Final Report, ESF Exchange Grant

Balázs Pozsgay
PhD student

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Host Institute: Institut de Physique Théorique CEA-Saclay
Host Professor: Hubert Saleur
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Purpose of the visit

The purpose of my visit was to start a new collaboration with Hubert Saleur in the field of integrable models. The original plan was to extend my previous work on finite volume form factors to non-diagonal scattering theories, which are relevant for condensed matter systems. However, we changed plans after my arrival. Following Saleur's proposal I decided to join him in his research on disordered systems with the use of supergroups. I started to work on the scattering theory approach to the disordered Ising model.

Details of the research project and Results

The common way to treat disordered systems is the so-called „replica trick". It maps a theory with a Gaussian random field to a certain limit of a deterministic theory. The two dimensional scaling Ising model is a free fermion theory; disorder can be represented here by a random, spatially dependent mass term. Making use of the „replica trick" one can show that the theory is equivalent to the $N \to 0$ limit of the Gross-Neveu (GN) model, where N is the number of fermion flavours. The GN model is well-known for $N \geq 2$, however its properties are less understood in the region $N < 2$, where the theory is massless and asymptotically free in the IR.

One possible way to understand this model is to find the exact S-matrix of the massless scattering theory. In [1] Cabra, Honecker, Mussardo and Pujol proposed that one should continue analytically the well-known S-matrix [2] of the $N > 2$ case to $N \to 0$. They developed a massless form factor approach to study the two-point function of the energy operator; their results were in agreement with perturbative calculations. However, their approach lacks any interpretation of the scattering theory.

A rigorous way to define the $N \to 0$ limit is through the introduction of supergroups. In the GN model with $OSP(2n, 2n)$ symmetry the fermionic and bosonic (ghost) degrees of freedom add up to $N = 0$. The scattering matrices of these theories obey the (graded) Yang-Baxter equations, which are formally solved by the usual $O(N)$-symmetric S-matrix [2] with $N = 0$. The supergroup structure manifests intself in the interpretation and the representation of these solutions.

Continuing Saleur's previous work on supergroups (see for example [3] [4] [5]) I studied a conjectured massless flow with $OSP(2,2)$ symmetry.

The Thermodynamic Bethe Ansatz (TBA) equations give information about the energy of the ground state in finite volume. In particular they can be used to evaluate the central charge of the CFT's which emerge as the UV and IR limits of the theory. I considered both these limits and I also worked on numerical simulations of the infinite system of coupled integral equations. A naive inspection of the TBA equations provided the correct IR central charge. However, a careful numerical investigation showed that for the given S-matrix the TBA does not allow any solution at all. The question of a possible massless flow into to GN model with $N = 0$ thus remains an open question and is subject to further research. The resolution of the contradiction between the results of [1] and our findings needs to be clarified as well.

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Balázs Pozsgay
Budapest, September 20, 2008.

References

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