Scientific report ESF Exchange Visit Grant Caltech – Walter van Suijlekom
Noncommutative geometry and gauge fields

This is the final report regarding an ESF exchange grant for a visit of Dr. W.D. van Suijlekom to the group of Matilde Marcolli at the California Institute of Technology in California, USA, during April 2011.

Background: applications of noncommutative geometry to gauge theories

Both the research of Matilde Marcolli and of Walter van Suijlekom are based in noncommutative geometry. This mathematical field was founded by Alain Connes [3] as a vast generalization of differential geometry and has seen many applications in other parts of mathematics as well as in physics. Concerning the latter, one of the most important applications is to gauge theories in theoretical physics. In fact, noncommutative geometry puts gauge field theories on similar geometrical footing as general relativity. This was accomplished in full detail recently by Chamseddine, Connes and Marcolli [2] (see also the book [4]). It unifies the gravitational interaction with the three remaining fundamental interactions described by the gauge theories that constitute the Standard Model, at least on the classical level.

The research of Walter van Suijlekom aims at understanding the (perturbative) quantization of gauge theories. One has the hope that noncommutative geometry sheds light not only on the classical form of gauge theories, but on the quantization problem of gauge theories as well.

Purpose of the visit

The purpose of the visit was to put together expertises and exchange ideas in the quest for a better understanding of the structure of quantum gauge theories, thereby making use of techniques from noncommutative geometry. In particular, the recent work of Marcolli et al. [5] on quantum gravity looked ideal for a cross-fertilization with noncommutative geometry. We will describe this in more detail in the next section.

Another purpose of the visit was to prepare for a visit of two Ph.D. students supervised by W.D. van Suijlekom from Nijmegen to Caltech in the near future. A plan has been set up for their short (future) visiting period at Caltech, providing a good embedding in the research groups there.

Description of the work carried out and main results

As said, a key role was played by the recent results of Marcolli et al. [5] on spin foams and noncommutative geometry. This treated the usual spin networks in loop quantum gravity in a categorical setup, yielding spin foams as cobordisms between such networks, at the same time incorporating the topological background data.

What Marcolli and Van Suijlekom have defined during April 2011 is an enrichment of this structure to incorporate also gauge theories. This is very much inspired by the aforementioned noncommutative geometrical description of gauge theories. Just as spin networks are *quanta of geometry*, a definition was set up to give *quanta of an almost commutative geometry*, the latter being a special case of a noncommutative manifold, relevant for the description of gauge theories already mentioned. Intriguingly, the techniques relied on quiver representations on the mathematical side, and used ideas from lattice gauge theory on the other.

The main results are the description of so-called gauge networks, extending spin networks to the noncommutative world. The category of such gauge networks used as morphisms a sort

of correspondence, based on algebra bimodules. This is very much as Kasparov's KK-theory forms a category on (C^*-)algebras.

Future perspective

With the setup of *gauge networks* as the quanta of noncommutative geometry in place, Marcolli and Van Suijlekom plan to continue their project in the direction of finding the physical content of their mathematical construction. In particular, the spectral action principle, introduced by Chamseddine and Connes [1] appears to play a prominent role and can already be related to the Wilson action natural for lattice gauge theories. Interestingly in the context of the 'Interactions of Low-Dimensional Topology and Geometry with Mathematical Physics (ITGP)' are the findings of relations with three-dimensional Chern–Simons theory through a dimensional reduction from the four-dimensional case.

Projected publications

An article of Van Suijlekom was finished during the visit to Caltech, which already appeared as a preprint [6].

Concerning the collaboration of Marcolli and Van Suijlekom, a draft manuscript has been setup by Van Suijlekom with the title "Gauge Networks in Noncommutative Geometry". It is now alternatingly worked on by Marcolli and Van Suijlekom.

References

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[4] A. Connes and M. Marcolli. *Noncommutative Geometry, Quantum Fields and Motives.* AMS, Providence, 2008.

[5] D. Denicola, M. Marcolli, and A. Z. al Yasry. Spin foams and noncommutative geometry. *Class.Quant.Grav.* 27 (2010) 205025.

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