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Eindhoven, December 7, 2005

Scientific report

The aim of the visit was to pursue our work on "Intermittency in a catalytic random medium" in collaboration with Professors Jürgen Gärtner and Frank den Hollander.
The goal of this project is to study intermittency for the parabolic Anderson model in a time-dependent random medium. The parabolic Anderson equation is a heat equation with a spatially random source-sink term that plays the role of a catalyst. It describes the diffusion of particles that branch or die at a rate that depends on the local value of the medium.
Intermittency is the phenomenon that the successive moments of the solution of the equation at a given site (under the law of the random medium) grow at different rates. If present, intermittency implies that the solution is highly irregular, which corresponds to the particles exhibiting a strong form of clustering. The aim is to describe this irregularity both qualitatively and quantitatively, i.e., to describe the height, the shape and the location of the dominant peaks in the solution as a function of time, which monitor the evolution of the clusters.
In the literature most of the work has concentrated on the time-independent case. Very little is known for the time-dependent case. In the present project we consider a random medium consisting itself of diffusion particles, of a second type, which trigger the branching and dying of the particles of the first type in a catalytic fashion. We are interested in computing various Lyapunov exponents monitoring the growth of the successive moments. We expect a subtle dependence on dimension. In particular, we expect that the system has different behaviors in low, intermediate and high dimensions.
When the catalyst is simple exclusion with a symmetric random walk transition kernel, we have already shown that these Lyapunov exponents are trivial when the random walk is recurrent, but display an interesting dependence on the diffusion when the random walk is transient. The purpose of the visit was to characterize the shape of Lyapunov exponents, as a function of the diffusion constant, when the speed of the diffusion goes to infinity. We partially solved this problem, except for the three dimensional case, for which the problem is still opened.
The redaction of this work is still in progress and the resulting paper will be submitted soon. We also plan to carry on our collaboration to solve the three dimensional open problem and also to investigate the same model in other media.