymers and self-interacting random processes
- Interfaces and surface phenomena
- Disordered media
- Relaxation to equilibrium and metastability
- Hydrodynamic behaviour of conservative systems
- Entropy production and fluctuations far from equilibrium
- Granular media and sandpile dynamics

Gibbsian versus non-Gibbsian spin systems

In many domains of statistical mechanics a description of the state of a thermodynamic system in terms of an effective Hamiltonian is employed, either explicitly or implicitly. Renormalisation-group theory is a prime example. It has been found that a description in terms of Gibbs measures is often impossible in circumstances where this was not expected. Part of the Gibbs theory survives in certain examples, either by making use of the notion of "weak Gibbsianness" or by analysing large deviation properties and associated entropy functions. Despite the existence of many important partial results, a general description is still lacking and good criteria for what is and what is not possible are unknown.

Work is needed to come to a Gibbsian characterisation of many-particle systems in a non-equilibrium state, for instance, in a quasi-equilibrium state that survives over extremely large time scales ("metastability"). A milestone would be to find a numerically observable criterion for transitions between Gibbsian and non-Gibbsian behaviour. Such transitions have recently been found to occur in Ising systems under a stochastic spin-flip dynamics.

Models with a "hard core" interaction do not fit into a Gibbsian description. Examples are dimer models, where dumbbell-shaped molecules cover a surface. Long-range interactions are typical for such models and lead to complex phase diagrams *(see Figure 1 – cover page, and caption p. 8)*.

Polymers and self-interacting random processes

Polymers and self-interacting random processes are important in the physical, chemical and engineering sciences. Many of the problems in the description of these systems are due to long-range interactions (such as excluded-volume-effects) and are mathematically very challenging. Examples are: obtaining bounds for the end-to-end distance in two- and three-dimensional polymers, understanding the nature of the collapse transition for polymers with mixed attractive and repulsive interaction, and describing the behaviour of hetero-polymers near wiggly interfaces.

There are spectacular developments in two dimensions, based on conformal invariance and the Stochastic Löwner Evolution. Key successes are the derivation of the intersection exponents of Brownian motion and the critical exponents for loop-erased random walk and percolation. Perspectives are opening up for solving long-standing open problems, like identifying the scaling limit of two-dimensional self-avoiding walk and constructing self-repelling random motion in continuous space and time, leading to new types of stochastic differential equations.

Related work concerns the identification, through the lace expansion, of the scaling limit for high-dimensional critical percolation in terms of super-Brownian motion *(see Figure 2)*. Another major challenge is to decide transience versus recurrence for high-dimensional reinforced random walk.

Interfaces and surface phenomena

Interfaces and surface phenomena are of fundamental importance in spatially extended systems. They arise from inhomogeneous or unstable initial conditions or from geometric constraints, typically in combination with conservation laws. Of crucial interest are wetting phenomena and droplet growth. Here, remarkable recent developments have been the discovery of a two-dimensional wetting transition and the construction of the three-dimensional Ising-spin droplet and lattice-gas droplet *(see Figure 3)*.

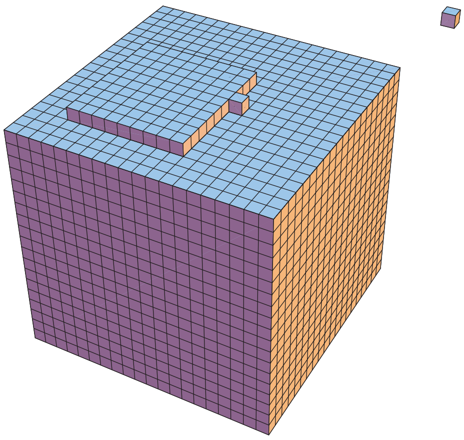


Figure 3: In a three-dimensional lattice gas with Kawasaki dynamics, particles perform independent random walks with two restrictions: no two particles can occupy the same site (exclusion); two neighbouring particles have a tendency to stick together (attraction).
In the limit of low temperature and low density the gas will take a long time to condensate. The threshold for condensation is the formation of a critical droplet that is large enough to absorb other particles rather than to evaporate. Attached to one side of the three-dimensional critical droplet appears a critical droplet for the two-dimensional lattice gas with Kawasaki dynamics.
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Other challenging problems arise from the interaction of random impurities with an interface, for instance, hetero-polymers near an oil-water interface. Of primary importance are further investigations on the nature of various associated phase transitions. Much is known in the deterministic case, like the behaviour of fronts in partial differential equations. But the random case, which is essential at the microscopic and the mesoscopic scale, still needs ripening, with notable results so far for Ising and solid-on-solid models in equilibrium.

