

INFTY Exchange Visit - Scientific Report

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Project Models of fragments of NF in suitable extensions of ZF.

I visited Prof. T.E. Forster to the University of Cambridge Set Theory Group in order to learn about Quine's NF ("New Foundations") set theory and to join this with my doctoral background on Zermelo-Fraenkel (ZF) set theory.

During my visit we have met regularly in a reading group concerning Forster's seminal paper "AC fails in the natural analogues of V and L that model the stratified fragment of ZF" and more recent published and unpublished results on this topic.

I focused my work on "stratified constructibility", shortly the construction of an analogue of Gödel's \mathbf{L} using stratified constructions in the sense of Quine's NF. What it arose from the discussions is that the different ways we can build the constructible universe \mathbf{L} are not obviously equivalent when we restrict ourselves to use only stratified notions. We have at least the choice between an hierarchy of stages as in Jensen's "Fine Structure" or an enumerating function as in Gödel (1940); and also a choice — at the successor stage of the hierarchy — between a closure operator built up from a finite set of stratified operations or from the set of all subset definable by means of stratified formulæ, like in Gödel (1939). In the \mathbf{L} case, all these approaches can be shown to be equivalent by an indirect argument, using the fact that \mathbf{L} is the "least inner model of ZF". But a similar property characterizing the universe of *stratified* constructible sets has to be find. So we have started the hard way directly comparing these different constructions.

Now we have some new insights about stratified constructibility — particularly thanks to the contributions of N. Bowler — but no definite results have been achieved. However I think I have clarified the picture enough to present this topic at my origin institution in the form of a cycle of seminars suitable for further discussions. In the same time I consider all the matter as a work in progress and so I planned to come back to Cambridge in May and to continue there these investigations.