Scientific Report on

COMPASS

COMPutational methods for Algebraic Spline Surfaces

An ESF Exploratory Workshop (EW 02–55)

Tor Dokken and Bert Jüttler (Co–Organizers)

September 29 – October 3, 2003

Contents

1 Executive Summary

The workshop, which was held at Landesbildungszentrum (State education center) of Schloß (Castle) Weinberg at Kefermarkt, took place from September 29 to October 3, 2003. In this executive summary, we provide information about the participants, the technical and social programme, and the planned publication of post–proceedings. We conclude it by analyzing whether or not the workshop has reached its objectives.

Structure of participants. The workshop was attended by 30 participants. In addition, two supporting doctoral students from Linz were present. The participants came from various European and non–European countries, making the workshop a truly international event. Both the groups of young and female scientists were well represented. See Section 5 for more details.

The participants also represented different fields of mathematics and Computer Science, such as Algebraic Geometry, Computer Aided Geometric Design, Computer Graphics, Differential Geometry, Numerical Analysis and Symbolic Computation. Due to this mixture, the workshop had a strong interdisciplinary character.

Technical Programme. The technical programme of the workshop consisted of 29 talks (with length varying between 30 and 45 minutes) and a panel discussion, which concluded the event. See Section 3 of this report for titles and speakers.

All talks had been carefully prepared and were presented at a very high standard. The speakers always tried to bring the ideas across, also to the part of the audience which had a different background than the speaker.

The technical sessions were characterized by lively discussions and exchange of ideas. The interest of the audience, as documented by the level of attendance, was very high: almost all participants were present throughout the entire programme.

By its very intention, the schedule of the programme was made to leave ample time for individual discussions, during the coffee and lunch breaks, and before dinner. Since all participants stayed together within the medieval castle, discussions frequently continued during dinner and even in the late evenings.

The programme was characterized by the interaction of the different fields, which were represented within the participants. It was demonstrated that closely related problems were attacked by participants from different fields by using different methods. For instance, Gravesen's talks on the differential geometry of implicitly defined surfaces was closely related to Elber's talk on curvature analysis and segmentation of trivariate data, which involved concepts from Computer Graphics. Other examples include curve and surface fitting, which was addressed by Thomassen, Wurm, and Wang, and problems from Computational Geometry (efficient computation of the intersection of two triangles), which played a prominent role in Sampoli's and Corvez' talks. Singularities and their classification were featured by Mourrain's talk in connection with visualization of surfaces, and the talks by Piene and Johannsen from the viewpoint of algebraic geometry. Last, but not least, approximate algebraic methods for curves and surfaces were used in various contexts by several speakers, ranging from Symbolic Computation (Perez, F. Winkler) to

Approximation Theory and Computer Aided Geometric Design (Aigner, Dokken, Wurm).

The meeting concluded with a panel discussion, which is documented in the Appendix to this report. The discussion circled around two questions which were asked to the audience.

The first one was whether or not it had made sense to bring together people from the different scientific communities of Computer Aided Geometric Design, Symbolic Computation, Algebraic Geometry, and others. The audience agreed that this was indeed the case, since it "forces you to live in a wider world" (J. Winkler). Also, the results of the other communities were widely seen as interesting and potentially useful.

In the second question, the audience was asked to identify open problems in their field, which could possibly benefit from the use of methods from the other fields. Starting from this question, a lively discussion emerged, which also included several suggestions for future research.

The panel discussion finally discussed whether a similar workshop should be held in the future. (This is not included in the transcript in the Appendix.) This was seen very positive. An attempt to organize such an event again could aim at 2005.

Social Programme. The workshop also included a trip to the monastery of St. Florian with an organ concerto at the Bruckner organ, and a guided tour. The conference dinner took place at a restaurant in Linz.

Proceedings. The two organizers have approached Springer Heidelberg, and discussed with them about a possible publication of post–proceedings. After consultation of the editors, Springer is inclined to publish them in its series entitled "Mathematics + Visualization" (edited by G. Farin, H.C. Hege, A.H. Hoffmann, C.R. Johnson and K. Polthier). Springer has expressed strong interest and will make the final decision after receiving the manuscript. So far, 23 participants have expressed interest in submitting a manuscript. January 15, 2004 has been chosen as the deadline for submissions. All submissions will be refereed. The process of refereeing and revising the submissions is expected to be finished by May 2004.

