



Research Networking Programmes

Short Visit Grant or Exchange Visit Grant

(please tick the relevant box)

Scientific Report

The scientific report (WORD or PDF file – maximum of eight A4 pages) should be submitted online within one month of the event. It will be published on the ESF website.

Proposal Title: Stratified Discrete Morse Theory

Application Reference N°: 4904

1) Purpose of the visit

Research activity with Neza Mramor Kosta, Primoz Skraba and Gregor Jerse on new perspectives for Discrete Morse Theory. The main purpose of this project is to characterize discrete Morse functions when defined on simplicial complexes that are triangulations of stratified topological spaces (e.g., real analytic spaces), and compute the homology through a Morse complex able to take information from the additional structure provided by the strata.

2) Description of the work carried out during the visit

I spent mostly of the first month in Ljubljana studying the foundational book on Stratified Morse Theory by Goreski and MacPherson, as well as different papers on the subject. I gave two seminars at the Faculty of Mathematics and Physics on the generalization of some of the main results in the classical Morse theory when spaces with singularities, instead of manifolds, are considered.

The second month was focused on meetings and long discussions with my colleagues on the right approach to be followed for a suitable description of the behaviour of a discrete gradient vector field along the strata and in the neighborhoods of critical simplices. I gave also an invited talk at the Department of Mathematics, University of Zagreb, entitled "A stable combinatorial distance for Reeb graphs of surfaces", on my recent research work join with Claudia Landi.

In the last month, we started to formalize the first definitions and results, and proceeded to speak and share our ideas. On 21st of January, I gave a talk at the Faculty of

Mathematics and Physics to show our preliminary results and share ideas with the audience of topologists. On 27th of the same month, I went to Institute for Medical Informatics, Medical University Graz, for an invited seminar on "Geometric-topological tools for shape description", and on 28th I gave an invited talk entitled "A stable combinatorial distance for Reeb graphs of surfaces" at the Institute of Science and Technology Austria, in Wien.

In these last weeks, we have also started to organize the ACAT Summer School 2015 that will be held in the next June in Ljubjana.

3) Description of the main results obtained

At the moment, we have formalized the concept of equivalence of paths in Discrete Morse Theory. This equivalence is induced by the inclusion of the stars of adjacent simplices, where a certain equivalence of pairs (i.e. elements belonging to the discrete vector field) occurs. In this work in progress, for each star, we are trying to figure out how to obtain the transversal information (leading to the definition of normal Morse data) in terms of equivalent incoming and outgoing paths. This would give information on the type of simplex we are considering: if it corresponds to a manifold or non-manifold point of the associate space X , and moreover, to a regular or critical point of the associated function. Then, we can pass from the local to the global schematical knowledge of the flow on the stratified object through equivalent paths which link two different critical simplices.

4) Future collaboration with host institution (if applicable)

Collaboration in the organization of ACAT Summer School 2015.

5) Projected publications / articles resulting or to result from the grant (*ESF must be acknowledged in publications resulting from the grantee's work in relation with the grant*)

This work will be continued with the purpose to write a paper on the obtained results, and (hopefully) an algorithm for determining the Morse complex associated with a discrete Morse function on a stratified simplicial complex. This would facilitate the computation of its homology groups.

6) Other comments (if any)

I just would like to thank ESF Research Network "Applied and Computational Topology" for this great opportunity.