Report: Visit to Professor Peter Gothen, University of Porto (Portugal)

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The geometry and topology of the moduli space of G-Higgs bundles $\mathcal{M}(G)$ have been studied for several complex reductive Lie groups and for some of their real forms. Morse-theoretic techniques for studying the topology of moduli spaces of Higgs bundles were introduced by Hitchin [7, 8]. The problem of counting the connected components of $\mathcal{M}(G)$ using these methods has been carried out for several reductive real Lie groups. Hitchin solved the problem for the groups $\mathrm{SL}(n,\mathbb{R})$ and $\mathrm{PSL}(n,\mathbb{R})$ in [8]. His methods were extended to $\mathrm{U}(p,q)$ and $\mathrm{GL}(n,\mathbb{R})$ with $n \geq 3$ by Bradlow, García-Prada and Gothen in [1, 2]. The problem for the symplectic group $\mathrm{Sp}(4,\mathbb{R})$ was studied by Gothen in [5] and by García-Prada and Mundet i Riera in [4], whereas the general case $\mathrm{Sp}(2n,\mathbb{R})$ was studied by García-Prada, Gothen and Mundet i Riera in [3]. The case of $\mathrm{PGL}(n,\mathbb{R})$ has been studied recently by Oliveira in [9].

In my Ph.D. Thesis, *The Geometry of* $\mathrm{SO}(p,q)$*-Higgs bundles*, supervised by O. García-Prada, we give important steps in the program of counting the connected components of the moduli space $\mathcal{M}(G)$ for $G = \mathrm{SO}_0(p,q)$. Since P. Gothen is one of the most important experts on this subject, during my thesis I have had the opportunity to work with him in many occasions and part of the research plan for these three months in Porto was to continue with this joint work with O. García-Prada and P. Gothen and to begin the preparation of a paper presenting and enlarging the research advance made in this thesis about the connected components of $\mathcal{M}(\mathrm{SO}_0(p,q))$. We have made some progress in this direction and we have also started the preparation of a paper about the particular case of $\mathrm{SO}_0(1,2m+1)$, which is one of the solved cases. Since the general problem remains open, it will be the subject of future collaborations.

Another topic treated in this thesis is the isomorphisms for special orthogonal groups of low rank. Using the list of isomorphisms between Spin Lie groups and other classical semisimple Lie groups described in [6, Theorem 14.1], we study the relation between the moduli space of polystable $\mathrm{SO}(n,\mathbb{C})$-Higgs bundles with $n = 3, 4, 5$ and 6 and the moduli space of polystable G-Higgs bundles, where $G = \mathrm{SL}(2,\mathbb{C})$, $\mathrm{SL}(2,\mathbb{C}) \times \mathrm{SL}(2,\mathbb{C})$, $\mathrm{Sp}(4,\mathbb{C})$ and $\mathrm{SL}(4,\mathbb{C})$ respectively. We also described the relations between the real forms $\mathrm{SO}_0(p,q)$ with $p+q=n$, for $n = 3, 4, 5, 6$, and the corresponding real forms of the Lie group G,

describing explicitly the morphisms between the moduli spaces of Higgs bundles.

These relations between moduli spaces of Higgs bundles for low rank Lie groups have been also studied by S. Bradlow, O. García-Prada and P. Gothen. These months in Porto we have started a joint work about this subject that will continue during several months and which will give rise to a paper.

References

[1] S. B. Bradlow, O. García-Prada and P. B. Gothen, *Surface group representations and* $\mathrm{U}(p,q)$*-Higgs bundles*, J. Diff. Geom. **64** (2003), 111–170.

[2] S. B. Bradlow, O. García-Prada and P. B. Gothen, *Representations of surface groups in the general linear group*, Publications of the RSME, vol. 7 (2004), 89–94.

[3] O. García-Prada, P. B. Gothen and I. Mundet i Riera, *Representations of surface groups in the real symplectic group*, arXiv:0809.0576v2 [math.AG].

[4] O. García-Prada and I. Mundet i Riera, *Representations of the fundamental group of a closed oriented surface in* $\mathrm{Sp}(4,\mathbb{R})$, Topology **43** (2004), 831–855.

[5] P. B. Gothen, *Components of spaces of representations and stable triples*, Topology **40** (2001), 823–850.

[6] F.Reese Harvey, *Spinors and Calibrations*, Academic Press, Inc. (1990).

[7] N. J. Hitchin, *The self-duality equations on a Riemann surface*, Proc. London Math. Soc. (3) **55** (1987), 59–126.

[8] N. J. Hitchin, *Lie groups and Teichmüller space*, Topology **31** (1992), 449–473.

[9] A. G. Oliveira, *Representations of surface groups in the projective general linear group*, arXiv:0901.2314[math.AG].