Programme title: **Contact And Symplectic Topology**

Programme acronym: **CAST**

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Keywords:

Contact and Symplectic Topology, Floer homology, Hamiltonian dynamics, Fukaya category, symplectic field theory, Stein manifolds, symplectomorphism and contactomorphism groups.

Abstract:

The goal of this network is to stimulate exchange between researchers from all branches of contact and symplectic topology, in order to create a comprehensive perspective on the field and make progress on some of the basic open questions. The European scale of the network reflects the global nature of these questions as well as the European strength in the subject. The planned activities include workshops, research collaborations, and the exchange of PhD students and postdocs.

Status of the relevant research field and scientific context:

Contact and symplectic structures have their origin in classical physics: Symplectic manifolds are the natural phase spaces for classical mechanics, while contact structures arise naturally on energy levels of autonomous Hamiltonian systems. Many basic concepts such as symplectic reduction, Hamilton-Jacobi equations, Legendrian and Lagrangian submanifolds were developed in this context and continue to play an important role to this day.

The modern subject of *contact and symplectic topology* emerged in the early 1980s through the work of Bennequin, Eliashberg, Gromov and others. Its birth may be dated to the following two events: Bennequin's proof that the standard contact structure on $\mathbb{R}^3$ is tight (1982), and Gromov's proof of his "non-squeezing theorem" (1985). Since then the subject has grown into one of the most active and vibrant areas of mathematical research, with profound contributions by many leading mathematicians. The most spectacular achievements include:

- Gromov's theory of pseudo-holomorphic curves in symplectic manifolds (1985);
- Eliashberg's classification of overtwisted contact structures (1989);
- Floer's work on the Arnold conjecture and the introduction of Floer homology (1989);
- Hofer's metric on the group of Hamiltonian diffeomorphisms (1990);
- the Fukaya category and its relation to mirror symmetry (1993);
- Kontsevich's introduction of the concept of a "stable map" and the subsequent emergence of Gromov-Witten theory (1995);
- *Donaldson*'s[1] theory of asymptotically holomorphic sections and Lefschetz pencils (1996);
- Taubes' correspondence between solutions of Seiberg-Witten equations and pseudo-holomorphic curves (1996);
- symplectic field theory by Eliashberg, Givental and Hofer (2000);
- *Giroux*'s open book decompositions for contact structures (2002);
- Legendrian contact homology by Chekanov, *Ekholm*, Etnyre and Sullivan (2002);
- the construction of quasi-morphisms on groups of Hamiltonian diffeomorphisms by *Biran*, *Polterovich* and *Entov* (2003);
- Heegaard Floer homology by Ozsvath and Szabo (2004);
- the discovery of exotic Stein structures on $\mathbb{C}^n$ by Seidel, *Smith* and *McLean* (2007);
- Taubes' proof of the Weinstein conjecture in dimension three (2007).

A distinguishing feature of contact and symplectic topology has always been its close interactions with other fields: complex and algebraic geometry, low-dimensional topology, and mathematical physics (classical mechanics, quantum field theories, gauge theories, string theory).

While a lot of progress has been made on some of the big problems in the field (Arnold conjecture, Weinstein conjecture in dimension 3, contact structures and Legendrian knots in dimension 3), many of the most basic questions are still wide open: existence of contact and symplectic structures above dimension 4, the structure of symplectomorphism and contactomorphism groups, and dynamical results beyond the existence of periodic orbits. Moreover, the methods in the field have become increasingly sophisticated and are sometimes not easily accessible (e.g. Fukaya categories and symplectic field theory).

[1] Throughout Section II of the proposal names of network members are printed in *slanted* font.

Objectives, envisaged achievements, and expertise in the Programme:

The goal of this network is to stimulate exchange between researchers from all branches of contact and symplectic topology, in order to create a comprehensive perspective on the field and make progress on some of the basic open questions.

In the following we describe in more detail some circles of questions which we plan to address. Each of these topics is strongly represented in the research of participants in the network. We expect substantial progress through broad collaboration of experts from various branches of the field.

Fukaya categories and mirror symmetry. Fukaya categories package a great deal of information about a symplectic manifold, but it is rarely obvious how to extract that information, so they have yet to become a widely used tool in symplectic topology. Sometimes computations are possible by using mirror symmetry or Lefschetz fibrations (or sometimes both). The over-riding goal now is to relate them to more classical areas in the subject.

