Scientific report

The purpose of my visit to Potsdam was to study two specific problems, that have been described in some details in the *Research Project*

A Cluster expansion approach to risk-sensitive control, in collaboration with Prof. Sylvie Roelly.

The main object of our research has been to develop tools for dealing with the following problem. Let $(x_t)_{t\geq 0}$ be a diffusion in $\mathbb{R}^d$ given as solution of the SDE

$$dx_t = b(x_t)dt + dB_t,$$

and let $c : \mathbb{R}^d \to \mathbb{R}$ be a measurable function. The aim is to prove the existence of a constant λ and a function $V(x)$ such that

$$\lim_{t\to+\infty} \log E_x \left[\exp\left(\gamma \int_0^t c(x_s)ds\right)\right] - \lambda t = V(x).$$

This has to be view as a first step in proving the existence of the value function in a risk-sensitive control problem in continuous time. We have observed that the function $V(x)$ can be in principle obtained via a *cluster expansion*, provided such expansion converges. If one assumes linear growth of $b(x)$ and at most quadratic growth of $c(x)$, convergence of the expansion follows by fast ergodicity of the diffusion and smallness of the *risk parameter* γ. The type of fast ergodicity needed here is a rather non-standard one, and can be summarized as follows:

- the diffusion has a unique invariant measure ν with at least Gaussian tails;
- if $p(t, x, y)$ is the transition density of the diffusion with respect to its invariant measure ν, then
$$|\log p(t, x, y)| \leq \epsilon(t)[|x|^2 + |y|^2 + 1],$$
where $\epsilon(t) \to 0$ as $t \to +\infty$.

Using also ideas of Prof. P. Cattiaux, that has been involved in the project, we have given sufficient conditions for such convergence to take place, and proved that the corresponding cluster expansion giving $V(x)$ and λ converges. A publication containing these results is presently in preparation. Next step in the research will be the extension on these results to the case in which the drift $b(x)$ depends on a control.

The monotonicity problem for continuous-time Markov Chains, in collaboration with Pierre-Yves Louis and Ida Minelli.

As specified in the Research Project, our aim was to study the so-called *monotonicity equivalence* problem, i.e. determining the partially ordered finite sets for which all monotone (continuous-time) Markov Chains can be realized as monotone *random dynamical systems*. Our results can be summarized as follows.

- Whenever equivalence holds for discrete-time Markov chains than it holds in continuous-time too.
- We provided several examples of partially ordered sets where equivalence holds in continuous time but fails in discrete-time.
- We provided several examples of partially ordered sets where equivalence fails in continuous time. We also gave several sufficient conditions for the failure of monotonicity equivalence.
- We gave a complete classification of partially ordered sets with less that seven points in terms of monotonicity equivalence.

Most results have already been gathered in the short note: P. Dai Pra, P.Y. Louis, I. Minelli, *The monotonicity equivalence for continuous-time Markov chains*, that has in these days been accepted in Comptes Rendus Acad. Science, Paris. A more extended article is in prepaparation.