ESF - Exchange Grant - Final Report

Purpose of the visit

I applied for the Exchange Grant in order to participate in the scientific program "Mathematical Logic: Set theory and model theory", which takes place in the Institut Mittag-Leffler in Djursholm from September 1 to December 15, 2009. This program is related to the INFTY activities. The purpose of my stay was to broaden my mathematical horizons, learn forcing and conduct research free of any teaching duty.

Description of the work carried out during the visit

Beside the ESF Exchange Grant I have been awarded a postdoctoral fellowship from the Institut Mittag-Leffler, which allows me to participate in the scientific program during the whole fall semester although the financial support from ESF covers only the first month of my stay.

During the first month of my stay I investigated some questions concerning $\mathcal{I}$-ultrafilters within the class of P-points and also the interaction between $\mathcal{I}$-ultrafilters and rapid ultrafilters, a topic which was opened and not completely solved in one of my previous papers. I considered several questions concerning analytic P-ideals, with particular attention towards the cardinal coefficients defined by Piotr Borodulin-Nadzieja. I looked also at some standard cardinal invariants like the splitting number and the distributivity number and particularly at their connection with products of sequentially compact spaces.

Most of the time I spent reading articles related to above mentioned questions and articles about various cardinal invariants, or learning forcing, since mastering forcing was one of my main expectations about this semestral stay. For this I read carefully the chapter on forcing in Kunen's book "Set Theory – An Introduction to Independence" and also several papers about applications of the forcing method e.g. about Laver forcing or Mathias forcing.

In the end of the month I gave a talk in the local seminar, in which I presented some of my older results.

Description of the main results obtained

Although I focused on studying and learning new methods (forcing) during the first month of my stay, I obtained some new results.

$\mathcal{I}$-ultrafilters and P-points

It is known that an ultrafilter on ω is selective if and only if it is a P-point and a Q-point. Since Q-points are weak thin ultrafilters, it is not at all surprising that selective ultrafilters can be also characterized as P-points, which are thin ultrafilters. I considered the question whether it is possible to replace thin ultrafilters in the previous characterization by another class of $\mathcal{I}$-ultrafilters for an ideal $\mathcal{I}$ on ω. The following theorem claims that the class of P-points, which are at the same time $\mathcal{I}_{1/n}$-ultrafilters, is consistently strictly larger than the class of selective ultrafilters and even semiselective ultrafilters, which were described as rapid P-points.

Theorem 1. (MA_{ctble}) *There exist a P-point which is an $\mathcal{I}_{1/n}$-ultrafilter, but not rapid.*

Subsequently, I introduced two ideals on ω, which are contained in $\mathcal{I}_{1/n}$, in order to refine the classfication of various ultrafilters between selective ultrafilters and P-points. I proved that these two ideals

$$\mathcal{S}_0 = \{A \subseteq \mathbb{N} : \lim_{n\to\infty} \frac{1}{\sqrt[n]{a_n}} = 0\} \qquad \mathcal{S} = \{A \subseteq \mathbb{N} : \limsup_{n\to\infty} \frac{1}{\sqrt[n]{a_n}} < 1\}$$

determine the same class of $\mathcal{I}$-ultrafilters, which is contained in the class of all $\mathcal{I}_{1/n}$-ultrafilters and contains all thin ultrafilters. Unfortunately, the exact relation between those ultrafiters (in particular, whether the inclusions are consistently strict) has not been cleared yet.

"Expansive ideals"

For a given ideal $\mathcal{I}$ on ω we define $s(\mathcal{I}) \subseteq \mathscr{P}(\omega)$ as a family consisting of those increasingly enumerated sets $A = \{a_n : n \in \omega\}$ for which there exists $B \in \mathcal{I}^+$ such that $\{a_n : n \in B\} \in \mathcal{I}$. It is easy to see that $s(\mathcal{I}) \supseteq \mathcal{I}$. We call an ideal $\mathcal{I}$ *expansive* if $s(\mathcal{I}) \setminus \mathcal{I} \neq \emptyset$.

The summable ideal $\mathcal{I}_{1/n}$ or the density ideal $\mathcal{Z}_0$ are expansive, van der Waerden ideal is not expansive. The quest for a characterization which ideals are expansive and which are not, is in progress.

Future collaboration with host institution

Future collaboration with the Institut Mittag-Leffler is conceivable only if a scientific program in an area close to my research interest (set theory, set-theoretic topology) is organised there, which is not expected in the near future.

Projected publications/articles resulting or to result from your grant

An article based on my new results concerning $\mathcal{I}$-ultrafilters and P-points is projected. Further investigation of ”expansive ideals” may eventually lead to another article. Moreover, I assume that the newly adopted knowledge of forcing enables other publications in the future.