Fukaya categories provide one route to obtaining information about the Lagrangian submanifolds of a symplectic manifold. Typical applications (due to Fukaya-Seidel-*Smith* respectively Abouzaid-*Smith*) include restrictions on the topology and intersection properties of exact spin Lagrangian submanifolds of Maslov class zero in a cotangent bundle, or of low-genus Maslov zero Lagrangian surfaces in the four-torus. Critically, in these cases one can describe explicit Lagrangian submanifolds which generate or split-generate the Fukaya category; this means you can express any Lagrangian in terms of these basic objects, resulting in a powerful "geometric Fourier theory". We hope to use this theory to attack other questions, not just Lagrangian embedding questions.

There is scope for progress on a wide range of questions, relating directly to other research areas; we give two examples.

• Conjecture: The boundary connect sums of an exotic Stein manifold (diffeomorphic to Euclidean space) with itself are all pairwise distinct.

This follows if the wrapped Fukaya category is generated by ascending manifolds of a plurisubharmonic Morse function, and shows one can hope to use the Fukaya category to derive "refined Morse inequalities" in Stein geometry.

• When do symplectomorphisms of Calabi-Yau manifolds which act trivially on cohomology and have zero flux have fixed points?

This question suggests a "refined Arnol'd conjecture". It holds in some cases, e.g. the product of a K3 surface and a torus, by studying deformations of Fukaya categories (Abouzaid-Seidel-*Smith*). There are obvious relations to Lagrangian submanifolds and it would be interesting to find relations to Hamiltonian dynamics – does the Fukaya category see entropy?

The most interesting questions are at the interfaces of areas. To compute the Fukaya category one probably needs mirror symmetry (*Joyce, Thomas*) or knowledge of Stein handle presentations (*Bourgeois, Cieliebak, Ekholm*). To use it to study symplectomorphisms, one should think about stability conditions (*Smith, Woolf*) and about entropy (*Schlenk*). It is natural to combine constraints on Lagrangian embeddings obtained this way with the constraints coming from other areas (*Biran, Damian*), and to refine these by inputting string topology (*Cieliebak, Latschev*). Hochschild homology of the Fukaya category relates to symplectic homology (*Viterbo*) and Gromov-Witten theory (*Coates*). Expicit generators for Fukaya categories of moduli spaces in low-dimensional topology – and the mirrors of those moduli spaces – are not yet known (*Friedl, Lisca, Thomas*).

Floer homology and Hamiltonian dynamics. Hamiltonian dynamics and major conjectures therein such as the Arnold conjecture and the Weinstein conjecture have been a major driving force in the development of methods and theories in the context of symplectic rigidity phenomena, most notably Floer homology and its contact geometrical analogon of contact homology and, more generally, symplectic field theory.

Whereas the Arnold conjecture and the 3-dimensional Weinstein conjecture are settled by now, the methods developed in this course have been extended further and directed towards a broad spectrum of new applications.

Major goals for these developments are:

• Better multiplicity results for periodic solutions of Hamiltonian systems in the presence of particular symmetries such as for autonomous systems, reversible systems or in the presence of other finite symmetry groups. For example, are there always n or infinitely many periodic orbits on a star-shaped hypersurface in $\mathbb{R}^{2n}$?

This requires in particular suitable equivariant versions of Floer homology or similar methods based on holomorphic curves.

• A deeper, systematic understanding of the symplectic invariants developed, for example the relation of Floer homology to loop space topology and string topology operations, its functoriality, and its interrelation with gauge theory.

• A deeper analysis of Hamiltonian systems of physical type on cotangent bundles, in particular in the presence of magnetic fields, with the methods from holomorphic curves, and its relation to other important dynamical invariants such as entropy or volume growth.

• New applications to symplectic rigidity phenomena such as packing problems or displacement problems.

• Further extending the analytical foundations for the method of holomorphic curves.

The expertise for these various research projects is represented by the following network members: *Abbondandolo, Bourgeois, Bialy, Biran, Cieliebak, Ekholm, Izydorek, Latschev, Mohnke, Oancea, Paternain, Polterovich, Salamon, Schlenk, Schwarz, Siburg*, and *Zehnder*.

