

FINAL REPORT FOR THE INFTY EXCHANGE GRANT 2535

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Period of the visit: 1 January 2010 – 1 April 2010

Title of the project:

Axiom A revisited

Realization of the research plans.

My research plans concerned answering some open problems in the area of forcing, particularly connected with forcing notion satisfying Axiom A.

In an attempt to generalize the well-known results concerning Forcing Axioms, like PFA and BPFA to wider classes of forcing notions, I was examining the Bounded Forcing Axioms for α -proper forcing notions. This was motivated by the characterization of Axiom A provided by Ishiu, which says that Axiom A is equivalent to $< \omega_1$ -properness. One of the interesting problems here was the question whether these forcing axioms are equivalent. This was motivated by earlier work of Weinert, who proved that BPFA is not equivalent to the Bounded Forcing Axiom for Axiom A forcing notions. Another important question in this area concerns the impact of these forcing Axioms on the size of the continuum. Motivated by the celebrated result of Moore for BPFA, I was trying to determine whether these weaker forcing axioms also imply that $2^{\aleph_0} = \aleph_2$. The latter is known to follow from the unbounded version of the forcing axiom even for forcing notions which are two-step iteration of a σ -closed forcing and a ccc forcing. On the other hand,

the latter is connected to the old conjecture of Baumgartner saying that every Axiom A forcing notion is completely embeddable into such iteration.

During my visit in Vienna I answered the question of Baumgartner in the negative. Namely, I showed that there is an Axiom A forcing notion which is not completely embeddable into σ -closed*ccc iteration.

Moreover, together with Asperó, Mota and Friedman, we showed that the Bounded Forcing Axioms for α -proper forcing notions are not equivalent for indecomposable countable ordinals α . To separate them, we used a version of Club Guessing Principles.

The question about the size of the continuum remains, however, open, even for the Bounded Forcing Axiom for ω -proper forcing notions.

The first result yields, however, a new interesting question: whether the Bounded Forcing Axiom for Axiom A forcing notions is equivalent to the Bounded Forcing Axiom for forcing notions embeddable into σ -closed*ccc iteration.

I am going to continue the work in this subject, focusing on the remaining problems.