Lublin October 5-th of 2005.

The report on the visit to Université Paris IX (Dauphine), CEREMADE by Tomasz Komorowski UMCS, Lublin

1. Dates of the visit: September 15 – 30, 2005.
2. Host: Professor Stefano Olla.
3. Purpose of the visit: This visit served two main purposes. First, it was meant to maintain the collaboration between me and Professor Stefano Olla in the subject of transport in random media. Our joint work in this topic has been carried on for the last five years.

Secondly, I became now a part in the project of writing a book concerning the fluctuation phenomena in statistical mechanics. Another participant, besides me and Stefano Olla is Professor Claudio Landim from IMPA Rio de Janeiro, CNRS, Univ. de Rouen. I am responsible for writing the part of the book dealing with the homogenization of diffusions in random media. We plan to finish the book by the end of the year 2006.

4. Description of the work carried out during the visit. The main results obtained. The work we have focused on is a follow up to our article concerning superdiffusive behavior of a passive tracer in an incompressible flow of a turbulent fluid, which appeared in **[KO]**. In the passive tracer model the trajectory of a particle is given by a solution of a stochastic differential equation

$$\left\{\begin{array}{l} d\mathbf{x}(t)=\mathbf{V}(\mathbf{x}(t))\,dt+\sqrt{2\kappa}\,d\mathbf{w}(t),\quad t\ge 0,\\ \mathbf{x}(0)=\mathbf{0},\end{array}\right. \tag{0.1}$$

where $\mathbf{w}(t)$ is a d-dimensional standard Brownian motion and $\mathbf{V}(\mathbf{x})$ an independent of $\mathbf{w}(\cdot)$, spatially homogeneous, zero mean random field with divergence free realizations. The main subject of interest is the long time asymptotic behavior of the tracer and in particular whether its trajectory satisfies the Central Limit Theorem (C.L.T.), i.e. whether the laws of the random variables $t^{-1/2}\mathbf{x}(t)$ converge, as $t\to+\infty$, to a normal distribution. It has been shown, see e.g. **[Os]**, that this is indeed the case when (among some other assumptions concerning the regularity of $\mathbf{V}(\mathbf{x})$) the Péclet number of the flow Pe:$=\int\mathcal{E}(d\mathbf{k})|\mathbf{k}|^{-2}$ is finite. Here, $\mathcal{E}(d\mathbf{k})$ is the power energy spectrum of the drift $\mathbf{V}(\mathbf{x})$. In **[KO]** we have considered a family of Gaussian, spatially isotropic velocity flows whose power energy spectrum has a density given by

$$\mathcal{E}(d\mathbf{k}) := \frac{a(k)\, d\mathbf{k}}{k^{2\alpha+d-2}}, \tag{0.2}$$

where $k = |\mathbf{k}|$ and $a : \mathbb{R}_+ \to \mathbb{R}_+$ is compactly supported bounded, measurable function such that $0 \in \text{supp } a(\cdot)$. We have shown that then for $\alpha \in (0,1)$ the motion of the tracer is superdiffusive, i.e. there exists an exponent $\gamma > 0$ such that $\limsup_{t \to +\infty} t^{-1-\gamma}\mathbb{E}|\mathbf{x}(t)|^2 > 0$. Here $\mathbb{E}$ denotes the mean. We note that $\alpha < 0$ implies that the Péclet number is finite, so the motion then is diffusive. The assumption $\alpha < 1$ is needed in order to avoid an infrared divergence of the power–energy spectrum.

In our present work, we consider also the case when $\alpha > 1$. This is in fact the most interesting situation from the point of view of applications to statistical hydrodynamics, because when $d \geq 3$, according to the Kolmogorov-Obukhov theory, we have $\alpha = 4/3$. Then, in order to avoid the issue of divergence of the power–energy spectrum we introduce the cut-off in the wave-number domain. We consider therefore the case $a(k) = \chi_{[\delta,1]}(k)$, where $\delta \ll 1$ and $\chi_{[\delta,1]}(k)$ is the indicator function of the interval $[\delta, 1]$. We have shown that in such a situation the motion of the tracer particle stays superdiffusive, provided that we take an appropriate scale of observations. Namely, we have proved that one can find a certain $\kappa_* > 0$ such that for any $\kappa \in (0, \kappa_*]$ there is $\gamma > 0$ for which

$$\limsup_{\delta \to 0} \delta^{\kappa(1+\gamma)}\mathbb{E}\left|\mathbf{x}\left(\frac{t}{\delta^\kappa}\right)\right|^2 > 0. \tag{0.3}$$

One can however observe that for κ big enough, e.g. $\kappa \geq 2$, the lower bounds we have obtain are insufficient to prove the statement expressed in (0.3). This could be related to the fact that by removing the correlations of the field $\mathbf{V}(\cdot)$ on the scales that are greater than δ^{-1} and focusing our observation of the particle trajectory on the much longer spatial scales (i.e. $\delta^{-\kappa/2}$) we are bound to see again the diffusive behavior of the tracer. This is a conjecture we have reached from our work so far and we wish to keep on pursuing this direction in the future. Proving such a behavior of the passive tracer motion in an isotropic divergence free model would highlight a sharp contrast of this model with the shear layer case considered in **[AM]** and **[GZ]**. Recall that then the superdiffusive behavior persists regardless of the ratio of the cut-off to observation scales.

Bibliography

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5. Future collaboration with host institution. Because our current work has not been completed during my recent stay at CEREMADE, me and Professor S. Olla plan to work on it in the near future. We hope to meet in January of 2006 in order to finish this work. Since Professor Claudio Landim will also visit at CEREMADE in January 2006 we hope that all the participants of the book project could also meet then in order to compare and unify the parts of the book we are currently working on.

6. Projected publications/articles resulting or to result from your grant. I am preparing, together with Professor S. Olla, an article under the working title *A note on the superdiffusive behavior of passive tracer in an isotropic random flow,* which will be submitted for a publication in a peer refereed journal upon its completion. The topic covered by this publication has been extensively discussed in part 4 of this report.