Symplectic field theory. Symplectic field theory (SFT) stands for a program to create a unified theory of holomorphic curves in symplectic manifolds. Building on the seminal work of Gromov and Floer, this program was first formulated by Eliashberg, Givental and Hofer in 2000. In its ultimate form, SFT will incorporate all existing holomorphic curve theories such as Gromov-Witten theory, Floer homology, Legendrian contact homology, and the Fukaya category. Applications of SFT cover such diverse areas as Hamiltonian dynamics, symplectic and contact topology, smooth topology, complex and algebraic geometry, and fluid dynamics. Here is a list of important open problems on SFT on which we expect progress within the next 5 years:

• Compute the SFT of Stein manifolds.

This is a central problem of SFT; in view of *Biran*'s decomposition, its solution will allow the computation of SFT for all closed symplectic manifolds. For the special case of cylindrical contact homology the problem is solved by a result of M.-L. Yau and the surgery exact sequence of *Bourgeois*, *Ekholm* and Eliashberg.

• Describe the invariants of smooth manifold pairs obtained from SFT of conormal bundles. Do they distinguish smooth structures?

For example, Ng has combinatorially described the invariant for knots in $\mathbb{R}^3$ resulting from Legendrian contact homology of its conormal bundle. Is this a complete knot invariant?

• Develope relative SFT and study applications to the classification of Legendrian submanifolds and multiplicity results for Reeb chords.

Based on ideas of *Cieliebak*, *Latschev* and *Mohnke*, Ng has recently found a combinatorial description of relative SFT for Legendrian knots in $\mathbb{R}^3$. In higher dimensions, relative SFT will involve string topology in an essential way.

• Seidel has conjectured a precise relation between symplectic homology and the Fukaya category. Prove this, and understand more generally the relation between SFT and the Fukaya category.

• Is there an SFT interpretation of the contact class in Heegaard Floer homology?

• Overtwisted contact 3-manifolds have vanishing SFT. What about the converse? What does vanishing of SFT mean geometrically in higher dimensions?

• Find SFT obstructions to fillability of contact manifolds and Legendrian knots.

• Find obstructions to Lagrangian embeddings or immersions using the relation between SFT and string topology discovered by *Cieliebak* and *Latschev*.

Contact Topology. Contact topology has recently made great progress, thanks to *Giroux*'s description of contact structures in terms of open book decompositions. It is now the meeting point of many research areas: low dimensional topology, foliation theory, dynamical systems, symplectic topology, and Floer homology theories.

In dimension three, there is a fundamental dichotomy between tight and overtwisted contact structures. Overtwisted contact structures are well-understood by work of Eliashberg. A relatively good understanding of tight contact structures is given by a rough classification theorem of *Colin, Giroux* and Honda: A closed irreducible 3-manifods carries infinitely many tight contact structures if and only if it contains an incompressible torus. Several fundamental questions that we plan to address are however still open.

• We know that legendrian surgery preserves any kind of symplectic fillability. Does it preserve tightness?

• There are irreducible 3-manifolds that do not carry a positive tight contact structure. Does every irreducible 3-manifold carry a tight contact structure?

One of the most useful tools in contact topology is the contact class, which associates to any contact structure an element in Heegaard-Floer homology. The contact class vanishes for overtwisted contact structures and is nonzero for symplectically fillable ones. In the case of tight contact structures, *Ghiggini*, Honda and Van Horn-Morris, and *Lisca* and *Stipsicz* pointed out the role of *Giroux*'s torsion in the vanishing of the contact class.

• Does a tight contact structure on an atoroidal manifold have nonzero contact class?

We also expect to make progress on the following existence and classification problems.

• Existence of positive tight contact structures on 3–manifolds obtained by surgery along a knot in the 3-sphere. Partial results were obtained by *Lisca* and *Stipsicz*.

• Classification of tight contact structures on closed, Seifert fibered 3-manifolds. Partial results were obtained by *Ghiggini, Lisca* and *Stipsicz*.

• Classification of Legendrian and transverse knots in fixed knot types. At the moment we have complete classification only in simple cases like torus knots and the figure-8; this could be extended to twist knots, for example.

• Classification of symplectic fillings of links of isolated surface singularities, and the comparison of these fillings with smoothings of the singularities. Partial results were obtained by *Nemethi* and *Popescu-Pampu*, *Lisca, Stipsicz* and Szabò, Wahl, and Bhupal.