Scientific content. Summing up, the two organizers are convinced that the workshop has achieved its main objectives. First, it has provided an overview about the state-of-the-art in approximative implicitization and its application, including both the theoretical basis and the existing computational techniques. It is hoped that this will encourage and promote the use of approximate implicitization for solving geometric problems in computer-aided design and related fields.

Second, the discussions at the workshop have helped to identify a number of problems (both theoretical and practical ones) which need to be addressed by the different research communities, in order to exploit the potential of implicit representations.

Finally, the workshop has achieved its goal to support the mutual exchange of ideas between the various research communities, and to promote interdisciplinary research. It has been demonstrated that the interactions between different mathematical disciplines such as approximation theory, numerical analysis, classical algebraic geometry and computer aided geometric design will play an essential role for exploiting the new idea of approximate algebraic geometry.

2 Assessment of the results, contributions to the future direction of the field

One of the goals of this workshop was to give an overview about the state-of-the-art in approximative implicitization and its applications, including both the theoretical basis and the existing computational techniques. This was the subject of the talks by E. Wurm on algebraic curve and surface fitting, by J. Thomassen on the use for detecting and analyzing intersections and self–intersections, and by T. Dokken on weak approximate implicitization. In addition, a number of talks focused on closely related problems from algebraic geometry and computer aided geometric design, such as the talks by R. Martin on visualizing algebraic curves and surfaces, by R. Krasauskas on toric surfaces and spline curves on them, and many others. This survey of approximate algebraic methods will help to promote these new ideas for solving geometric problems in computer-aided design and related fields.

Second, the discussions at the workshop have helped to identify a number of problems (both theoretical and practical ones) which need to be addressed by the different research communities, in order to exploit the potential of implicit representations. In addition to the talks by Vibeke Skytt on challenges for surface–surface intersections, and B. Mourrain on symbolic–numeric techniques for curves surface, this was also addressed by the lively panel discussion which is documented in the Appendix to this report.

The third group of presentations was devoted to closely related problems from Applied Geometry, Numerical Analysis, Computer Graphics and others. They also included additional possibilities for applications of approximate algebraic methods.

The organizers are convinced that the workshop has achieved its goals. It has provided a summary of recent results in the emerging field of approximate algebraic methods and their applications. This was not only limited to computational techniques, but it also included the theoretical basis of this approach. It has been demonstrated that approximate implicitization is a valuable and powerful approach for solving geometric problems in computer-aided design and related fields, therefore promoting further research in this field. In addition, the workshop has discussed a number of possible problems that need future research. In addition to several talks, this was also addressed by the lively panel discussion. Clearly, this will encourage future research. Finally, the organizers are inclined to the view that the workshop has contributed to the exchange of ideas between the various research communities, and to promote interdisciplinary research. Cooperation of different mathematical disciplines such as approximation theory, numerical analysis, classical algebraic geometry and computer aided geometric design may lead to interesting new results, which may help to exploit the new idea of approximate algebraic geometry.

3 Final Programme

Monday September 29

09:00–09:30 Registration, Opening remarks, Announcements

09:30–10:15 Ralph Martin (Cardiff, Wales)
A Recursive Taylor Method for Algebraic Curves and Surfaces

10:15-10:45 Coffee break

10:45–11:15 Marc Daniel (Marseille, France)
Discrete curvatures for CAGD

11:15–12:00 Rimvydas Krasauskas (Vilnius, Lithuania)
Spline curves on toric surfaces

12:00–14:00 Lunch

14:00–14:30 Elmar Wurm (Linz, Austria)
Approximate Implicitization via Surface Fitting

14:30–15:00 Wenping Wang (Hongkong, China)
Spline shape approximation without data parameterization

15:00–15:30 Coffee break

15:30–16:00 Jens Gravesen (Copenhagen, Denmark)
Invariants of implicit surfaces

16:00–16:30 Joab R Winkler (Sheffield, England)
Numerical and algebraic properties of Bernstein basis resultant matrices

18:00–19:00 Dinner

Tuesday September 30

09:00–09:45 Gershon Elber (Haifa, Israel)
Global Curvature Analysis and Segmentation of Volumetric Data Sets Using Trivariate B-spline Functions

09:45–10:15 Carlo Traverso (Pisa, Italy)
Approximate Gröbner bases

10:15-10:45 Coffee break

10:45–11:15 Pal Hermunn Johannsen (Oslo, Norway)
What algebraic geometry can do for you: the case of the tangent developable

