Exchange visit of Piotr Koszmider at the University of Paris-Diderot
Hosted by Boban Velickovic
Final report

After initial discussions we focused our joint research on two topics which are described in the following two sections.

1. Automorphisms of $\wp(\mathbb{N})/Fin$ and automorphisms of ℓ_∞/c_0

The main general problem here is the impact of the structure of the automorphisms of the Boolean algebra $\wp(\mathbb{N})/Fin$ on the automorphisms of the Banach space ℓ_∞/c_0. The link, of course, is that $C(K_{\wp(\mathbb{N})/Fin})$ is isometric to ℓ_∞/c_0, where $K_{\mathcal{A}}$ denotes the Stone space of $\mathcal{A}$. In particular we investigated in what sense one could have trivial and non-trivial automorphisms of the Banach space ℓ_∞/c_0.

We noted that the notion of a trivial automorphism of ℓ/c_0, should not refer to operators on c_0 if it was in some analogy to the notion of a trivial automorphism of $\wp(\mathbb{N})/Fin$. This is because we could construct many different automorphisms of ℓ_∞ which are the identity when restricted to c_0. So, probably a better notion of a trivial automorphism of ℓ_∞/c_0 is the one that can be lifted to an automorphism of a finite codimensional subspace of ℓ_∞ onto a a finite codimensional subspace of ℓ_∞. We focused on such automorphisms which can be lifted to an automorphisms of ℓ_∞.

It turned out that under some extra set-theoretic assumptions there are plenty of automorphisms of ℓ_∞/c_0 which behave quite differently than automorphisms of $\wp(\mathbb{N})/Fin$. In particular we constructed in ZFC more than 2^ω automorphisms of ℓ_∞. So there is no hope for a reasonable description even of the automorphisms of ℓ_∞/c_0 which can be lifted. In particular the use of OCA as in the case of the automorphisms of $\wp(\mathbb{N})/Fin$ is questionable and restricted to those automorphisms which potentially could have liftings which preserve c_0 (allowing for some Borel description).

We also showed that some nontrivial automorphisms of $\wp(\mathbb{N})/Fin$ induce nontrivial (without liftings to ℓ_∞) automorphisms of ℓ_∞/c_0.

2. A construction of a Boolean algebra

We considered a new construction of a Boolean algebra of cardinality ω_2 which could have some interesting applications. In particular we tried to prove that it solves the small diagonal problem, i.e., its Stone space is nonmetrizable but it has a small diagonal (any uncountable collection of points in the square has uncountable subcollection which is separated from the diagonal). This algebra would be obtained by highly nontrivial stepping-up. We considered several modification which still need to be improved.

3. Conclusions

We obtained an interesting progress in two directions and plan to continue research exploiting the partial results obtained during the exchange visit.