In higher dimensions, the topological characterization of closed manifolds which admit contact structures is a fundamental problem which is completely open. One approach to this problem is the following: According to work of *Giroux* and Mohsen, every closed contact manifold has an open book decomposition whose pages are Stein domains and whose monodromy map is a symplectic diffeomorphism which is the identity near the boundary. On the other hand, and old result by Lawson states that every closed odd-dimensional manifold admits an open book whose pages have the homotopy type of a half-dimensional cell complex. This description has already been used by *Bourgeois* to show that every odd-dimensional torus carries a contact structure. Proving existence of contact structures on more general manifolds thus requires the following two steps:

• Use Eliashberg's topological characterization of Stein manifolds to find an open book whose pages are Stein domains.

• Understand if the monodromy of this open book is isotopic (relative to the boundary) to a symplectomorphism.

Even in the case when the Stein domain is a ball, the latter problem is interesting and non trivial. For instance, the following questions are open:

• Does the standard symplectic ball admit a symplectomorphism which is smoothly, but not symplectically, isotopic to the identity? Does it admit a symplectomorphism which is not smoothly isotopic to the identity?

Complex geometry and Stein manifolds. Stein manifolds have played a central role in the development of complex and algebraic geometry in the 20th century, with deep contributions by H. Cartan, Grauert, Hörmander and many others. The main techniques in their study were Cartan's theory of coherent analytic sheaves and Hörmander's theory of the $\bar{\partial}$-equation. In 1991 Eliashberg provided an entirely new view on the subject by giving a purely topological characterization of all manifolds of even dimension at least 6 admitting a Stein structure. His proof relies on a range of techniques from differential, symplectic and contact topology. Moreover, Eliashberg introduced a natural notion of homotopy of Stein structures and posed the following question.

• When are two Stein structures on the same manifold homotopic?

The first examples of non-homotopic Stein structures were found bei Seidel and *Smith* in 2005. In 2007 *McLean* constructed infinitely many non-homotopic Stein structures on $\mathbb{C}^n$, $n \geq 4$, that are distinguished by the ring structures on symplectic homology. His work raises the following question:

• Is the symplectic homology of a Stein manifold always finitely generated as a ring?

Recent work by *Bourgeois* and *Oancea* on the relation between symplectic and contact homology suggest the following

• Conjecture: The contact structures on S^{2n-1} arising on the sphere at infinity in *McLean*'s examples are pairwise distinct.

In a forthcoming book *Cieliebak* and Eliashberg prove that subcritical Stein structures are homotopic provided some obvious topological conditions are satisfied. So the whole complexity of Stein structures lies in the critical cells, i.e. cells of half the dimension in a cell decomposition arising from a plurisubharmonic function.

• Can one estimate the number of critical cells from below? For example, does the number of critical cells tend to infinity in *McLean*'s examples?

Besides the theory of Stein manifolds, two other important topics at the interface of complex geomery and symplectic topology arose in the work of *Donaldson*.

The first topic concerns the existence of Kähler metrics of constant scalar curvature on complex projective varieties. *Donaldson* used an action-angle coordinates framework developed by Guillemin and *Abreu* to study this problem for toric varieties, completely solving it in the case of toric surfaces. This action-angle coordinates framework has been extended to toric symplectic cones and we expect to be able to use it to study in this context an analogue of the above general problem:

• Characterize toric complex cones which admit a scalar flat Kähler metric.

The second topic is *Donaldson-Thomas* theory, which can be viewed as a complexification of Floer theory in the same way that Picard-Lefschetz theory is a complexification of Morse theory. This theory and its relation to Gromov-Witten theory is an area of active current research that is also of great interest in theoretical physics.

To address these questions, the network includes, besides specialists in symplectic geometry and topology, the following experts in complex geometry: *Barraud, Gayet, Loi, Opshtein, Shevchishin, Siebert, Viterbo*, and *Voisin*.

Topology of symplectic manifolds. Closed Kähler manifolds have many special homotopical properties (e.g. even odd-dimensional Betti numbers, hard Lefschetz property, vanishing Massey products, formality). On the other hand, Cavalcanti, Fernandez, Ibanez, *Muños*, Rudyak, *Tralle* and Ugarte showed over the recent years that all known homotopic restrictions on Kähler manifolds are violated by symplectic manifolds. This adds evidence to the (still open) conjecture that symplectic manifolds have no distinguished cohomological properties:

• Conjecture (Thurston 1976): For any graded-commutative finite dimensional algebra satisfying Poincaré duality and with an element of degree 2 whose top power is nonzero, there exists a closed symplectic manifold whose cohomology algebra is isomorphic to the given one.