11:15–12:00 Franz Winkler (Linz, Austria)
Computer Algebra and Parametrization

12:00–14:00 Lunch

14:00–14:30 Panagiotis Kaklis (Athens, Greece)
On the Local Shape Effect of A Moving Control Point

14:30–15:00 Kestutis Karciauskas (Vilnius, Lithuania)
Construction of multisided tensor–border surfaces

15:00–15:30 Coffee break

15:30–16:00 Jan Brede Thomassen (Oslo, Norway)
Self-intersection problems and approximate implicitization

16:00–16:30 Vibeke Skytt (Oslo, Norway)
Challenges in surface-surface intersections

18:00–19:00 Dinner

Wednesday, October 1

09:00–09:45 Fernando Carreras (Santander, Spain)
Sturm–Habicht sequences

09:45–10:15 Zbynek Šir (Prague, Czech Republic)
Statistical error propagation in geometrical transformations

10:15-10:45 Coffee break

10:45–11:15 Martin Peternell (Vienna, Austria)
Computation of the convolution of surfaces from special classes

11:15–11:45 Martin Aigner (Linz, Austria)
Robustness of implicit representations

11:45–12:30 Lunch

Afternoon: Excursion to the monastery of St. Florian with the Bruckner organ

19:00–21:30 Conference dinner at the Pöstlingberg at Linz

Thursday, October 2

09:00–09:45 Ragni Piene (Oslo, Norway)
Classification of real algebraic curves and surfaces

09:45–10:15 Solen Corvez (Rennes, France)
An efficient algorithm for computing the real intersection points of two planar cubic A-splines

10:15-10:45 Coffee break

10:45–11:30 Pavel Chalmoviansky (Linz, Austria)
Algebraic subdivision

11:30–12:00 Bernard Mourrain (Nice, France)
Symbolic-numeric methods for curves and surfaces

12:00–14:00 Lunch

14:00–14:30 Sonia Perez (Madrid, Spain)

Rational Parametrizations of Approximate curves and Surfaces

14:30–15:00 Josef Schicho (Linz, Austria)
Del Pezzo Surfaces

15:00–15:30 Coffee break

15:30–16:00 Lucia Sampoli (Siena, Italy)
Minkowski sum of LN surfaces

16:00–16:30 Ibolya Szilagyi (Linz, Austria)
Symbolic-numeric techniques for cubic surfaces

18:00–19:00 Dinner

Friday, October 3

09:00–09:45 Tor Dokken (Linz, Austria)
Weak approximate implicitization

09:45–10:15 Frank Lenzen (Innsbruck, Austria)
Automatic detection of arcs formed by gravitational lensing

10:15-10:45 Coffee break

10:45–11:45 Panel discussion
Interaction of Computer Aided Geometric Design, Algebraic Geometry and Symbolic Computation

11:45–12:00 Concluding remarks

12:00–13:00 Lunch

4 List of participants

Participants marked with an asterisk (⋆) are "young scientists".

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5 Statistical information on the participants

The 30 participants (plus 2 supporting students) represented the following 10 European and 2 non–European countries: Austria (7 + 2 supporting students), Norway (6), France (4), Italy (2), Lithuania (2), Spain (2), United Kingdom (2), China (1), Czech Republic (1), Denmark (1), Greece (1), and Israel (1).

While 11 participants (36.7 %) can be considered as "young scientists" by the ESF definition (this does not include the two supporting students), 5 of them (16.7%) were female.

6 Date and signature

Linz, November 19, 2003

Bert Jüttler

Appendix: Panel discussion

The following is the transcript of the panel discussion which took place at the last day of the workshop.

Contributors: Marc Daniel (MD), Tor Dokken (TD), Jens Gravesen (JG), Panagiotis Kaklis (PG), Rimvydas Krasauskas (RK), Ralph R. Martin (RM), Bernard Mourrain (BM), Josef Schicho (JS), Carlo Traverso (CT), Joab R. Winkler (JW)

Moderator: Bert Jüttler (BJ)

BJ: I have prepared a few questions to this audience for this panel discussion. Of course, these are just suggestions; any other comments questions are welcome.

This workshop has brought together experts from different communities: Computer Aided Geometric Design, Symbolic Computation, and Algebraic Geometry.

Do you think that this combination has made sense? Did you benefit from the presentations and discussions at this workshop, also if they were not fro your "native" field?