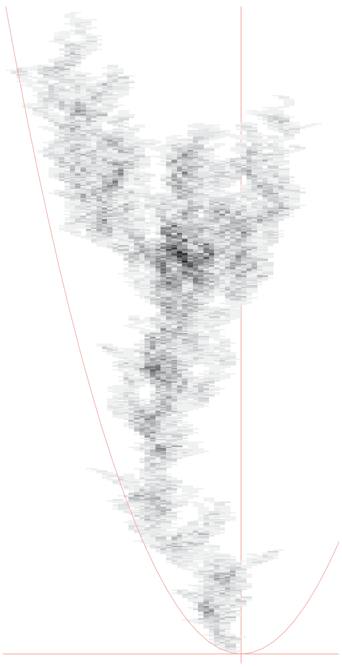

Figure 2: The figure shows the evolution of a random mass distribution, with time running vertically and space running horizontally. Darker shadings represent higher mass, and the parabola shows the spatial scaling.
The mass distribution arises from an embedding of a large critical branching tree. A superposition of all the mass onto the spatial axis produces a random mass distribution whose total mass is a random variable, say K. This superposition, with mass rescaled by $1/K$ and space rescaled by $1/K^{1/4}$, has the same distribution as "Integrated Super-Brownian Excursion".
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A unification of different problems in statistical physics is envisioned through the study of equilibrium interface fluctuations at the microscopic scale. Surprisingly, various quite different two-dimensional models of one-dimensional interfaces can be mapped onto a single class of mathematical models. Examples come from percolation theory, transport theory, quantum theory, random matrix theory, and combinatoric studies in computer science. A new universality is showing up in the scaling behaviour, with the Wigner semi-circle distribution and the Tracy-Widom distribution appearing as attractors.

Disordered media

The application of statistical mechanics to disordered systems (such as amorphous materials, biomolecules, and neural networks) has proven to be a mathematically enormously challenging problem. For instance, controversies about the nature of the phase structure in realistic spin-glass models are caused, in part, by the tremendous difficulties that come up when doing laboratory experiments or numerical simulations. These difficulties are typically provoked by long-range interactions, causing a complex "free-energy landscape" that gives rise to intricate transients and slow relaxation to equilibrium. In this situation heuristic methods often fail to give conclusive results and rigorous mathematical methods are pivotal. The fact that these methods are effective was demonstrated, for instance, in the resolution of the controversy around the lower-critical dimension for the random-field Ising model.

One of the most challenging problems at present is to understand the phenomenon of replica symmetry breaking in realistic spin-glass models and to investigate dynamical aspects of complex disordered systems, in particular, the phenomenon of aging. There has been dramatic progress in the understanding of mean field spin-glasses over the last few years, and a universal random mechanism based on continuous-state branching processes emerges as a candidate for explaining the asymptotic fluctuations in these systems. Still, a rigorous proof of this connection for the Sherrington-Kirkpatrick model remains the benchmark problem in the field. Based on the progress here, as well as on some new tools developed for the analysis of metastability, a number of rigorous results on aging in the stochastic dynamics of spin-glass models are also emerging. These issues are particularly important, as it was found recently that some key problems in computer science, artificial intelligence, biology, economics and communication technology can be mapped onto problems of spin-glass type. Examples are the turbo-codes, which are spectacularly successful but are mathematically ill understood.

Closely related problems concern the behaviour of random walks in random environments, where the last years have brought impressive progress, in particular, in high dimensions. Here, large deviation theory has again been the driving force. Other hard open problems are proving mixing properties for hard-ball dynamics and related properties for diffusion in random media, where spectacular progress has been made in the past decade, but which is mathematically full of pitfalls.

