

THE SCIENTIFIC REPORT ON THE INFTY EXCHANGE VISIT
BY MIRNA DŽAMONJA TO BOBAN VELIČKOVIĆ, MARCH
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I have been supported for a four week visit to the University of Paris VII to work with Prof. Boban Veličković. This visit in fact was spread over approximately eight weeks since during the month of March I worked in Norwich on a three days per week basis, and during the month of April I was away for one week.

Let us call a tree a *Väänänen* tree if the height and the cardinality of the tree is ω_1 , while there is no uncountable branch in the tree. The class of all such trees properly includes the class of Aronszajn tree. By an *embedding* between such trees T and T' we mean any function $f : T \rightarrow T'$ which preserves the strict order (in particular, such an embedding maps branches to branches). If there is such an embedding we say that $T \leq T'$. One of the main problems considered during the visit was the question of the possible existence of one Väänänen tree T^* which embeds all the others. Let us call such a tree universal. It is known that a universal Väänänen tree cannot exist under CH . Mekler and Väänänen in [1] have claimed that under $MA + \neg CH$ there is such a tree, in fact an Aronszajn tree. They have soon realised that their proof was incorrect and the question has been open since.

One of the results we obtained during the visit was that under $MA + \neg CH$ it is indeed true that every Väänänen tree embeds into an Aronszajn tree. However, it was proved by Todorćević in [3] that under the same assumption $MA + \neg CH$ there is a strictly \leq -increasing ω_2 -sequence of Aronszajn trees cofinal in the class of Aronszajn trees. As a consequence, there is no universal Väänänen tree under $MA + \neg CH$.

We are continuing our research into the possibility of the existence of a universal Väänänen tree. It seems that it is very relevant to first understand the possible existence of a universal linear order of size \aleph_1 in the absence of CH . A theorem claiming the consistent existence of such a linear order was published by Shelah in [2]. Unfortunately, the proof is not complete. There has been a concentrated effort in the community to understand this proof, including Abraham, Džamonja, Geschke, Thompson and Veličković. The proof is still not there and this research continues.

I continue to actively collaborate with Veličković on a very regular basis, on these and other questions. We plan to write the results obtained under the support obtained from INFTY in a paper to be published as a chapter of the conference proceedings devoted to the Väänänen 60th birthday. This should be written next year. Support from INFTY will of course be acknowledged. I use the occasion to thank my colleagues on the Steering Committee for granting me this support.

REFERENCES

1. A. Mekler and J. Väänänen, *Trees and Π_1^1 -subsets of ${}^{\omega_1}\omega_1$* , The Journal of Symbolic Logic **58(3)** (1993), 1052–1070.
2. S. Shelah, *Independence results*, Journal of Symbolic Logic **45** (1980), 563–573.
3. Stevo Todorćevic, *Lipschitz maps on trees*, J. Inst. Math. Jussieu **6** (2007), no. 3, 527–556.

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