An important theme in symplectic topology is the relation between the group of symplectic diffeomorphisms and the whole diffeomorphism group.

• Problem (Gromov-McDuff): Is any symplectomorphism of a standard symplectic torus which acts trivially on homology isotopic to the identity?

Hajduk and *Tralle* have proposed a program to solve of the Gromov-McDuff problem. This program requires a deeper understanding of moduli spaces of pseudoholomorphic curves and will use the expertise on this subject present in the network (*Biran, Schwarz,* *Salamon, Siebert* and many others). Moreover, *Hajduk* and *Tralle* have related this problem to the following.

• Do exotic tori admit symplectic structures?

An substantial portion of research in symplectic topology in recent years has been devoted to the study of Lagrangian submanifolds. We have already mentioned above the work by Fukaya, Seidel and *Smith* on exact Lagrangians in cotangent bundles. Recent work by *Biran* and Cornea on monotone Lagrangians leads to the following question.

• Is it true that for all monotone Lagrangian submanifolds the Floer homology either vanishes, or is isomorphic to the singular homology?

Groups of symplectomorphisms and contactomorphisms. The symplectomorphism and contactomorphism groups play a central role in symplectic and contact topology. Here are a few proposed directions of study regarding the topology, algebra and geometry of these groups.

In the case of rational and ruled surfaces and their blow-ups *Abreu, Anjos* and their coauthors have provided a complete description of the homotopy type of the symplectomorphism groups. In addition, *Kedra* has shown that for small blow-ups of certain 4-manifolds the group of symplectomorphisms has infinitely generated cohomology. We expect to make further progress on these questions for blow-ups of $\mathbb{C}P^2$. *Kedra* and McDuff have shown that a natural action $SU(n) \to Ham(M,\omega)$ on a generalized flag manifold induces a monomorphism on the rational homology. A general open problem is the following:

• Understand to which extent the topology of the symplectomorphism group is determined by compact subgroups arising from Lie group actions.

Using Floer theory, *Entov* and *Polterovich* have constructed quasi-morphisms on the groups of Hamiltonian symplectomorphisms which give rise to symplectic quasi-states. These objects have numerous applications in symplectic topology and are related to physics. A different construction of a quasi-state for T^*T^n has been found by *Viterbo* in the framework of his symplectic homogenization theory.

• What is the connection between these two constructions of quasi-states?

• Study quasi-morphisms on groups of symplectic homeomorphisms (*Entov-Polterovich*-Py, *Kotschick*).

• Study actions of discrete groups on symplectic manifolds and extend *Polterovich*'s results on the Zimmer conjecture for actions of non-uniform lattices on symplectically aspherical manifolds to other groups and manifolds.

Gal and *Kedra* have constructed a 1-cocycle on the group of Hamiltonian diffeomorphisms of a symplectically aspherical manifold which obstructs actions of non-uniform lattices. It would be interesting to study the Bockstein differential of the above cocycle which defines a real second cohomology class of the group of symplectomorphisms.

The topology of contactomorphisms groups of contact 3-manifolds has been studied by *Giroux, Geiges-Gonzalo* and *Bourgeois*, using convex surface theory and contact homology. Eliashberg, Kim and *Polterovich* have shown that contactomorphism groups either carry a partial order or contain a semigroup of positive contractible loops.

• Explore this dichotomy for pre-quantizations of closed symplectic manifolds and develop tools for calculation of this semigroup (the answer is unknown even for the 3-

sphere). This will require further study of algebraic properties of contact homology of domains.

European context and expected benefit from European collaboration:

In recent years a large number of young researchers in contact and symplectic topology have returned to Europe and established active research groups in their home countries. Thus Europe has become a stronghold of the subject. This is reflected by an increase in workshops and conferences held in Europe. In particular, several continuing series of workshops have been established:

- Workshops on Symplectic Geometry, Contact Geometry and Interactions I-III, organized by J.-C. Alvarez-Paiva, *J.F. Barraud, F. Bourgeois, M. Damian, A. Oancea* and *C. Viterbo* (2007 in Lille, 2008 in Bruxelles, 2009 in Strasbourg). These workshops bring together 60-70 researchers for exchange of recent results and ideas.
- Workshops on Symplectic Field Theory I-III, organized by *K. Cieliebak, K. Mohnke* and *M. Schwarz* (2005 and 2006 in Leipzig, 2008 in Berlin, 2009 planned in München). They consist of lecture series on specific topics by international experts (so far Hofer, Eliashberg, and Givental), preceded by a 2-day precourse to prepare for the lectures, and they draw large audiences (about 100 participants in 2008) ranging from undergraduate students to senior researchers.
- GESTA Workshops, organized by *I. Mundet, V. Muñoz* and *F. Presas* (held in Madrid and Barcelona for the last 8 years). Their goal is to offer to students and young researchers a series of mini-courses on contact and symplectic topology. The GESTA workshop became a satellite conference of the ICM 2006 held in Madrid.

