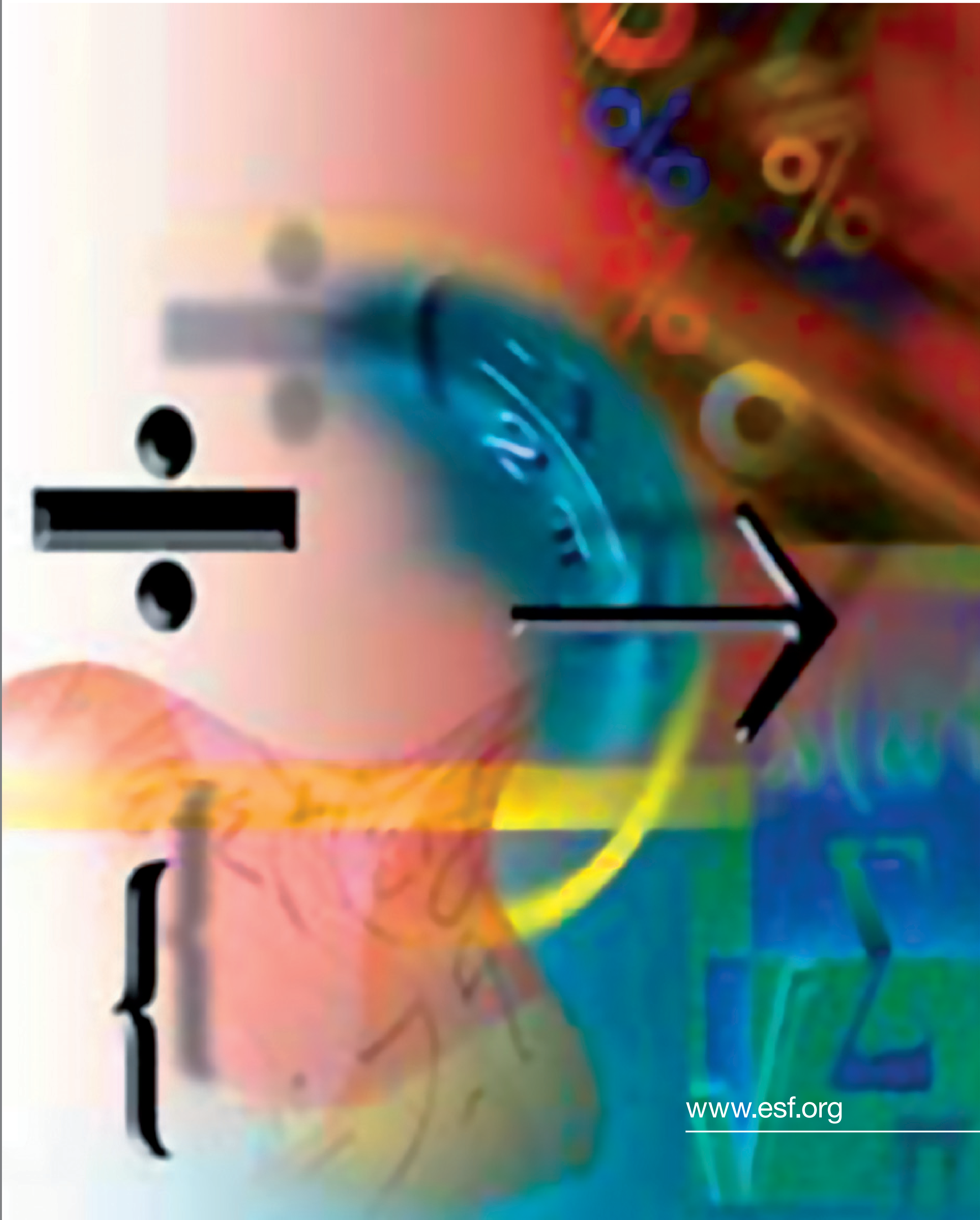
EUROPEAN SCIENCE FOUNDATION
SETTING SCIENCE AGENDAS FOR EUROPE

RESEARCH NETWORKING PROGRAMME

HARMONIC AND COMPLEX ANALYSIS AND ITS APPLICATIONS (HCAA)

