I. EXCHANGE VISIT GRANT - SCIENTIFIC REPORT OF ROBERT KONIK

A. Visit to Universiteit van Amsterdam, Feb. 11-26 and Mar. 3-7, 2008

The purpose of my visit to Universiteit van Amsterdam was to initiate a collaboration with J.-S. Caux, a member of the university's Institute for Theoretical Physics. There, Dr. Caux and I focused upon exploring the effects of trapping upon the spectrum and correlation functions of a cold atomic gas.

At the heart of this effort was a synthesis of two aspects of our respective research programmes. I, myself, have recently developed a semi-numerical methodology to studying both the spectrum and correlation functions of non-integrable perturbations of exactly solvable models [1, 2] based on the truncated conformal spectrum approach [3, 4]. This approach combines numerical renormalization group technologies together with data coming from integrable models. A key ingredient of the approach are matrix elements of the perturbing operators of interest. In a model that has a *relativistic continuum* description, these matrix elements are readily available. However for exactly solvable models that are not relativistic or have a lattice representation, the computation of these matrix elements is a complex problem. And so in a happily complementary fashion, Dr. Caux has been focused upon using the algebraic Bethe Ansatz to obtain the correlation functions of exactly such one dimensional integrable systems models [5–13]. A key part of his efforts has been the computation and subsequent categorization of the matrix elements that arise in a Lehmann expansion of a typical response function. Because of both the complexity of the matrix elements together with the large number of matrix elements needed to form the Lehmann expansion, this has required the development of sophisticated computerized algorithms. During my visit, we thus combined my numerical renormalization group approaches with his ability to compute matrix elements in non-relativistic integrable models.

A one dimensional atomic Bose gas in a trapping potential is a system that can then be studied through this synthesis. The untrapped gas is described through the continuum but non-relativistic Lieb-Liniger model, an exactly solvable model. The trap then can be treated in my numerical renormalization group approach as a perturbation. However to do so the numerically complex matrix elements of the density operator in the unperturbed theory are needed.

The primary accomplishment of my visit was then to produce a code that combines Dr. Caux's previous algorithm development for computing matrix elements with code that I myself have written implementing the numerical renormalization group procedure. With this code we have initiated studies of the following problems:

1) The study of the spectrum of the gas in the trap.

2) Computation of the temporal evolution of a gas if the shape or depth of the trap is changed.

3) The computation of the single particle spectral function of the gas in the trap.

We have preliminary results for the first two problems and we are in a good position to begin obtaining results on the third problem. We expect to have a preprint shortly.

Collaboration with J.-S. Caux beyond the visit has continued and is expected to continue. The three problems listed above that we made progress towards solving are but a few of the problems the synthesis of our research efforts can tackle. In particular we plan to examine problems involving perturbations of Heisenberg spin chains.

B. Visit to SISSA, Feb. 26-Mar. 1, 2008

The purpose of my visit to SISSA of Giuseppe Mussardo was to discuss a set of problems in statistical mechanics that were perhaps amenable to my development of a numerical renormalization group for the truncated spectrum approach. These problems fall into two categories:

i) Perturbations of minimal conformal models.

ii) Polynomial perturbations of free bosons;

With respect to i), I developed computer code incorporating my renormalization group procedure able to study arbitrary perturbations of the minimal models.

With respect to ii) Giuseppe Mussardo and I, together with his student Luca Lepori and his post-doc Gabor Toth, had extensive discussions on how to analyse polynomial perturbations of free bosons. The behaviour of such models has not been fully and rigourously established and so I believe these discussions will thus lead to further collaborative efforts.

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