Amenability of percolation subgraphs and the Integrated Density of States

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Host: Dr. Florian Sobieczky, Technische Universität Graz, Austria
Visitors: PD Dr. Daniel Lenz, Technische Universität Chemnitz, Germany
and
Dr. Ivan Veselić, Technische Universität Chemnitz, and
Emmy-Noether-Programme of the DFG, Germany

The purpose of the visit was to study amenability properties of (random) graphs and the integrated density of states of the corresponding discrete Laplace operators.

To facilitate the exchange of ideas the two guests have given talks in the "Strukturtheorie" seminar at the Mathematics departement of the Technische Universität Graz.

D. Lenz gave an exposition on "Aperiodic order and uniform existence of the integrated density of states" and I. Veselić on "Anderson-Percolation Hamiltonians and finitely supported eigenstates". In particular, the construction of the integrated density of states by an exhaustion procedure was explained. This approach is applicable in the case of amenable graphs using a Følner sequence. For discrete Laplace operators arising in different geometric settings (graphs, Delone quasi-crystals) it turns out that the topologies of the convergence along the Følner sequence are different.

Methods and ideas presented in these talks rely on amenability features of the underlying graphs, transitivity properties of the groups, and decomposition techniques.

The overall intention of the project is to combine these methods with the the expertise of the host F. Sobieczky on random walks on percolation graphs to study amenability features of graphs.

Beside the intrinsic interest in amenability of graphs this should lead to larger classes of graphs on which the integrated density of states can be defined. In a subsequent step, the integrated density of states can then be used to understand the underlying graphs in more detail. One might also hope to achieve a better

understanding of what may be the integrated density of states in a non-amenable situation by having specific examples with weak amenability properties at hand.

Along this line, we identified several models of graphs whose study should lead to the construction of interesting examples with intermediate amenability properties:

1. Trees,
2. horocyclic products of regular trees such as Lamplighter graphs or more general Diestel-Leader graphs,
3. horocyclic products of non-regular trees which satisfy certain (almost) periodicity properties
4. suitable percolation subgraphs of the examples (i) to (iii).

We consider the visit a succes since the host and the visitors were able to learn new techniques from the respective other field. While the duration of five days was too short to carry out some in depth analysis, it sufficed to set up a framework of relevant models and decide on some concrete questions to be tackled next. In order to do so, the intention is to invite F. Sobieczky to the Technische Universität Chemnitz in the summer. Likewise, it would be desireable that D. Lenz, I. Veselić and PhD students from Chemnitz working on percolation visit Graz again for a continuation of the work carried out during this research stay.