(Many:) Yes.

CT: During the programme of the workshop, new challenges for symbolic computation have been identified. We have seen examples where other approaches [from CAGD, B.J.] fail, and where symbolic techniques might help. It is unusual to see the failures of others in conferences, but it helps to identify new problems.

JW: This combination forces you to live in a wider world, not just in your native field.

MD: Currently, we are experiencing a great change in CAGD. While the traditional approaches relied almost exclusively on parametric representations, now implicitly defined curves and surfaces start to play a greater role. It is very interesting to exploit the advantages of both representations, and we will have to work to make the most out of it.

RM: In the past, symbolic methods were often regarded as being too slow to be of any practical use for geometric applications. This workshop has shown us that combining symbolic and numerical methods can be done in a useful way, to make practical algorithms — for example, being able to replace rationals with reals for most of the calculation, and only use more precise methods when specifically required.

JS: The workshop was a good opportunity to meet people who one does not meet usually at workshops and conferences in my field; I have enjoyed this very much.

JW: It has been stated by a leading mathematical authority that numerical analysis and computational linear algebra will become increasingly important because many problems only have a numerical solution, and thus a discrete (eg linear algebraic) rather than continuous (eg integral) equation is used to obtain the solution. The CAGD community must be aware of these numerical issues in order that reliable solutions to difficult problems be obtained.

TD: It seems that the CAD industry is currently not very interested in new ideas. Due to the concentration process CAD market, now very few vendors dominate the market, and they are trying to maximize their profits, not necessarily to improve their products. As another trend, high–end CAD systems are now being sold to everybody, since powerful computers are now available to everyone. Still, many things in CAD systems have to be improved, such as the robustness of intersection algorithms, and the data exchange. It is currently not clear to me whether the CAD industry is ready to address these issues.

BJ: It is hoped that the combination of knowledge from different fields helps to solve real problems and to gain new insights.

Which problems from Computer Aided Geometric Design, Symbolic Computation, or Algebraic Geometry could benefit from the use of methods or results originating in the other fields?

Are there any (obvious) (more or less) new questions in one of these fields, which are motivated by the other ones?

RM: Is there a way to parameterize an implicitly defined curve or surface by functions involving not only rational expressions, but also square roots, or even more general functions? More usefully, can algorithms be devised for doing this? Methods to decide if and when this is possible or not would be useful, but even more useful would be algorithms to generate the parameterization.

As another question, Sturm sequences and their multivariate generalizations can be used for counting the number of real zero-dimensional roots in a box. What if the solutions have higher dimension? Is an algorithm possible to count the number of (real) separate connected solution pieces of each degree which are contained in a box?

BM: The answer is yes! This is answered by Hermite's Theorem.

CT: No, the situation becomes more difficult if one is interested in real solutions.

MD: Is there still a future for NURBS surfaces? Is it still interesting to concentrate on these surfaces? Very often one starts from points, and it is not always straightforward to generate surfaces. Perhaps it would be better to continue working with the points, instead of bothering to generate surfaces.

BM: As an advantage, NURBS surfaces encode the geometry in a very compact way.

TD: NURBS surfaces are in the standards! However, NURBS are currently used only to represent relatively simple shapes. As a matter of fact, NURBS can represent much more complex shapes, but currently we do not know how to do this. We should try to develop techniques for fully exploiting the potential of NURBS representations. This is closely related to the parameterization problem, that is, to the problem of parameterizing implicitly defined surfaces.

JS: In Bernard Mourrain's talk, we have seen that it is difficult to correctly visualize surfaces, especially in the neighborhood singularities. What precisely is the mathematical information needed for visualizing surface singularities correctly?

TD: A new approach, which exploits exploit capabilities of recent hardware, is to avoid triangulation completely. Instead, the surface can be evaluated directly.

PK: I propose a "life–cycle" philosophy for CAD: During their life, from conceptual design via numerical analysis and simulation to detailed construction and manufacturing, curves and surfaces may need different representations, but the different representations should "talk to each other". What we need is research on different representations, also in order to explore the transformations between them. Obviously, different representations of curves and surfaces are differently well suited for certain applications in the product life cycle.

TD: Currently, a big problem is to build the results of numerical simulations into an existing CAD model. More precisely, the results of a simulation has to be reflected in the model. Currently, this is a very difficult problem in industry, and I expect that this will not be fully solved within the near future.

