Report for short visit ESF grant 420
by Dimitri Petritis

Purpose of the visit

DP visited Prof. Mikhail Menshikov, Department of Mathematical Science, University of Durham (UK) from 6 to 17 June 2005 to carry collaborative work on the general topic of asymptotic behaviour of Markov chains in random scenery. These Markov chains arise naturally in the description of realistic queueing models.

Description of the work carried out during the visit

We studied a queueing model of d stations and one server. Each station has its own infinite capacity queue; when the server serves one of the queues $i \in \{1, \dots, d\}$ only one additional queue $j \neq i$ receives customers beyond the queue i. While the server is at station i, it serves all customers waiting at i in a batch, even those having arrived after the beginning of the service. At the end of the service, the server instantly jumps at station j; the station i momentarily seizes receiving customers, while a new station $k \neq j$ opens to receiving customers, the choice of k being made randomly according to a stochastic $d \times d$ stochastic matrix. During each batch of services, the arrival and serving rates are defined afresh, as independent random variables, identically distributed according to a given law. These parameters as well as the random choice of the next queue constitute the random scenery of the process. By sampling the queueing system only at the times when one customer arrives or finishes to being served, it is mapped to a Markov chain with bounded jumps on a multi-sheet manifold of 2-dimensional complexes in random scenery. During the visit of DP at Durham, we studied the asymptotic behaviour of this system.

Description of the main results obtained

The main results that have been obtained concern the complete classification of the system for almost sure (with respect to the scenery) positive recurrence / null recurrence / transience. What was quite surprising for us is that contrary to the conventional wisdom, the null recurrence region is a thick set of the parameter region. More precisely, the null recurrence region of parameter space can be seen as the manifold of coexistence of two different regimes: non recurrence and non transience. Now in most known systems, both in statistical mechanics and in critical phenomena associated with stochastic processes, the null recurrence region is of zero Lebesgue measure with respect to the Lebesgue measure in dimension given by the number of parameters needed to explore the whole phase space. Here, we need two parameters to explore the whole behaviour of the system ranging from positive recurrence to transience and the region of parameters leading to almost sure null recurrence has a strictly positive 2-dimensional Lebesgue measure.

Future collaboration with host institution

It is intended to study in the future the queueing model with d stations when all sta-

tions are simultaneously open to receiving customers. The corresponding change for the sampled system is that the Markov chain evolves now in a d-dimensional complex instead of the multi-sheet 2-dimensional complex studied here. Moreover, the random scenery will no longer act through a sequence of random scalar variables but through random matrices.

Projected publication

An article on the topic of the collaboration is in progress. A first draft of the article is expected to be completed before the end of July. Of course acknowledgement to the ESF support will be explicitly mentioned on this article.