Relaxation to equilibrium and metastability

The problem of relaxation to equilibrium goes back a very long time, but even today detailed mathematical results are missing, most notably for conservative dynamics. The simplest model is that of a lattice gas dynamics reversible with respect to the canonical Gibbs measure. Although in recent years substantial progress has been made in the analysis of the relaxation in this model, many problems remain open, like determining the shape of large growing droplets. One of the goals is the extension of the "diffusive scaling of the relaxation time", known to be valid at high temperature, to the whole one-phase region. The major obstacle here is the fact that, due to fluctuations, there are regions in the gas where the local particle density has anomalous values that lie in the phase-coexistence interval. Thus, there is a need to study how anomalous local fluctuations in the density profile relax away.

Such a problem is also encountered in the study of high-temperature continuous systems, in low-temperature plane rotators with a Glauber dynamics, and in dynamics of solid-on-solid surfaces. Its solution would open up the door to several important developments. Other related problems of current interest come from the theory of turbulence and the spatial-temporal relaxation of driven stochastic Navier-Stokes equations.

In the last decade there has been a remarkable development in the theoretical study of metastability and nucleation. Following the "pathwise approach" to metastability, it has proved to be possible to obtain sharp asymptotics for the relaxation time from the metastable to the stable state and to clarify the mechanisms of transition in the case of the two- and three-dimensional stochastic Ising model at low temperature. A milestone would be the extension of these results to conservative dynamics. Conservation laws cause long-range dependence and therefore change the relaxation pattern. An important goal is the description of the relaxation from a metastable state in a large volume, in particular, the growth and the motion of supercritical droplets and the depletion of the supersaturated vapour around them.

Another major challenge is to understand catalytic and mutually catalytic interacting particle systems, where particles of one type evolve in a way that depends on the presence of particles of another type, and vice versa *(see Figure 4).* Here, slow transient behaviour and clustering phenomena play a central role.

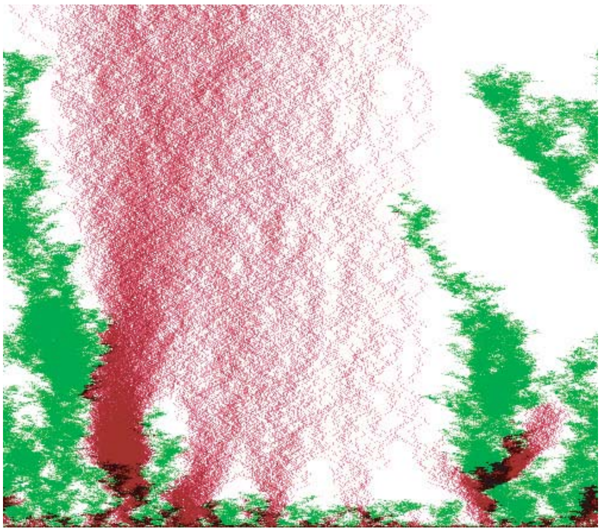

Figure 4: The figure shows a computer simulation of a catalytic branching random walk. Initially there are 2000 green particles and 2000 red particles, placed on an array of 500 sites with a periodic boundary condition that turns the array into a loop. Time runs over 750 units. The evolution is as follows: at rate 1, green particles move to the left, move to the right, or stand still and split into 0 or 2 new green particles (each with probability 1/2); red particles do the same, except that the splitting occurs at a rate proportional to the number of green particles present at the same site. Thus, the green particles act as catalyst for the red particles. Without the green particles, the red particles would perform independent random walks at rate 1. © Achim Klenke (Cologne)

Hydrodynamic behaviour of conservative systems

One of the basic problems of non-equilibrium statistical mechanics is the derivation of hydrodynamic equations. On the proper space-time scales, interacting particle systems develop an autonomous behaviour for a collection of conserved quantities, such as energy, momentum and density. Progress has been fast over the past decade, due to new probabilistic and analytic techniques. It is believed that applications of these techniques will help to clarify a number of phenomena that are currently still in the dark. Particularly urgent here are a definite mathematical discussion of multi-component systems of hyperbolic conservation laws, which typically arise in the hydrodynamic limit for systems of particles of different types and which are notoriously difficult to analyse.

Another task is to investigate the diffusive behaviour of non-reversible interacting particle systems (like the asymmetric simple exclusion process, interacting Ornstein-Uhlenbeck processes, and Hamiltonian systems with noise), in particular, to solve the bulk- and the self-diffusion-problem. More effort is needed to investigate systems for which the invariant measure is not known explicitly, as is the case for the majority of transport problems.

