A Scientific Report On The Research Visit

By Žiga Virk

The purpose of the visit was to establish a collaboration between Virk and prof. Edelsbrunner’s group. Virk started his career as a general topologist but has shifted his research focus to computational topology. Being an established researcher on the fields of wild spaces, coarse geometry and continuum theory, Virk indends to use his previous experience to make an impact on the field of computational topology. In particular, the aim is incorporate into the existing framework the ideas of hyperbolic geometry and generalizations of the metric structure.

A majority of the work has been carried on with the collaboration of Edelsbrunner and Wagner on the project of the topological data analysis of text documents. During the three months of the visit the project has consistently increased in its volume and potential impact. In its current form in consists of several results:

* An introduction of the Bregman divergence into the standard framework of the computational topology, including the persistence, by proving that the standard constructions are mostly preserved in a non-symmetric divergence setting
* An introduction of the Logarithmic transform of the term vectors resulting in a similarity measure, which is applied in order to study the persistence.
* The establishment of a connection to the standard tools of other fields, such as the Kullback-Leibler divergence.

Virk has contributed significantly to these development, providing mathematical proofs (such as the proof of convexity of the Log transform function; connection and establishment of the triangle inequality and its failure in the basic case; comparison of the Rips and Cech construction in various settings; … ) and developing new aspects of the project, which resulted in additional directions of research (such as the connection to the hyperbolic geometry; connection to the curvature of the graph; …). The project is to continue by applying the developed techniques to text documents and other large data sets. At least one paper, and possibly one or two more, are expected to be published as a result of this work.

The second project initiated during the visit is the study of persistence in the setting of hyperbolic geometry. A few basic observations on the subject were noted by Chezal et al. Virk’s work on the subject has produced a general overview of intrinsic differences between the Euclidean and the hyperbolic setting. In particular, the following results are either obtained or currently in the phase of development:

* The restriction of the persistence diagram on the initial band, determined by the hyperbolicity constant.
* The logarithmic bound on the life span of a cycle, depending on the generator length (as opposed to the polynomial bound in the case of Euclidean space).
* The effect of the exponential volume growth in the hyperbolic setting.

The obtained results are expected to result in at least one paper. The potential experimental application may result in additional papers. Furthermore, the current results are the beginning of a general data analysis setting in the hyperbolic setting which hasn’t been developed yet and which Virk is expected to work on. For example, in geometric group theory and in geometric topology the hyperbolic structure turns out to be perhaps the most powerful intrinsic invariant of the field. It is therefore reasonable to expect that influential results may be obtained by the study of some datasets in the hyperbolic instead of the standard Euclidean setting.

As mentioned above, both projects are currently under way and will be worked on in the future. In fact, the collaboration was so fruitful that Virk is to come to the IST Austria in the fall of 2015 for two years to join the Edelsbrunner’s group. The aim is to complete the mentioned projects and then start working on new projects as well.