SCIENTIFIC REPORT (REF. NR. 2585)
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Infty Exchange Visit to the
Kurt Gödel Research Center, Vienna, Austria
Oct. 1, 2009 – Dec. 31, 2009

In July 2009, I was awarded an Exchange Visit Grant (ref. nr. 2585) from the European Science Foundation through the INFTY research networking programme to visit the Kurt Gödel Research Center for Mathematical Logic at the University of Vienna (KGRC) from October 1, 2009 until December 31, 2009. This is the corresponding scientific report, submitted on February 8, 2010.

Purpose of Visit

Over the last few years my work and that of others has shown that strongly unfoldable cardinals can serve as a highly efficacious substitute for supercompact cardinals and several large cardinal phenomena such as indestructibility [Joh08] and forcing axioms [HJ09], [Miy98]. This is significant, as strongly unfoldable cardinals are rather low in the large cardinal hierarchy; they are relatively consistent with $V = L$. The goal for my visit to the Kurt Gödel Research Center as stated in my proposal from July 2009 was to continue the investigation of such smaller large cardinals in the context of forcing axioms, indestructibility, or new directions, and possibly start new collaborations with members of the KGRC.

Description of work carried out during the visit

In October 2009, I presented my new, joint work with Joel Hamkins on what we call the *Resurrection Axioms* [HJ] in the weekly research seminar of the Kurt Gödel Research Center. The Resurrection Axioms have striking implications on the size of the continuum, they generalize bounded forcing axioms such as MA or BPFA, and various instances of these axioms are equiconsistent with an *uplifting cardinal*, a large cardinal notion that is very low in the large cardinal hierarchy, well below the existence of a Mahlo cardinal. A key advance during the last few months was our discovery that an uplifting cardinal, which

is characterized by the existence of certain elementary embeddings, is always accompanied by an ordinal-anticipating Laver function; if $V = L$ this implies the existence of a true Laver function, a surprising discovery given how small uplifting cardinals are.

During the talk in October several members of the KGRC got very interested in the subject and raised intriguing questions that have led to fruitful discussions and starting collaborations, in particular with Sy Friedman, Lyubomyr Zdomskyy, and Marcin Sabok; we are investigating possible variations (e.g. Σ_n-Resurrection) and applications of these new axioms (e.g. to forcing axioms for *Axiom A* forcing), and to the investigation of models with $\mathfrak{c} > \aleph_2$ that may satisfy weak fragments of PFA. Moreover, during my 3-month visit to the KGRC, Hamkins and I obtained several new results, such as

- the existence of an ordinal-anticipating Laver function,
- the relative consistency of RA(ccc),
- the inconsistency of RA(CARD) , and
- the consistency of PFA($\aleph_2$-preserving) relative to wRA(proper) (an answer to a question from [HJ09]).

On February 15, 2010, I will give an invited lecture at the Young Set Theory workshop in Raach, Austria, on the Resurrection Axioms, presenting several of these new results. Moreover, Joel Hamkins gave on February 6, 2010 a talk on the Resurrection Axioms including our new results at the recent MAMLS meeting at the University of Boulder, Colorado.

In November 2009, I received the referee's report for our article *Indestructible Strong Unfoldability* [HJ10], accepting it for publication in the Notre Dame Journal of Formal Logic. This paper improves on the results obtained in [Joh08] and presents a detailed analysis of all that is known about indestructibility for these interesting small large cardinals. While making final revisions to the article, I discussed parts of the paper with some members of the KGRC and had several fruitful discussions that may lead to further collaborations (with Lyubomyr Zdomskyy, Marcin Sabok, Mohammad Golshani) on these and other smaller large cardinals, and with Radek Honzik from Charles University, Prague, Czech Republic. With Zdomskyy we extended the usual definition of a κ-proper poset to the case when $\kappa^{<\kappa}$ is *larger* than κ, thereby extending the well-known definitions of [Eis03], [Ros05] and [Joh08] to this trickier case. Radek Honzik [Hon10] recently used the Gitik-Shelah [GS89] method of making strong cardinals indestructible by Prikry-like forcing; we have discussed the related question for strongly unfoldable cardinals and are both interested to pursue it.

Projected Publications resulting from the grant

During my visit, I revised the accepted article *Indestructible Strong Unfoldability* [HJ10], which will appear in 2010 in the Notre Dame Journal of Formal Logic. Furthermore, I worked continuously on the article *The Resurrection Axioms* [HJ], which Hamkins and I plan to submit for publication soon. I will acknowledge in each of these publications the ESF's generous support of my visit through an Exchange Visit Grant from the INFTY research networking programme; this support has been extremely valuable for the revisions of the first, and the conception of the second article. Furthermore, I started collaborations with several members of the KGRC (e.g. Friedman, Zdomskyy, Sabok) and expect that some of these collaborations will lead to publications.

Final comment

The Kurt Gödel Research Center has been a very inspiring research environment for me, and I made several advances during my stay. I will stay at the KGRC until March 2010 and hope to receive another INFTY Exchange Visit Grant for the three months starting on April 1, 2010; this would allow me to prolong my visit to the KGRC through the end of June 2010 and continue my work and collaborations on several already started projects.

References

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