Research Networking Programmes

Exchange Visit Grant

Scientific Report

Proposal Title: **Conley index applied to switched controlled systems**

Application Reference N°: ***4879***

Many systems encountered in practice involve a coupling between continuous dynamics and discrete events. Systems in which these two kinds of dynamics coexist and interact are usually called hybrid. For example, the following phenomena give rise to hybrid behaviour: a valve or a power switch opening and closing; a thermostat turning the heat on and off; biological cells growing and dividing; a server switching between buffers in a queueing. network; aircraft entering, crossing, and leaving an air traffic control region; dynamics of a car changing abruptly due to wheels locking and unlocking on ice. Hybrid systems constitute a relatively new and very active area of current research. They present interesting theoretical challenges and are important in many real-world problems. Due to its inherently interdisciplinary nature, the field has attracted the attention of people with diverse backgrounds, primarily computer scientists, applied mathematicians, and engineers.

Researchers with a background and interest in continuous-time systems and control theory are concerned primarily with properties of the continuous dynamics, such as Lyapunov stability. A detailed investigation of the discrete behavior, on the other hand, is usually not a goal in itself. In fact, rather than dealing with specifics of the discrete dynamics, it is often useful to describe and analyze a more general category of systems which is known to contain a particular model of interest. This is accomplished by considering continuous-time systems with discrete switching events from a certain class. Such systems are called switched systems and can be viewed as higher-level abstractions of hybrid systems, although they are of interest in their own right.

If the continuous part is nonlinear, it exhibits extremely complex behavior with respect to both the system variables and parameters. Such complex behavior proven in theoretical work has to be contrasted with the capabilities of application; in the case of modeling multi-scale processes, for instance, measurements may be of limited precision, parameters are rarely known exactly and nonlinearities are often not derived from first principles.

This contrast with the linear case suggests that extracting robust features which persist over a range of parameter values is of greater importance than a detailed understanding of the ne structure at some particular parameter.

On the other hand, there exists the Conley index and the Morse decomposition approach, is used to obtain a coarse yet robust description of the global (nonlinear) dynamics at a resolution specified a priori. A crude but rigorous characterization of the local dynamics is given via the Conley Index - an algebraic topological invariant. The idea is to try to use this description for navigate between modes of the switching systems and then to stabilize it.

The work focuses on the automatic determination of a maximal stability zone of a (differential) system, and its associated (discrete) controller via Morse decomposition and Conley Index.

Then the main goal of this exchange is to bridge the gap between classical mathematical control theory, the interdisciplinary field of hybrid systems and algebraic topology, the formers being the point of departure. More specifically, algebraic topology tools will be used to analyze and synthesize systems that display quite nontrivial switching behavior and thus fall outside the scope of traditional control theory. In that it will be a little revolution and a new application field of the algebraic topology.

In the beginning of the exchange we worked on a better definition of the problem and establish what exactly in the Conley - Morse theory could be used to solve our problem. We also discovered some old works on the application of the Morse Decomposition on Control Theory.

After that we worked on the conditions on a process described by the hybrid system to give real dynamical system. From that work we found those conditions and start to play/ apply with success the Conley index to find and characterize invariant sets for some toy/ simple examples of hybrid systems like the Boost DC-DC Controller with two subsystems.

We were also interested in trying to find isolating neighborhoods for flows using set theoretic methods, other than interval arithmetic. We have an experience with polyhedra, zonotopes, linear and non-linear templates etc. and how to use them to give over-approximation of functions. Also, we have thought a new ways to "solve" Inv(B) \subset int(B). In particular given such "shapes". But of course, if we use homological methods, we then have to see how to compute homology of slightly more complex spaces (than the very specific grid shapes cubical sets).

From that research is born the most significant advance of this exchange : the use of real semi-algebraic geometry to find isolating blocks (which are special isolating neighbourhoods with nice properties) as semi-algebraic sets for flows and compute the homological Conley index from a theorem which gives a triangulation of semi-algebraic sets.

After that exchange my advisors went to Mrozek's team in Poland to continue the collaboration and came back in France with new ideas that we also hope to develop with the Mrozek's team.

Indeed during this first collaboration we only used the first part of the theory (only the Conley index), and now we hope to work together on the Morse decomposition of hybrid systems to give more details on the structure of their invariants.

We are currently writing two articles in parallel : "Conley index computing in semi-algebraic sets" and "Approach based on isolating neighborhoods/blocks as some generalized Lyapunov method for switching systems".