

Purpose of the visit (for 4 weeks) The open-open game was introduced by P. Daniels, K. Kunen and H. Zhou [1]. Two players take turns playing on a topological space X ; a round consists of player I choosing a nonempty open set $U \subseteq X$; and player II choosing a nonempty open set $V \subseteq U$; a round is played for each ordinal less than ω . The player I wins the game if the union of open sets which have been chosen by II is dense in X . Let X be a topological space equipped with a topology \mathcal{T} and let \mathcal{T}^α denotes the set of all sequences of length α consisting of elements of \mathcal{T} . The space X is called *I-favorable* whenever there exists a function

$$\sigma : \bigcup \{ \mathcal{T}^n : n < \omega \} \rightarrow \mathcal{T}$$

such that for each sequence $\{B_n : n < \omega\} \subseteq \mathcal{T}$ with $B_0 \subseteq \sigma(\emptyset)$ and $B_{n+1} \subseteq \sigma(\{B_0, B_1, \dots, B_n\})$, for each $n \in \omega$, the union $\bigcup \{B_n : n \geq 1\}$ is dense in X . The function σ is called a *winning strategy*. I have obtained a characterization of a I-favorable space as an inverse limit of a σ -complete system $\{X_\sigma, \pi_\sigma^\sigma, \Sigma\}$ with $w(X_\sigma) \leq \kappa$ in which all X_σ are metrizable and all bounding maps π_σ^σ are skeletal surjections. (a joint paper with Sz. Plewik,[2]). I would like to examine a question about representation of topological space with II player strategy or without I player nor II player strategies. I have also obtained some results about open-open games of uncountable length. I would like discuss about my results with Prague mathematicians and I hope to find some examples and counterexamples concerning open-open games of uncountable length.

Description of the work carried out during the visit I took a part at three seminars in each week, one of prof. P. Simon second of prof. B. Balcar and third of young researchers. I gave two lectures on this seminars about my recent results. I discussed about this and tried to solve problem about representation of topological space with II player strategy or without I player nor II player strategies. I heard about a new result in topology and set theory which get some people in Prague.

Description of the main results obtained I establish that the following result get M. Scheepers, [3]

Theorem . [3, Th. 14] *Let X be compact Hausdorff space. II player has winning strategy in open-open game if and only if there is countable many dense families $\{\mathcal{A}_n : n \in \omega\}$ consisting of open sets such that for every sequence $\{U_n : n \in \omega\}$ with $U_n \in \mathcal{A}_n$ we have $\text{cl} \bigcup \{U_n : n \in \omega\} \neq X$*

In the proof of this Theorem there are some gap and I have done the proof and get direct proof that if II player has winning strategy in open-open game then there is countable many dense families $\{\mathcal{A}_n : n \in \omega\}$ consisting of open sets such that for every sequence $\{U_n : n \in \omega\}$ with $U_n \in \mathcal{A}_n$ we have $\text{cl} \bigcup \{U_n : n \in \omega\} \neq X$. I find easy example in ZFC (in [3, p. 29] there is that it is consistent that there is such example) that the answer on [1, p.220 Question 4.3] is NO. The authors ask if a player has a winning strategy in the game G if, and only if, the same player has a winning strategy in the game G_7 . I establish a connection between tiny sequences in the sense of Szymański[4] and open-open game.

Future collaboration with host institution I have taken a part in Winter School in Abstract Analysis organized by prof. P. Simon and prof. B. Balcar, often. "Traditionally, the school is an informal meeting where emphasis is put on the joy of doing mathematics. Questions and discussions are welcome and there is plenty of space for them outside of the talks." I plane to take a part in the next Winter School in Abstract Analysis, section Set Theory, where I like present my recent results and exchange of news in mathematics.

Projected publications/articles resulting or to result from the grant

After discussion I have changed something in my recent work about "Open-open games of uncountable length" and I plane to publish it. Now I write some note about spaces with II player strategy in open-open game, and connection with tiny sequence but in view of [3, Theorem 14] I don't know will be possible to publish it.

Other comments Due to logistic reasons travel dates were slightly changed from the ones proposed on the original application. Travel from Katowice to Prague was done the 3 October 2010 and travel from Prague to Katowice was done the 31 October 2010. These changes did not reduce the length of the stay (4 weeks).

REFERENCES

- [1] P. Daniels, K. Kunen and H. Zhou, *On the open-open game*, Fund. Math. 145 (1994), no. 3, 205 - 220.
- [2] A. Kucharski and Sz. Plewik, *Inverse systems and I-favorable spaces*, Topology Appl. 156 (2008), no. 1, 110 - 116.
- [3] M. Scheepers, *Combinatorics of Open Covers (V): Pixley-Roy spaces of sets of reals, and ω -covers.*, Topology Appl. 102 (2000), no.1, 13-31.

- [4] A. Szymański, *Some applications of tiny sequences*, Proceedings of the 11th winter school on abstract analysis (Železná Ruda, 1983). Rend. Circ. Mat. Palermo (2) 1984, Suppl. no. 3, 321 - 328.

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