

SCIENTIFIC REPORT

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Visitor : Professor Ali Enayat, Washington, DC, United States

Host: Professor Anders Björner, Institut Mittag-Leffler, Djursholm, Sweden

Unit(s): PESC / SCH

Activity Title: New Frontiers of Infinity: Mathematical, Philosophical and Computational Prospects

Activity Acronym: INFTY

1. Purpose of the visit. I was an invited participant in the research program *Mathematical Logic: Set Theory and Model Theory* during Fall 2009 at the Institut Mittag-Leffler of the Swedish Academy of Sciences. I spent two months at the Institut Mittag-Leffler (September 1 - November 1, 2009). This stay provided me with the precious opportunity to spend an extended period of uninterrupted time to work on my research program, with the added singular benefit of open access to a most distinguished international group of mathematical logicians. I am grateful for this opportunity to concentrate on my research, and to be able to interact in an informal but highly effective environment with my colleagues.

2. Description of the work carried out during the visit. My activities during my visit can be categorized as follows.

A. Preparation of a substantially revised draft of a joint paper with Prof. Shelah: *An improper arithmetically closed Borel subalgebra of $\mathcal{P}(\omega)$ mod FIN*. This paper has been submitted to the preprint series of the Institut Mittag-Leffler and is submitted for publication.

B. Preparation of the first draft of the paper *Which models of PA arise as the standard model of some model of ZF?* The results of this paper were obtained during the first two weeks of my stay. Discussions with Prof. Kossak and Prof. Väänänen were highly influential in shaping this paper.

C. Preparation of the first draft of the paper *Borel structures and models of PA*. This paper benefited from key conversations with Prof. Kossak, Prof. Baldwin, Prof. Veličković, and Dr. Schlicht.

D. Preparation of the first draft of the paper *Automorphisms of models of set theory, a survey*.

E. Prof. Kossak and myself co-organized a weekly informal seminar on models of arithmetic and set theory. I presented 4 talks at the seminar: the first talk dealt with the results of the paper mentioned above in item B. My second talk reported on some recent work on satisfaction classes and interpretations; and the last two talks dealt with the results of the paper mentioned in item C above.

3. Description of the main results obtained. The main new results obtained during my stay at Institut Mittag-Leffler relate to the papers mentioned in items B and C above. Here is the summary of the key results of each paper.

- The paper *Which models of PA arise as the standard model of some model of ZF?* provides an axiomatization of the theory PA^{ZF} of the class of models of arithmetic that arise as the standard model $\mathbb{N}^{\mathcal{M}}$ of arithmetic of some model \mathcal{M} of ZF. It is well-known that in general $\mathbb{N}^{\mathcal{M}}$ is either isomorphic to the standard model \mathbb{N} of arithmetic, or $\mathbb{N}^{\mathcal{M}}$ is recursively saturated. It turns out that the converse is true not only for countable models, but also for Borel models, i.e.,

Theorem. *Suppose \mathcal{A} is a recursively saturated model of PA^{ZF} such that \mathcal{A} is Borel. Then there is a model \mathcal{M} of ZF such that $\mathcal{A} \cong \mathbb{N}^{\mathcal{M}}$.*

It is also shown that “Borel” cannot be removed from the above result.

- The paper *Borel structures and models of PA* provides a unified, perspicuous framework for the construction of Borel structures using certain well-known techniques for building nonstandard models of arithmetic. In particular, it is shown that one can employ Gaifman’s “minimal types” to construct totally Borel uncountable models of any theory T with an infinite model; this result was originally established during the 1980’ by H. Friedman, and (later but independently) by Malitz-Mycielski-Reinhardt. Moreover, as shown in the paper, one can use Schmerl’s classical construction of a conservative elementary end extension of models of the form $(\mathbb{N}, X)_{X \in \mathcal{G}}$, where \mathcal{G} is any collection of mutually generic Cohen reals (\mathcal{G} is allowed to be uncountable) to establish that if the Stone space of T is uncountable, then there is a

Borel model of T that realizes uncountably many types; this result, in its current form, is the generalization of a theorem of Woodin about uncountable Borel ω -models of \mathbf{ZF} , and was first established in the collaborative work of Woodin-Knight-Jockush-Simpson (unpublished) in 1985.

4. Projected publications/articles resulting or to result from the grant. The 4 projected publications that were either facilitated (two items) or directly resulted by the grant (two items) are the papers specified in items (A)-(D) above.