Standing Committee for Physical and Engineering Sciences (PESC)



The European Science Foundation (ESF) was established in 1974 to create a common European platform for cross-border cooperation in all aspects of scientific research.
With its emphasis on a multidisciplinary and pan-European approach, the Foundation provides the leadership necessary to open new frontiers in European science.
Its activities include providing science policy advice (Science Strategy); stimulating cooperation between researchers and organisations to explore new directions (Science Synergy); and the administration of externally funded programmes (Science Management). These take place in the following areas: Physical and engineering sciences; Medical sciences; Life, earth and environmental sciences; Humanities; Social sciences; Polar; Marine; Space; Radio astronomy frequencies; Nuclear physics.
Headquartered in Strasbourg with offices in Brussels, the ESF's membership comprises 75 national funding agencies, research performing agencies and academies from 30 European countries.
The Foundation's independence allows the ESF to objectively represent the priorities of all these members.

Introduction

Mathematics is continually changing and there are no restrictions on what direction it will take in any particular area. The HCAA programme is a synthesis of several subjects including Theoretical and Mathematical Physics and Mechanical and Engineering applications. It is a forum for two communities: analysts with a broad background in Complex and Harmonic Analysis and Mathematical Physics, and specialists in Physics, Mechanics and Engineering Sciences.

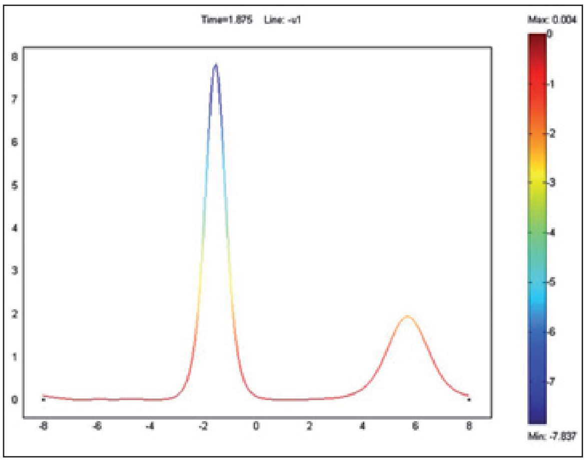
Physical and Engineering science prediction and intuition are sometimes braked by technical constraints, e.g. progress in elementary particle studies depends on the power of modern accelerators. Here, Mathematics provides new ideas and develops new concepts that must then be confirmed experimentally. Particular topics that will be considered by this programme include Conformal and Quasiconformal Mappings, Potential Theory, Banach Spaces of Analytic Functions and their applications to the problems of Fluid Mechanics, Conformal Field Theory, Hamiltonian and Lagrangian Mechanics, and Signal Processing.

The running period of the ESF HCAA Research Networking Programme is for five years from April 2007 to April 2012.

Scientific Background

Looking back at the history of physics and mathematics one can see that many important discoveries made have been understood and recognised only decades later, and their mutual connections and positioning in modern science have been reasoned and caused by the development of advanced mathematical tools. Three examples of particular importance can be given. The following works carried out at the end of the 19th and beginning of the 20th century have influenced the development of modern mathematics and physics and caused a real boom in these fields at the end of the 20th century. They are (in chronological order) a paper by Diederik Johannes Korteweg and his student Gustav de Vries (published in *Philosophical Magazine, 1895*), a small note placed in *Nature, 1898* by Henry Selby Hele-Shaw, and a seminal work by Albert Einstein, one of the five papers that changed the face of physics, published in *Annalen der Physik, 1905*.

