

**Report: Visit to Professor Peter Gothen, University of Porto (Portugal)**

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The geometry and topology of the moduli space of  $G$ -Higgs bundles  $\mathcal{M}(G)$  have been studied for several complex reductive Lie groups and for some of their real forms. Morse-theoretic techniques for studying the topology of moduli spaces of Higgs bundles were introduced by Hitchin [8, 9]. The problem of counting the connected components of  $\mathcal{M}(G)$  using these methods has been carried out for several reductive real Lie groups. Hitchin solved the problem for the groups  $SL(n, \mathbb{R})$  and  $PSL(n, \mathbb{R})$  in [9]. His methods were extended to  $U(p, q)$  and  $GL(n, \mathbb{R})$  with  $n \geq 3$  by Bradlow, García-Prada and Gothen in [2, 3]. The problem for the symplectic group  $Sp(4, \mathbb{R})$  was studied by Gothen in [6] and by García-Prada and Mundet i Riera in [5], whereas the general case  $Sp(2n, \mathbb{R})$  was studied by García-Prada, Gothen and Mundet i Riera in [4]. The case of  $PGL(n, \mathbb{R})$  has been studied recently by Oliveira in [10].

In my Ph.D. Thesis, *The Geometry of  $SO(p, q)$ -Higgs bundles*, supervised by O. García-Prada, we give important steps in the program of counting the connected components of the moduli space  $\mathcal{M}(G)$  for  $G = SO_0(p, q)$  and in our paper *Higgs bundles for the Lorentz group*, we solve the problem of finding the number of connected components of the moduli space of  $SO_0(1, n)$ -Higgs bundles with  $n$  odd. Since P. Gothen is one of the most important experts on this subject, during my thesis I have had the opportunity to work with him in many occasions and part of the research plan for these weeks in Porto was to continue with this joint work with O. García-Prada and P. Gothen and to begin the preparation of a paper enlarging the research advance made in this thesis trying to solve the particular case  $\mathcal{M}(SO_0(2, n))$ . We have made some progress in this direction.

This particular case is strongly related to another topic treated in my Ph.D. thesis, which is the isomorphisms between some Spin Lie groups and other classical semisimple Lie groups described in [7, Theorem 14.1]. These relations between moduli spaces of Higgs bundles for low rank Lie groups have been also studied by S. Bradlow, O. García-Prada and P. Gothen. In these weeks in Porto we have been studying in detail the isomorphisms for  $SO(2, n)$  with  $n = 1, 3$  and 4.

## References

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