

Report on the visit to Groningen (June 19–23, 2006)

With Prof. Aernout van Enter we discussed two issues:

1) Analogue of non-Gibbsianness in quantum statistical mechanics.

Kozlov theorem tells us that, in classical statistical mechanics, non-Gibbsianness corresponds to the existence of a specification (regular system of finite-volume conditional probabilities) with kernels that are uniformly non-null (cylinders have non-zero measure uniformly on the exterior condition) and quasilocal (i.e., uniform continuity with respect to the external configuration). We wanted to determine the analogue of these properties in the quantum setting. Our ultimate goal would be to prove a quantum version of Kozlov's theorem for KMS states.

We focused on the detection of the analogue of quasilocality. To this end, we started to study some published examples of non-KMS states (Matsui and Ogata, Asbacher and Pillet). We also considered papers on the MacLennan-Zubarev ensembles (Tasaki and Matsui, Tasaki and Takahashi) that claim that KMS states are generic. At this point, we were unable to obtain a definite criterium. Non-KMS properties are derived from the failure to satisfy a variational property. In the classical case, this failure is more related with lack of non-nullness than of quasilocality. We would also like to study the hypotheses under which KMS states are proven to be generic. We suspect they are too strong for our general purposes.

2) Non-Gibbsianness of effective classical models.

The Falikov-Kimball limit of the Hubbard model includes both classical and quantum degrees of freedom. A standard approach to its study has been the integration of the quantum degrees of freedom so to obtain an effective classical interaction on the remaining variables. Following a question of Joel Lebowitz, we discussed the possibility of arriving to non-Gibbsian models as a result of the quantum integration. In the case of bosonic quantum particles, the classical ions should present this phenomenon if for some ionic configuration, the conditioned bosonic system exhibits a Bose-Einstein condensation. The resulting non-diagonal long-range order should act as a hidden mechanism connecting occupational probabilities of far away ions. This off-diagonal order is equivalent to the long-range magnetic order in the XY model (Aizenman et al). Therefore, the rigorous proof of this non-Gibbsianness would involve a perturbative version of the results of Aizenman et al, proving Bose-Einstein condensation of Bose-Hubbard models with zero chemical potential. Unfortunately, the argument of Aizenman et al is a rigid symmetry argument (based on reflection positivity) that does not admit a perturbative extension.

A simpler version of the same phenomenon could involve the integration of some degrees of freedom on a purely classical system with two types of particles or of random variables. This is probably more feasible, and we plan to look to the work of Chayes, Kotecky and Shlosman.

In addition, with Prof. C. Kulske we discussed the following two topics:

A) Rounding off of phase transitions in models of continuum spins. This is work in progress with a doctoral student, involving arguments based on, but simpler than, those in the paper by Aizenman and Wehr.

B) Convergence of finite-range non-Gibbsianness to mean-field non-Gibbsianness. The later has been the object of a recent paper by Kulske and Le Ny. We project to discuss

the convergences of decimated (non-Gibbsian) measures in two instances: finite-range to Kac models, and Ising models to mean-field as the dimensionality diverges.

Finally, I delivered a conference at the physics Department entitled "Mott transitions in lattice boson models"

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