RM: Many geometric problems can be viewed as finding the solutions to a set of algebraic (or more general implicit) equations in a set of unknowns. These may represent geometry, constraints on the geometry, and so on.

Implicitization and parameterization can be looked at as being specific questions regarding particular sets of equations: if we have multiple equations, how can we reduce them to a single equation; if we have a single equation, how can we introduce extra variables to help us e.g. draw the geometry?

In the more general setting, we can now ask a more general question — how can we transform the set of equations into a new set of more or fewer, or just different, equations, in more or fewer or just different variables? When it is better to decrease the number of variables, at the expense of a more complicated representation? When is it better to increase the number of variables, in order to obtain a simpler description?

Clearly, Groebner bases are related to this question, but are not the complete answer, I believe, as the issue of parameterization is not really addressed by Groebner bases.

JS: In algebraic geometry, these are the concepts of projection vs. unprojection.

RK: Can we identify a class of surfaces which have a "good" implicit and parametric representation at the same time? As a good class of candidates one could look at the class of del Pezzo surfaces of degree 4, which we have seen in Josef Schicho's talk.

JG: I do not think that this will help. For instance, for practical problems from physics, the geometry of an object might already be given. In this situation, the restriction to such a special class of surfaces would make the life much more difficult.

TD: I agree that simple surfaces might not be flexible enough for everything. However, I have observed that designers simply like certain shapes.In recent years, CAD systems have not seen much development in user interfaces. Still, what we have are mostly 2D interfaces! For instance, 3D curves still mainly generated by intersections. This is not satisfying at all, and other techniques would be much better, since designers still have big problems to get their ideas into the CAD system, and sometimes they simply give up trying. So, the current CAD systems pose limits to the designers' creativity, instead of inspiring them. Perhaps, certain classes of surfaces, of CAD models can contribute to shape modeling. But how can we achieve this? How can we explain better that certain shapes, something like a library of shapes, are available?

RK: This is a question of good control handles!

TD: We need to interface of different technologies, perhaps virtual reality.

As another issue, intersection algorithms are still difficult. I believe that a better theoretical basis for CAD will be needed in the future, since the growing demands for accuracy make problems like intersection even more difficult.

JG: Is there sensible way to restrict CAD systems to stable singularities? I feel that unstable singularities should be excluded beforehand, unless we are able to handle them in a reliable way.

JW: What is a stable singularity?

RM: Part of the issue is what is primary information, and what is derived information. An example of an unstable computation, given in the talk by Vibeke Skytt, is the attempt to compute intersections between tangential surfaces, e.g. a blending surface, and one of the base surfaces used to define it. Here the primary information is really the base surface,

and the trimming curve on the base surface which defines where the blend meets it. The blend surface should be derived from this information, rather than trying to compute the intersection curve from the two surfaces.

Some singularities can be avoided by being careful what operations the user interface allows the user to perform.

BM: I believe that, singularities cannot always be avoided. Instead, it is better to find the exact solution, and to deal with it directly.

JG: I agree.

TD: I have experienced that designers like singular shapes, and near-singular situations, such as surfaces meeting each other tangentially or almost tangentially. Thus, we will have to face these problems.

JG: This approach should be embedded in the underlying design philosophy, that is, in the user interface!

TD: According to my experience, trimmed surfaces often make problems.

RM: As a comment, the current version of STEP (the ISO standard for the exchange of CAD data) does not include the design history. The ISO certainly see this as a serious deficiency, and future versions will include such information. This will help to avoid some of the singularities and other problems caused by exporting data from one system and importing it into another with tighter tolerances. This symbolic information may also help systems to ascertain what the designer's intent was in singular cases.

TD: I agree, but converting design histories from one CAD system to another is much more difficult than converting models. This would imply to standardize not only the underlying representation, but even the available design tools? Already now, people simply cannot design certain shapes, due to the lack of available tools. A further standardization would make things worse.

JG: Here, open standards would do much better! It should be possible to include new methods, new types of surfaces or new classes of shapes into the standard.

TD: This may not be in the genuine interest of the CAD industry. Nevertheless, it will be important for other industries. In any case, this is a great challenge, this design history can sometimes be very, very difficult. For instance, if the design has partly be obtained by a numerical simulation.

As another remark, I feel that something like a "CAGD-hikers guide to algebraic geometry", and vice versa might be helpful. We use many similar concepts, but they come with different names.