Besides conferences and workshops, there are many flourishing research collaborations in this subject as well as exchange of PhD students and postdocs between European locations, facilitated by the geographical proximity.

The proposed network will provide a platform for European collaboration and exchange in contact and symplectic topology, thus increasing visibility and international competitivity of this important subject. This will facilitate exchange of students and researchers alike, and will allow us to attract international talents. Moreover, by bringing together experts from various branches to focus on specific questions, the network will become a catalyst for progress on some of the deep unsolved problems in the field.

The proposed network has a certain overlap with the existing ESF Research Networking Programm “Interactions of Low-Dimensional Topology and Geometry with Mathematical Physics (ITGP)”, and some researchers will be members of both networks. However, whereas the ITGP network covers a very broad range of topology, geometry and physics, the proposed network will be on a smaller scale, more coherent and focused on the subject of contact and symplectic topology. Of course, the proposed network will coordinate its activities with the ITGP network.

The proposed network also expects cooperation on certain activities with the following three researchers that recently received ERC Advanced Researcher Grants: Boris Dubrovin (FroM-PDE), Ludmil Katzarkov (GEMIS), and Ib Madsen (TMSS).

Proposed activities, key targets and milestones:

Over the duration of the networking programme we propose the following activities.

- One 2-week Summer School on Contact and Symplectic Topology held in the second year of the programme. It will consist of introductory lectures into various aspects of the contact and symplectic topology given by members of the network, with the goal of attracting students and young researchers to the subject. Depending on the financial possibilities, this summer school could have between 50 and 100 participants.

• Four 7-day workshops aimed primarily at graduate students and young postdocs in the subject. These will follow the successful scheme of the “Workshops on Symplectic Field Theory” (see above), consisting of a 2-day precourse and a 5-day lecture series by an international expert. The workshops will take place in years 1,3,4 and 5 of the programme and are expected to have 50-100 participants.

• An annual 3-day “Workshop on Symplectic Geometry, Contact Geometry and Interactions“ (see above), bringing together 60-70 senior researchers and advanced postdocs for exchange of recent results and ideas.

• Annual meetings of the steering committee to evaluate and plan activities in the network. These meetings will take place at the occasion of the ‘Workshops on Symplectic Geometry, Contact Geometry and Interactions“ and not require any extra funding.

• Five 5-day workshops with 10-15 participants to trigger cross-area collaboration on a specific open problem. The problems we plan to address are the following:
- Existence of contact and symplectic structures in higher dimensions
- Symplectic topology of Stein manifolds
- Relations between the Fukaya category and symplectic field theory
- Groups of symplectomorphisms and contactomorphisms
- Applications of holomorphic curves to Hamiltonian dynamics.

• Short scientific visits to continue existing collaborations and establish new ones.

• Exchange of PhD students and postdocs, enabling them to spend several months at a location in the network.

Duration: 60 months

Budget estimate (per year):

We expect that for the summer school and the workshops at least half the cost will be co-financed from other sources (as it has been the case for all past workshops). This leads us to the following budget estimates per year.

• Summer School on Contact and Symplectic Topology: 50.000 Euro / 5 =	10.000 Euro
• Four “Workshops on Symplectic Field Theory”: 4 × 25.000 Euro / 5 =	20.000 Euro
• Annual “Workshops on Symplectic Geometry, Contact Geometry and Interactions“	15.000 Euro
• Five 5-day workshops on specific open problems	10.000 Euro
• Short scientific visits	15.000 Euro
• Exchange of PhD students and postdocs	25.000 Euro
• Cost for setting up a database and creation and maintenance of a website for the network, as well as administrative costs (part-time assistant) for the project coordinator:	10.000 Euro
Total annual budget per year:	**105.000 Euro**

Steering Committee:
Belgium: Frédéric Bourgeois (Univ. Libre de Bruxelles)
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Germany: Kai Cieliebak (Ludwig-Maximilians-Univ. München)
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Global dimension:

The proposed network includes five key persons from the non-ESF country Israel:

Michael Bialy (Tel-Aviv Univ.) is an expert on dynamical systems and geodesic flows.

