**Research Report:**
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**Institution Visited: University City College, London, U.K.**
**Host Researcher: Prof. Sougato Bose.**
**Period of the visit: January 25 2009-February 27 2009 (5 weeks).**

**Topic of the collaboration:** ***Long Distance Entanglement in Spin Chains with Impurtities.***

***A) Motivations:***

By now the study of entanglement in many-body condensed matter systems and spin chains has developed into a topic of high interest [1]. In this context, long distance quantum correlations or entanglement are difficult to achieve, especially if they are required to be of substantial magnitude. In realistic models, for instance, entanglement between individual spins, usually quantified by concurrence [2], is non-zero only between nearest or next to nearest neighbours [3]. The few rare examples where substantial entanglement between distant points can be achieved are either through dynamics following drastic global changes of the Hamiltonian describing the system [4] or through perturbative effects in the ground state [5] and perturbative dynamics [6]. Besides its interest for fundamental physics, entanglement between distant points, say the end points, of a physical system will enforce their use as wires for conveying quantum information [7]. Our project is aimed at evidencing how features of condensed matter systems where non-perturbative phenomena and impurities are important, such as spin chains manifesting the Kondo effect [8] , may be exploited for creating very high long-distance entanglement.

Quantum spin chains have recently become a paradigm to investigate non-perturbative phenomena arising from the presence of impurities in condensed matter systems [9]. In this context, the Kondo model [10] arises when a magnetic impurity [11] is coupled to the end of an antiferromagnetic $J_1$-$J_2$ spin-1/2 chain, where $J_1$ ($J_2$) is the (next) nearest neighbour coupling [8,9]. At a critical value of $J_2$ the model admits a transition from a gapless Kondo phase to a gapped dimerized phase. We plan to investigate the emergence of a high quality long distance entanglement in the spin chain Kondo model and show how its qualitative behaviour is markedly different in the Kondo and the dimerized phases.

***B) Description of the work carried and of the main results obtained:***

We investigated the spin chain Kondo model with the purpose of characterizing the ground state entanglement of this system in both the Kondo regime and the dimer phase accessible to this model, following two different approaches:

In one approach, entanglement between the two end points of a finite Kondo spin chain is created through minimal, i. e, single bond, quench dynamics. For this purpose, we considered a finite Kondo chain in its ground state and, then, pertinently quenched the coupling at the opposite end of the impurity allowing for the dynamics to develop entanglement between the two ending spins. We found that the dynamical entanglement emerging from bond quenching is robust against temperature in both regimes for low temperatures and that bond quenching is a good probe for detecting the Kondo regime in a Kondo spin chain. Similar results have been obtained also for the spin 1/2 $XXZ$ model, which also supports a Kondo regime [12].

The other approach characterizes the entanglement of the ground state of a spin chain Kondo model by computing the negativity (13). We showed that the entanglement properties are very different in the gapless Kondo and the gapped dimerized phase. In both phases our approach allows to detect the size of the block of spins which are entangled with the magnetic impurity at the end of the chain: for this purpose we determine a distance, L*, after which the negativity, measuring the entanglement between the impurity and spins within that distance, is almost zero. We find that, in the gapless Kondo regime, L* scales with the strength of the Kondo coupling just as the the Kondo length does; furthermore, negativity is a homogeneous function of two ratios N/L* and L/N, where L is the size of the block and N is n the size of the whole chain. These results show that a measure of entanglement is able to detect the Kondo screening length in the gapless phase of the spin chain Kondo model. In the gapped dimerized phase, instead, L* is, for the same value of the impurity coupling, shorter than the corresponding Kondo length, while negativity is a function of the three independent variables L, N and L*.

***C) Projected Publications resulting from the grant:***

The results obtained are going to be summarized in, at least, two papers stemmed from the collaboration financed by the INSTANS ESF Program.

D***) Future Collaboration with host institution:***

A positive outcome of the visit has been also the possibility to foster an exchange program between the Quantum Information group directed by S. Bose at UCL and the Quantum Field Theory group directed by P. Sodano at the University of Perugia on topics related to the role of impurities in enhancing entanglement and, possibly, quantum state transfer in condensed matter systems relevant for the implementation of quantum information processing. There are many realistic condensed matter physics systems where a long distance entanglement between the ending points of a spin chain may be observed. In fact, spin chains may be implemented with ion traps [14], with trapped electrons [15], cold atomic systems [16] and Josephson junction arrays [17]. In a near future, we plan to address the emergence of long distance entanglement in the two latter systems: for this purpose, an exchange of graduate students between Perugia and London has been planned for 2009.

References:

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