Scientific report - Philipp Schlicht

November 2, 2009

My stay at the Institut Mittag-Leffler from September 1 - 30 was funded by an ESF exchange grant (reference number 2584) with project name 'Projective equivalence relations and descriptive set theory'. The purpose of the visit was to take part in the fall program 'Mathematical logic: Set theory and model theory' at the Mittag-Leffler institute and has been a great opportunity to learn about the newest research in set theory and model theory.

I will give a very brief summary of the work done during my stay in September. There is an ongoing project with Fredrik Engström, University of Göteborg, to prove a generalization of the Lopez-Escobar theorem which relates it to generalized quantifiers. The Lopez-Escobar theorem shows that an invariant Borel set is defined by a sentence in $\mathcal{L}_{\omega_1\omega}$. Replacing the symmetric group by a closed subgroup, say the automorphism group of a structure, we ask for a notion of definability for which a generalization of the theorem holds. If Q is a generalized quantifier of type $< 1 >$ which is not permutation invariant and $Aut(Q)$ is its automorphism group, we can show a version of the theorem for Q clopen or a finite Boolean combination of principal quantifiers, i.e. upward closures of single sets. The method generalizes to uncountable structures.

I further worked on a project which connects descriptive set theory and equivalence relations with the model theory of structures of size κ, for which the space κ^κ is a natural setting. I am interested in the structure of trees of size κ ordered by Lipschitz maps which is tied to facts about Σ_1^1 subsets of this space, and in September completed the proof (without cardinal arithmetic assumptions) that the partial order on trees of size $\aleph_n$ contains a copy of every partial order of size $\aleph_{n+1}$ for $n \geq 1$. Assuming the existence of a precipitous ideal on ω_2 with an additional property (this assumption was initially used by Hodges), I generalized the proof of the perfect set property for closed subsets of $\omega_1^{\omega_1}$ to Σ_1^1 subsets. I have made partial progress in the question whether a version of Silver's theorem for Π_1^1 equivalence relations holds for κ^κ by providing a counterexample for the full version assuming $2^\kappa > \kappa^+$, but it is still possible that a restricted version is true. This project is related to the research of Jouko Väänänen, Tapani Hyttinen, Vadim Kulikov, and Teppo Kankanpää and it was quite helpful to discuss this project . I will collaborate with Jouko Väänänen to find out if the perfect set property holds for Π_1^1 sets (with a set theoretic assumption).

I am currently working on a paper with the title 'Non-permutation invariant Borel quantifiers' together with Fredrik Engström which is based on work done during my stay.