Paul Biran (Tel-Aviv Univ.) has done outstanding work on many aspects of symplectic topology including: symplectic packings, topology of Lagrangian submanifolds, Floer homology, interactions between symplectic and complex algebraic geometry.

Leonid Polterovich (Tel-Aviv Univ.) is one of the world leaders in symplectic topology whose work covers the whole breadth of the subject. For example, he has pioneered the following topics: topology of Lagrangian submanifolds, symplectic fibrations, symplectic packings, symplectomorphism and contactomorphism groups, quasi-states and quasi-morphisms, and relations to Hamiltonian dynamics.

Michael Entov (Technion – Israel Inst. of Technology) is a leading expert on symplectomorphism groups, the theory of quasi-states and quasi-morphisms, and C^0-rigidity phenomena.

Jake Solomon (Hebrew Univ. of Jerusalem) is working on open Gromov-Witten invariants.

Clearly, the expertise of these researchers will be invaluable for the networking programme. Moreover, the geographical proximity facilitates collaborations and exchange with other European members. Thus the Israeli members regularly participate in European workshops and have collaborations with the following network members: K. Cieliebak, E. Giroux, V. Humilière, G. Paternain, D. Salamon, F. Schlenk, K.-F. Siburg, C. Viterbo.

The country representative has informed the Israel Science Foundation (ISF) about the proposed networking programme. As soon as the programme has been approved by the ESF, the Israeli members will apply to the ISF for funding on their part.

International collaborators:

Of course, the network will also maintain close interactions with the international community in contact and symplectic topology. To name but a few, network members have ongoing collaborations with the following international researchers: M. Abouzaid (Clay Research Institute), D. Auroux (MIT), Y. Chekanov (Moscow), O. Cornea (Montreal), Y. Eliashberg (Stanford), J. Etnyre (Georgia Institute of Technology, Atlanta), K. Fukaya (Kyoto Univ.), V. Ginzburg (Santa Cruz), H. Hofer (Courant Institute), K. Honda (USC, Los Angeles), Y. Karshon (Toronto), F. Lalonde (Montreal), D. McDuff (Barnard College, New York), L. Ng (Duke Univ.), K. Ono (Hokkaido Univ.), P. Ozsvath (Columbia Univ., New York), R. Pandaripande (Princeton), P. Seidel (MIT), Z. Szabo (Princeton).

CV Frédéric Bourgeois

Education:

1993-1998 Université Libre de Bruxelles, Engineering;
1993-1997 Université Libre de Bruxelles, Licence in mathematics;
1998-2002 Stanford University, PhD in mathematics.

Positions:

1997-1998 Teaching assistant at Université Libre de Bruxelles;
1998-2002 FNRS Research Fellow at Université Libre de Bruxelles, on leave at Stanford University;
Summer 2002 Liftoff Mathematician, Clay Mathematics Institute;
2002-2003 EDGE Postdoc, Ecole Polytechnique, Palaiseau;
since 2003 Chargé de cours at Université Libre de Bruxelles.

Research Grants:

2005-2007 FRFC Grant 2.4655.06/1 "Study and applications of contact homology";
2007-2009 FRFC Grant 2.4655.06/2 "Study and applications of contact homology".

Organized workshops and conferences:

FNRS contact group in differential geometry (president since 2005);
Workshop on Symplectic Geometry, Contact Geometry and Interactions I-III (2007-2009, with J.-C. Alvarez-Paiva, J.F. Barraud, M. Damian, A. Oancea and C. Viterbo).

5 recent relevant publications:

F. Bourgeois, *Odd dimensional tori are contact manifolds*, Int. Math. Res. Not. **2002**, no. 30, 1571–1574 (2002).

F. Bourgeois, Y. Eliashberg, H. Hofer, K. Wysocki and E. Zehnder, *Compactness results in Symplectic Field Theory*, Geom. Topol. **7**, 799–888 (2003).

F. Bourgeois, *Contact homology and homotopy groups of the space of contact structures*, Math. Res. Lett. **13**, no. 1, 71–85 (2006).

