FINAL REPORT FOR THE INFTY EXCHANGE GRANT 2581

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Title of the project:
Idealized forcing, Mathias model and the Covering Property Axiom

Realization of the research plans.

My research plans concerned answering some open problems in the area of idealized forcing and extending some exisiting results in this field.

Particularly, I was interested in introducing a version of the forcing axiom CPA for the Mathias forcing and finding applications of this version of CPA which would be interesting to other researchers studying the iterated Mathias model. Secondly, I wanted to answer the following open problem, posed by Pawlikowski, Zapletal and others: what is the descriptive complexity of the family of Ramsey-null Borel sets (in the sense of Borel codes)? The conjectured answer here is that this set is $\mathbf{\Pi}^1_2$-complete.

As to the first problem, I was able to formualate and prove an appropriate version of the axiom CPA which holds in the iterated Mathias model. Moreover, I found an interesting application of the axiom, namely a simple and short proof of the fact that the cardinal invariant h_2 (the distributivity number of the Boolean algebra $(P(\omega)/\mathrm{fin})^2$) is equal to ω_1 in the iterated Mathias model. This has been already

proved by Shelah and Spinas (answering a question of Balcar and Simon) but the existing proof is complicated and not well-understood. The proof from CPA is much shorter and, in my opinion, explains much better this problem. Of course, the result saying that $h_2 = \omega_1$ follows from CPA is more general than that of Shelah and Spinas.

As to the second problem concerning the descriptive complexity of the family of Ramsey-null sets, I only have some partial results. The original question is still open but I am going to work on this further.

Cooperation with other reaserchers.

During my stay in the Mittag-Leffler Institute I began fruitful cooperation with Daisuke Ikegami (a Ph.D. student from the group of Prof. Väänänen). In particular, the results mentioned above were obtained with cooperation with Daisuke Ikegami. They should appear in a forthcoming paper.

Other advantages from the visit.

Among other advantages of the visit in the Mittag-Leffler Institute I can mention the meetings with outstanding mathematicians, like Professors Shelah, Woodin, Todorcevic, Larson, Enayat, Magidor, Farah, Schindler and many others who answered many of my questions and showed me beautiful mathematics. To mention just a few examples, I have learned about connections of OCA and C^*-algebras from Prof. Farah, about the Ω-logic from Prof. Larson, about Fine Structure Theory from Prof. Schindler and about Keisler's absoluteness theorem from Prof. Enayat.