All these great results and discoveries are intrinsically related and they could not have been made without development and progress in mathematical techniques and models, in particular in Harmonic and Complex Analysis in the broad sense. Harmonic Analysis provides deep insight into a vast array of modern topics of Real, Complex and Geometric Analysis and the associated topics of Mathematical Physics and Mechanics. A wide variety of problems of Mathematical Physics and Fluid Mechanics can be intrinsically modelled in terms of geometric and analytic methods, in particular when viewed under Hamiltonian formalism. A strong interplay between Partial Differential Equations, Integrable

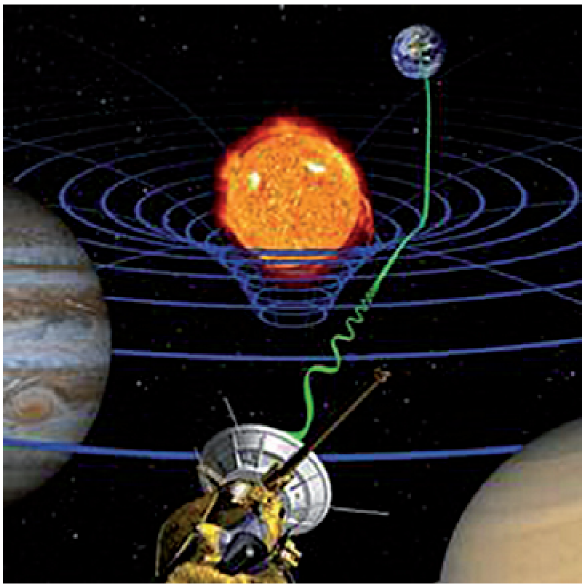


The KdV equation that originally described the 'solitary wave' discovered by the Scottish engineer John Scott Russell about half a century earlier, becomes the condition under which the potential of the spectral stability of the Schrödinger operator can be seen, eigenvalues of which lead to energy quantisation for bound systems. The existence of quantised energy levels is determined experimentally by observation of the energy emitted or absorbed when the system makes a transition from one level to another.

Scientific Background

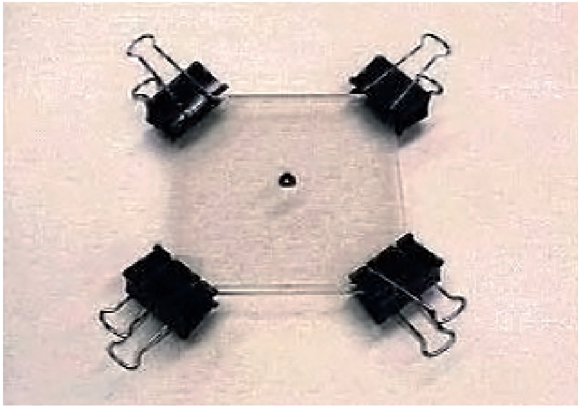
State of Arts

Systems, manifold structures (Riemannian and non-Riemannian), Field Theory on one side and Complex and Harmonic Analysis and Geometric Function Theory on the other allows us to concentrate our efforts. This project is dedicated to creating new approaches and general methods of considering all spectra of problems as well as to applications of these methods in Physics and Mechanics. Particular topics which will be considered by this programme include Conformal and Quasiconformal Mappings, Potential Theory, Banach Spaces of Analytic Functions and their applications to the problems of Fluid Mechanics, Conformal Field Theory, Hamiltonian and Lagrangian Mechanics and Signal Processing.



Einstein's special, and subsequent general, theory of relativity explaining gravity as the curvature of spacetime has the strong and simply formulated Einstein's equation. It is a dynamical equation which describes how matter and energy change the geometry of spacetime, this curved geometry being interpreted as the gravitational field of the matter source. The solutions to this equation are metrics. In the two-dimensional case, Einstein's equation with a cosmological term describes a metric of constant negative Gaussian curvature. Recent advances in the theory of gravitation include Polyakov's discovery that first-quantised bosonic strings propagating in R^d can be described as the theory of d free bosons coupled with with the 2-D quantum gravity, the theory formulated as a quantum geometry of Riemann surfaces, where the Liouville theory plays the principal role.