F. Bourgeois and A. Oancea, *Symplectic Homology, autonomous Hamiltonians, and Morse-Bott moduli spaces*, arXiv preprint (math.SG/0704.1039), to appear in Duke Math. J.

F. Bourgeois and A. Oancea, *An exact sequence for contact- and symplectic homology*, arXiv preprint (math.SG/0704.2169), to appear in Invent. Math.

CV Kai Cieliebak

Education:

1986-1988 Ruhruniversität Bochum;
1988-1989 Beijing University;
1989-1993 Ruhruniversität Bochum, diploma in mathematics;
1993-1996 ETH Zürich, PhD in mathematics.

Positions:

1996-1997 Benjamin Pierce Assistant Professor at Harvard University;
1997-1998 Risk management at Swiss Bank Corporation, Basel;
1998-1999 Postdoc at IBM Zurich Research Laboratory;
1999-2001 Szegö Assistant Professor at Stanford University;
since 2001 Professor for Differential Geometry (C3) at Ludwig-Maximilians-Universität München.

Research Grants:

1997-2000 NSF Grant DMS-9700209 "Symplectic manifolds with boundary";
2000-2003 NSF Grant DMS-0072267 "Moment maps and J-holomorphic curves";
2003-2009 DFG Grant CI 45/1 "The symplectic vortex equations and applications";
2003-2009 DFG Grant CI 45/2 "Punctured holomorphic curves in symplectic geometry" (with K. Mohnke).

Organized workshops and conferences:

Stanford Contact Geometry Quarter (Sep-Dec 2000, with Y. Eliashberg and J. Etnyre);
Contact Topology Workshop (München, Feb 2003); Workshops on Symplectic Field Theory I-III (2005-2008, with K. Mohnke and M. Schwarz);
Workshop "Towards Relative Symplectic Field Theory" (New York 2007, with T. Ekholm, Y. Eliashberg, K. Fukaya, D. Sullivan and M. Sullivan).

5 recent relevant publications:

K. Cieliebak, *Handle attaching in symplectic homology and the chord conjecture*, J. Eur. Math. Soc. (JEMS) **4**, no. 2, 115–142 (2002).

K. Cieliebak, I. Mundet and D. Salamon, *Equivariant moduli problems, branched manifolds, and the Euler class*, Topology **42**, no. 3, 641–700 (2003).

K. Cieliebak and K. Mohnke, *Compactness for punctured holomorphic curves*, J. Symp. Geom. **3**, no. 4, 589–654 (2005).

K. Cieliebak and D. Salamon, *Wall crossing for symplectic vortices and quantum cohomology*, Math. Ann. **335**, no. 1, 133–192 (2006).

K. Cieliebak and K. Mohnke, *Symplectic hypersurfaces and transversality in Gromov-Witten theory*, J. Symplectic Geom. **5**, no. 3, 281–356 (2007).

CV Ivan Smith

Education:

1991-1994 New College, Oxford (Mathematics B.A.);
1994-1995 Trinity College, Cambridge (Certificate of Advanced Study);
1995-1997 New College, Oxford (D. Phil);
1997-1998 Stanford University, California (Visiting Research Scholar).

Positions:

1998-2001 G.H. Hardy Junior Research Fellow, Oxford;
2001-2003 Marie-Curie postdoctoral fellow, Ecole Polytechnique, Paris;
2003-2007 University Lecturer, Cambridge, UK;
2007-present University Reader, Cambridge, UK.

Research grants:

2004 Nuffield Foundation Award;
2004-2006 EPSRC First Grant Scheme award;
2008-2012 European Research Council Starting Investigator grant.

5 recent relevant publications:

R. Thomas, I. Smith and S.-T. Yau, *Symplectic conifold transitions*, J. Diff. Geom. **62**, 209–242 (2002).

S. Donaldson and I. Smith, *Lefschetz pencils and the canonical class for symplectic 4-manifolds*, Topology **42**, 743–785 (2003).

P. Seidel and I. Smith, *The symplectic topology of Ramanujam's surface*, Comment. Math. Helv. **80**, 859–881 (2005).

P. Seidel and I. Smith, *A link invariant from the symplectic geometry of nilpotent slices*, Duke Math. J. **134**, 453–514 (2006).

K. Fukaya, P. Seidel and I. Smith, *Exact Lagrangian submanifolds in simply-connected cotangent bundles*, Invent. Math. **172**, 1–27 (2008).