SCIENTIFIC REPORT

Host: Viviane Baladi (CNRS and Institut Henri Poncaré).

Period: June 6-11 and June 17-26 (the slight change of the dates compared to the originally planned one is the consequence that later I received an invitation for a conference, held in Edinburgh between June 12 and 17)

My host, Viviane Baladi is not only a leading expert in the theory of hyperbolic dynamical systems, but she is also the author of the monography Positive Transfer Operators and Decay of Correlations. The techniques treated in her monography have, in particular, a fundamental role in applying Young's tower construction, so efficient for obtaining stochastic properties (correlation decay, limit laws, etc.) of hyperbolic dynamical systems with singularities. This is also true for an alternative suggestion of Liverani based on invariant-cone-field methods. Moreover, Baladi has been one of the organizers of the trimester Time at Work held between April 18 and Jult 13 in the Institut Henri Poincaré, where several experts gathered with whom I was also able to discuss actual problems of the theory. Beside Viviane Baladi I had particularly interesting discussions with D. Dolgopyat (U. of Maryland), S. Gouëzel (ENS, Paris), M. Lenci (Stevens Institute, N. Y.), C. Liverani (U. of Roma), O. Sarig (PennState U.) and L. S. Young (NYU).

The most extensive discussions and the strongest progress I have made toward answering the following problem. The Lorentz gas (or Lorentz process) is a much interesting model both of dynamical systems and of statistical physics since it is almost the only physical system which can also be rigorously treated by mathematical methods. Starting with the 1980/81 twin papers of Bunimovich and Sinai, but more intensively by the 1998 paper of Young, several results have been obtained for the two-dimensional gas with a periodic configuration of scatterers. For further application, however, it is utmost important to get rid of the condition of the periodicity. But, even the case of a small perturbation of just one scatterer raises deep new problems, and so far no results are known (except for the recurrence statement of Lenci in the finite horizon case and the analogous one of Szasz-Varju in the infinite horizon case). The big aim is to understand the decay of correlations and to prove the conjectured diffusive behaviour. I was able to discuss this problem with several people, and, in fact, with D. Dolgopyat we have started a coordinated attack on the problem. The arguments of Szasz-Telcs, 1981 providing diffusion for simple symmetric random walks with local impurities (RWwLI) are far too week here though they are still helpful in certain details. This approach uses heavily a ratio limit theorem of Kesten-Spitzer, 1963 valid for planar recurrent random walks (PRRW). One important task is to generalize it to the Lorentz gas. Another important property of a RWwLI is the trivial very weak dependence of this process between successive returns to a finite domain, containing the impurities. The proof of the analogous

statement for the Lorentz process is far from trivial but a most challenging task in itself. These elements, if I succeed to prove them, can hopefully be combined with the strategy and strong dynamical arguments of Dolgopyat to provide the desired result.

Finally I emphasize the quite interesting discussions with Sarig about the connection of phase transitions on the one hand and non-central limit theorems on the other hand for dynamical systems.

July 5, 2005

Domokos Szász