Scientific Report on the project: Expected Return Probability of Random Walks on IDLA-clusters (Program RDSES, ref. 733)

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1 Purpose of the visit

While the theory of random walks on finite graphs has been thoroughly studied, the more specialized question about the *expected* return probability of a simple random walk on a finite, *random* graph occurs in the context of percolation theory and the analysis of random walks in random environments. It is not clear, which of the well known results of finite Markov chains carries over to the (time-)non-homogeneous case, if the state space is generated randomly. A special case of a random state space of a non-homogeneous Markov chain is given by the IDLA clusters, where it is just a random walk which belongs to their defining construction. It is the purpose of the present project to ask the question of the expected return probability in this case. IDLA has been studied in detail by S. Blachère in his thesis and subsequent publications. A method to bound the expected return probability was introduced by F.Sobieczky, in a work following ideas of his thesis. First ideas reveal the IDLA-process on the two-dimensional Euclidean lattice as the most interesting case.

2 Work carried out during the visit

· inspection of the possible occurring limiting cases in the case of growing 'balls' on $\mathbb{Z}^d$, and identification of a 'slowly-' and a 'fast-growing' regime; The intermediate regime, where the radius of the ball grows like the square root of the time shows to be the essential, i.e. interesting case.

· identification of the open question of recurrence and transience of the simple random walk on a three-dimensional Euclidean lattice, restricted to a ball of radius growing in time at rate proportional to the square root of time; identification of possible methods to answer this question;

· In going beyond the scope of the project, and in discussion with other probabilists at *l'université Aix-Marseille*, in particular with P. Mathieu, the open problem of where the *speed* of a simple random walk on a percolative subgraph of a non-amenable Diestel-Leader Graph should switch between zero and finite on the percolation cluster ('phase transition') should be studied with respect to methods known about deterministic graphs, in particular with respect to 'anchored expansion'.

3 Description of the main results obtained

In homogeneous and inhomogeneous random walks, one of two cases may occur: either recurrence or transience. The influence on the random walker of the specific rate at which the state space is enlarged in time may also be *at least* one of two cases: either the 'bounding walls' are reached quickly and the 'return-trip', back to the origin is undertaken early enough to ensure recurrence, or the probability of reaching the origin ever again is less than one, due to too many possibilities to escape. It shows that a transient random walk starting at a fixed starting point with infinite state space, which is restricted to growing balls around the origin will show a change towards recurrence as the rate of growth of the balls' radius is reduced from at least linear in time to less than a *critical rate*. The results obtained indicate this growth rate to be essentially the square root of time.

4 Future collaboration with host institution

It is our intention to give a detailed answer to the question of whether there is a specific and different behavior of the simple random walk at the critical growth rate, compared to the extreme situations, in which the state space is either growing fast, or slowly, in the later of which the behavior is that of a finite random walk. Moreover, the results shall be used to derive statements about the simple random walk on IDLA-clusters, taking into account that the limiting shapes of them are balls. Furthermore, a research project with P.Mathieu concerning the speed on percolation clusters is planned.

5 Projected publications

It is intended to pursue the project with the aim of a publication, possibly with the title "Random Walks on IDLA-clusters".