ESF - Short Visit Grant n°1068

Phase Transitions and Fluctuation Phenomena for Random Dynamics in Spatially Extended Systems

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Report on the visit :

Study of equilibrium measures for hard spheres with infinite range interaction.

The purpose of the visit was to develop our previous works on random dynamics for infinite systems of interacting Brownian balls. We study an infinite number of interacting hard spheres with diameter $r > 0$ evolving in $\mathbb{R}^d$. The interaction φ is infinite as soon as the distance between the centers of two particles is smaller than r. A natural modelization of such a situation is the following infinite-dimensional reflected stochastic differential equation : For $i \in \mathbb{N}$, $t \in [0, 1]$,

$$dX_i(t) = dB_i(t) - \frac{1}{2}\sum_{j\neq i} \nabla\varphi(X_i(t) - X_j(t))dt + \sum_{j\neq i}(X_i(t) - X_j(t))dL_{ij}(t). \qquad (1)$$

The L_{ij} here are local times, i.e. positive continuous additive functionals which increase only when the distance between particles i and j is equal to r, and the B_i are d-dimensional independant Brownian motions.

During the visit, we concentrated our study on the behaviour of these dynamics when the underlying interaction potential φ has infinite range. The existence and uniqueness of the solutions of (1) with finite range interaction was already proved (see [1]) and we knew similar techniques also work for potentials with exponential decay. But from a physical point of view, exponential decay assumptions are too strong. So we developed a new proof and were able to construct the solution of (1) with deterministic initial conditions for potentials verifying :

$$\exists\, a, b > 0 \text{ s. t. } \forall (\xi_j)_j \text{ hard spheres configuration } \sum_{\{j:|\xi_j|>R\}} |\nabla\varphi(\xi_j)| \leq \frac{1}{R^{a(\log R)^b}}$$

This decay rate is quasi-polynomial, and much slower than exponential.

Our other concern during the visit was the characterization of the set of equilibrium measures when potential φ has infinite range. We were able to prove that this set coincides with the set of canonical Gibbs measures for potentials satisfying the above quasi-polynomial decay assumption. That is, we obtained the same conclusion as in [2], with more natural assumptions.

This joint work resulted in the preprint [3], which will be submitted for publication.

We are looking forward for further collaboration and we are orienting our work into new investigations about hard core processes : How can we compare the different hard core processes, e.g. their mean densities ?

References :

[1] M. Fradon and S. Roelly. *Infinite system of Brownian balls with interaction : The non-reversible case.* to appear in the Proceedings of the conference "Stochastic Analysis and Mathematical Finance" (Paris, june 2004) ESAIM 2006.

[2] M. Fradon and S. Roelly. *Infinite system of Brownian balls : Equilibrium measures are canonical Gibbs.* Stochastics and Dynamics, Vol. 6 No. 1 (2006) 97-122.

[3] M. Fradon and S. Roelly. *Brownian hard balls submited to an infinite range interaction with slow decay.* Preprint 2006/01, Mathematische Statistik und Wahrscheinlichkeitstheorie, Institut für Mathematik Potsdam.