I visited Prof. Veličković at University of Paris 7 from Apr.12 to Jun.18 (2
months). The purpose of my visit was to discuss with Prof. Veličković about vari-
ous reflection principles on \( \mathcal{P}_{\omega_1}(\lambda) \) such as the stationary reflection principle (SR),
the semi-stationary reflection principle (SSR) and the Fodor-type reflection prin-
ciple (FRP). In particular we aimed to make the relationship among these reflection
principles clearer by examining their consequences on combinatorial principles.

SR has been studied extensively by many set theorists, but SSR and FRP have
not been studied as well as SR. SSR is a weakening of SR and is equivalent to that
every \( \omega_1 \)-stationary preserving poset is semi-proper. It was shown in [3] that SSR
is strictly weaker than SR. FRP is shown in [1] and [2] to be equivalent to many
reflection principles on topological spaces. FRP is implied by Axiom R, which is a
strengthening of SR, and it is known that FRP does not imply SSR.

In my research proposal I suggested the following open questions among other
things:

- Does SSR implies the failure of \( \Box(\lambda) \) for every regular cardinal \( \lambda \geq \omega_2 \)?
- Does SSR implies the singular cardinal hypothesis (SCH)?
- Does FRP implies the failure of \( \Box(\lambda) \) for every regular cardinal \( \mu \geq \omega_2 \)?

(It is known that FRP implies SCH.)

During my stay at Paris, we discussed these questions and could solve the above
problems:

**Theorem 1.**

1. **SSR implies the failure of \( \Box(\lambda) \) for every regular cardinal \( \lambda \geq \omega_2 \).**

2. **SSR implies SCH.**

3. **FRP(\( \lambda \)) is consistent with \( \Box(\lambda) \) for any regular cardinal \( \lambda \geq \omega_2 \).**

In particular, it turned out that FRP differs from SR and SSR in that it is con-
sistent with \( \Box(\lambda) \).

We could find a new difference between SR and SSR, too. Weiss [4] introduced
combinatorial principles \( (\kappa, \lambda) \)-TP and \( (\kappa, \lambda) \)-ITP for a regular cardinal \( \kappa \geq \omega_2 \) and a
cardinal \( \lambda \geq \kappa \). These principles are generalizations of characterizations of strongly
compact and supercompact cardinals, due to T. Jech and M. Magidor. In fact the
following holds:
For an inaccessible cardinal $\kappa$,
- $\kappa$ is strongly compact if and only if $(\kappa, \lambda)$-TP holds for every $\lambda \geq \kappa$,
- $\kappa$ is supercompact if and only if $(\kappa, \lambda)$-ITP holds for every $\lambda \geq \kappa$.

In [4] the following are proved:

- $(\kappa, \lambda)$-ITP implies $(\kappa, \lambda)$-TP.
- PFA implies $(\omega_2, \lambda)$-ITP for every $\lambda \geq \omega_2$.
- $(\omega_2, \lambda)$-TP implies the failure of CH.

We investigated the relationship between reflection principles and these new combinatorial principles. We obtained the following results:

**Theorem 2.**
1. SSR together with Martin’s axiom for Cohen forcing implies that $(\omega_2, \lambda)$-TP holds for every $\lambda \geq \omega_2$.
2. SR together with Martin’s axiom for Cohen forcing implies that $(\omega_2, \lambda)$-ITP holds for every $\lambda \geq \omega_2$.
3. SSR together with Martin’s axiom for Cohen forcing does not imply $(\omega, \lambda)$-ITP for any $\lambda \geq \omega_3$.

Thus we found that the difference between SSR and SR is similar as those between strongly compact and supercompact cardinals.

We are planning to publish our results containing Thm. 1 and 2 above.

We do not know whether the assumption of “Martin’s axiom for Cohen forcing” in Thm. 2 (1) and (2) can be weakened to “the failure of CH”. This is one of our future works. Moreover I suggested to discuss generalizations of various reflection principles on $\mathcal{P}_{\omega_1}(\lambda)$ to those on $\mathcal{P}_{\omega_3}(\lambda)$. This remains to be our future work, too.

During my stay at Paris, we could also discuss many other problems, and I am very pleased that I could visit Paris. I thank the ESF research networking program “New frontiers of infinity” so much for supporting my visit.

## References