A detailed analysis of the hydrodynamic behaviour of lattice gases under a conservative dynamics can be carried out in the regime where phase segregation takes place. Here the main goal is to analyse the motion of the Wulff droplet of the minority phase in a large volume with suitable boundary conditions and to obtain bounds for its diffusion constant.

Entropy production and fluctuations far from equilibrium

There has been a recent burst of activity in the statistical mechanics of steady-state non-equilibrium systems. Inspiration comes from new ideas in the theory of dynamical systems and from recent extensive numerical work employing so-called thermostated dynamics. The area is witnessing the slow emergence of a microscopic theory, from which not only the thermodynamics of irreversible processes close to equilibrium can be derived, but which promises to go far beyond the linear regime. A central role is played by the notion of entropy production. In the phenomenological theory this quantity appears as the dissipated heat in a driven non-equilibrium system.

Since the recent work of Cohen and Gallavotti, a symmetry has been established for the fluctuations in steady-state non-equilibrium systems, the consequences of which are thought to be similar to those of the Ward identities in field theory. This novel development promises to profoundly affect the area of non-equilibrium statistical physics, which has a much less solid mathematical foundation than equilibrium statistical physics.

In a recent approach, based on a Gibbsian hypothesis, the space-time distribution of the particle trajectories are Gibbsian and the entropy production is identified with that part of the space-time action functional that breaks the time-reversal invariance. In this setting, the fluctuation symmetries turn out to be an immediate consequence of the Dobrushin-Lanford-Ruelle conditions characterising Gibbsianness. The most immediate applications lie in the theory of non-reversible interacting particle systems, where there is now good hope to establish local fluctuation theorems. This will represent important progress, because the resulting symmetry laws are expected to be very general. Further input is needed from the theory of large deviations for spatially extended stochastic dynamics.

Granular media and sandpile dynamics

Granular media have become an active domain of research over the last few years. Both computer simulations and laboratory experiments have revealed a wealth of new phenomena and often unexpected behaviour from the traditional point of view of kinetic theory and soft condensed matter. Mathematical work is far behind. It suffices to compare the simulation work on sandpile dynamics with what is known rigorously, to appreciate that a bundling of the expertise present in the current programme is badly needed to give this topic a proper mathematical impetus.

A key problem is that of the thermodynamic limit of sandpile dynamics. The major obstacle here is to beat the strong long-range dependence in the dynamics, which make the role of boundary conditions and finite-volume effects unclear. A milestone would be to obtain a rigorous derivation of some of the basic features of the infinite-volume standard abelian sandpile process in high dimensions, such as anomalous scaling, non-Gaussian fluctuations and “self-organised criticality”.

Activities

The activities of the RDSES programme are:

- **Workshops**
  Each year, two thematic workshops will be organised (20-30 participants) and one brainstorm meeting (10 participants). The topics are selected by the Steering Committee. Suggestions are welcome and should be directed to the chair, Frank den Hollander.

- **Exchange visits**
  A visitor exchange programme is active for visits of up to two weeks, with an electronic open call four times a year. Applications should be directed to the ESF administrator of the programme, Catherine Werner. The deadlines for application are: 1 January, 1 April, 1 July, 1 October.

- **Summer schools**
  Three summer schools are planned during the running period of the programme; one large summer school (in the Summer of 2005) and two small summer schools (one in the Spring of 2003).

- **Website**
  A homepage is maintained on the ESF website, where all the relevant information can be found.

- **Newsletter**
  Twice per year, a newsletter will be sent around, starting in 2003.

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For the latest information on this programme consult the *RDSES* home page: www.esf.org/rdses

Figure 1 (cover page):
In the dimer model (or domino-tiling model) one considers the coverings of a two-dimensional lattice with dimers that cover pairs of adjacent sites. When the energy assigned to the dimers is spatially periodic, the phase diagram can be computed rigorously as a function of a two-component magnetic field. In the phase diagram there are 12 distinct phases: 6 outer phases (unbounded coloured regions), which are "frozen" and have no fluctuations; 5 inner phases (bounded coloured regions), which are "gaseous" and display exponential decay of correlations; 1 central phase (white region), which is "liquid" and displays polynomial decay of correlations.
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