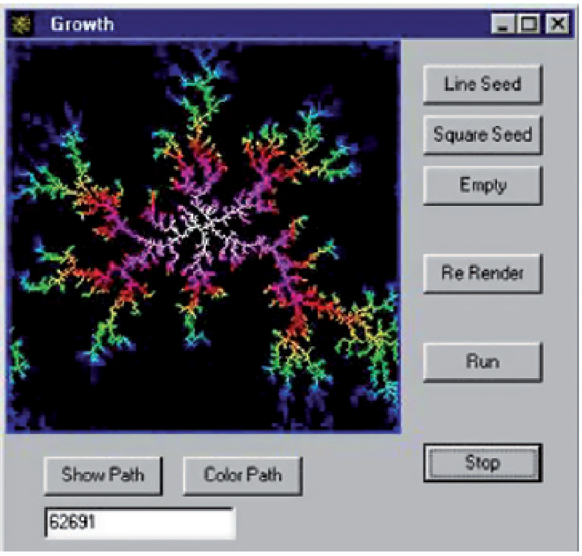
Harmonic analysis and its influence on differential equations and complex and potential analysis has a long and distinguished history starting with the Laplacian equation. While the main ideas of this proposal are inspired by three seminal works by Korteweg and de Vries, by Hele-Shaw and by Einstein, the main development and understanding of the topic has taken place only in the last couple of decades. It arises from certain classes of nonlinear differential equations known as *evolution equations*.



An important evolution equation characterising Laplacian Growth is concerned with the problem that arises from the analysis of viscous flow in plane parallel shells, which one can consider as the motion of petroleum in sections of porous medium. The problem of the boundary deformation in two dimensions is known as the 'free boundary problem' (Laplacian Growth) in Hele-Shaw cells.

The usual Dirichlet problem possesses new and unexpected features under the supposition of a moving boundary. Hele-Shaw first described his famous cell, that became a subject of intense investigation only more than 50 years later. Probably the most important characteristic of flows in such a cell is that when the Reynolds number based on gap width is sufficiently small, the Navier-Stokes equations averaged over the gap reduce to a linear relation analogous to Darcy's law, and hence to a Laplace equation for pressure. Through the similarity in the governing equations, Hele-Shaw flows are particularly useful for visualisation of saturated flows in porous media, assuming they are slow enough to be governed by Darcy's law. The next important step after Hele-Shaw's pioneering note was made independently by Polubarinova-Kochina and Galin who developed a complex variable approach to the Hele-Shaw problem.

The article on KdV was based on De Vries' PhD thesis in which he proved that the 'solitary wave', discovered and described by the Scottish engineer John Scott Russell about half a century earlier, really could keep its form. The KdV equation remained in obscurity until



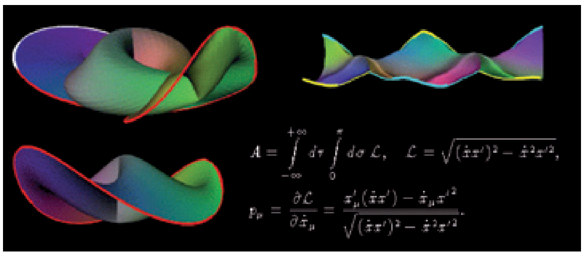
Diffusion Limited Aggregation simulation in Borland C++.

1965, when Zabusky and Kruskal discovered that two such solitary waves, or ‘solitons’, emerge unchanged from a collision. The discovery of this remarkable stability property caused a still ongoing tide of research (sometimes called the ‘soliton revolution’). Two years later, Gardner, Greene, Kruskal and Miura invented the Inverse Spectral Transform to solve the Cauchy problem for KdV, and with this method Zakharov and Faddeev proved, in 1971, the complete integrability of KdV. Also, in 1970, Kadomtsev and Petviashvili introduced the more general KP equation, used to study transversal stability of the soliton solutions to the KdV equation. Many applications (including Laplacian Growth) can be thought of as the dispersionless limit of this or of the integrable Toda hierarchy constrained by a string equation. Whereas many applications of KP theory are in the field of fluid mechanics, also vast areas of mathematics (algebraic curves, theta functions, vector bundles, Jacobian and Prym varieties, infinite-dimensional Grassmanians, commuting partial differential operators, algebraic and symplectic geometry, Lie group theory, differential geometry) and theoretical physics (quantum field theory, string and conformal field theory, quantum gravity, classical general relativity) opened up as a consequence of the basic research into the KP equation. It is worth remarking that the inverse scattering problem for dispersionless KP remains open.

Complex transition functions appear naturally in the theory of evolution equations. The original formulation of quantum Liouville theory through a path integral has been given by Polyakov in 1981. A thorough mathematical treatment appeared later in the works of Takhtajan and Zograf.

Another important contribution has been made by Saffman and Taylor in 1958 who discovered the long time existence of a continuum set of long bubbles between two parallel walls in a Hele-Shaw cell that have since been called the Saffman-Taylor fingers. A real burst of interest in this problem arose recently, as the great number of bibliographical references testifies. Nowadays, the Hele-Shaw cell provides a powerful tool in several fields of natural sciences and engineering, in particular, matter physics, materials science, crystal growth and, of course, fluid mechanics.

It is not very surprising that several quantum features appear in nonlinear problems of hydrodynamics. The ubiquitous nature of integrable systems and evolution equations, in particular, those implied by the KdV, KP (or analogous), Liouville theory, and Laplacian Growth (Hele-Shaw problem), is by now well established. Nevertheless, the search for their common origin has not yet been successful. The present project will bring together and generalise a number of concepts of PDE, Harmonic, Complex, and Geometric Analysis that may have been implicitly known and applied by practitioners in the field of integrable systems and evolution equations (Laplacian Growth, in particular) but which have not yet been placed on a broader and sound basis. The main idea of this project is to establish a fruitful cooperation between two scientific communities: analysts with a broad background in Complex and Harmonic Analysis and Mathematical Physics, and specialists in Physics and Applied Sciences.



The Liouville action and the Liouville equation play a key role in two-dimensional gravity. They are based on a Lagrangian given on the Riemannian geometry of the underlying space. Liouville action describes a highly non-trivial dynamics in quantum field theory and appears in Feynman’s path integral that represents the transition amplitude between two quantum states of a system expressed as a sum over contributions from possible classical histories of that system.

Research Topics

- Complex Analysis and Hamiltonian systems
- Free boundary problems
- Hamiltonian mechanics on non-Riemannian structures
- Inverse scattering problems for nonlinear configurations
- Spectral problems for Landau equations with impurities

Activities

Conferences
Three programme conferences will take place in 2007 (Norway), 2010 (Germany), and 2012 (Spain).

Conference Collaboration
The HCAA programme will collaborate with existing conferences to allow for a wider scope of researchers to attend. In particular:

2007
- Trends in Harmonic Analysis Strobl07, University of Vienna, Salzburg, Austria 18-22 June 2007
- International School and Conference on Geometry and Quantisation, Steklov Mathematical Institute, Moscow, Russia, 9-23 September 2007
- Potential Theory and Applications, Royal Technology Institute, Stockholm, Sweden 15-17 October 2007

2008
- Geometric and Stochastic Methods in Geophysical Fluid Dynamics, Jacobs University, Bremen, Germany, 7-11 January 2008
- Geometric Analysis and its Applications, University of Bern, Bern, Switzerland, 21-24 January 2008
- Second Winter School in Complex Analysis and Operator Theory, University of Sevilla, Seville, Spain, 5-9 February 2008

Exchange Visits
Two types of grants for exchange visits are available:
- Short visit grants of up to 15 days
- Exchange grants from 15 days to three months

Website
A homepage is maintained on the ESF website, in addition to the programme's own home page which can be found at: **http://hcaa.uib.no**

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For the latest information on this Research Networking Programme consult the HCAA websites:
http://hcaa.uib.no and **www.esf.org/hcaa**

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