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Simulating electron-phonon dynamics
using ultracold Bose-Fermi mixtures

Final Report

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Abstract

The aim of the original proposal (6 months) was to combine my expertise with those of the host institution in order to build a theoretical tool to study ultracold Bose-Fermi mixtures and their connection with polaron physics. The results achieved during my 3 months stay are twofold: on one side we developed a better theoretical understanding of the connections between the well established Holstein description of polarons and the Bogoliubov approach to the bosonic component in Bose-Fermi mixtures. On the other side and in order to validate quantitatively this connection, we then implemented a computer code which provides static and dynamic properties of bosonic and fermionic degrees of freedom in a finite cluster.

1 Introduction and Motivations

The problem of electrons interacting with lattice vibrations is a long standing one in condensed matter and, despite of the advances achieved in the theoretical understanding of this subject, a quantitative agreement between theory and experiments is still missing, particularly in systems in which electron-electron correlations are relevant.

There are several reason behind these disagreements. Indeed a fair description of real materials, where a number of phononic modes are excited simultaneously, requires very complex theoretical models. On the other hand, even the simplest models introduced so far for describing electron-phonon interactions, like e.g. the Holstein model [1] where phonons only couple to the electronic density, defied so far an exact solution in the whole parameter range. In order to disentagle these two sources of error one would clearly need better theoretical results about these models, together with experiments focusing on systems so simple to be well described by these simple models.

As mentioned in the original proposal, it has already been shown (e.g. in [2]) that ultracold atomic gases are the ideal candidates to investigate similar issues, since the experimental parameters can be tuned to regimes where simplified model Hamiltonians provide a very accurate description of the system. The same kind of control is hardly attainable in analogous condensed matter experiments.

Bose-Fermi mixtures with spin-polarized fermions tuned to a suitable parameter regime are the natural candidates to realize the Holstein model in experiments. Indeed, assuming the mixture to be well described by a Bose-Fermi Hubbard model[1], whose Hamiltonian is given by $\hat{H} = \hat{H}_B + \hat{H}_F + \hat{H}_{BF}$ and

$$\hat{H}_B = -J_B \sum_{<i,j>} \hat{b}_i^\dagger \hat{b}_j - \mu_B \hat{N}_B + \frac{U_{BB}}{2} \sum_i \hat{n}_i^B (\hat{n}_i^B - 1) \tag{1}$$

$$\hat{H}_F = -J_F \sum_{<i,j>} \hat{c}_i^\dagger \hat{c}_j - \mu_F \hat{N}_F \tag{2}$$

$$\hat{H}_{BF} = U_{BF} \sum_i \hat{b}_i^\dagger \hat{b}_i \hat{c}_i^\dagger \hat{c}_i \tag{3}$$

it is immediately evident that the bosonic atoms are coupled to the fermionic density as in the Holstein model. However, as already remarked in [3], the

[1] See [3] for a detailed explanation of the notation in use.

analogous role of the phonons in condensed matter is not played by the original bosonic particles (atoms), but by the Bogoliubov modes of the condensate. In this respect therefore the *phonons* in the Bose-Fermi mixtures are dynamic quasiparticles whose properties are well defined only in suitable parameter regimes. For example, as shown in [3], due to the interplay between the condensate depletion and the thermal population of the Bogoliubov modes for increasing temperature, the temperature dependence of the polaronic damping is in principle richer than in the corresponding condensed matter case, where there is no softening of the phononic modes with the temperature. Therefore the connection between the polaron problem and ultracold Bose-Fermi mixtures has to pass through the Bogoliubov approach of the bosonic component in the Bose-Fermi mixture. This point of view was already taken in Ref. [3], where however we chose to restrict our considerations to a specific regime, in order to focus rather on the connection with the current experiments in ultracold gases. A better understanding of this issue in the general case was the main focus of the analytic part of the results (see Sec. 2.1) during my stay in Lund.

Since most of the experimental methods proposed so far to study polarons in Bose-Fermi mixtures are based on observing the system dynamics (typically cloud expansions or trap oscillations), an extension of the existing approaches to the real-time domain is needed in order to directly compare with real experiments. To this aim we developed a numeric approach which is able to address both equilibrium and non-equilibrium properties of a Holstein model on a finite cluster. This will allow in the next future both to benchmark the analytic approach and its assumptions in the static case, and also to address real-time properties. Details about the numeric approach and the computer code we developed are provided in Sec. 2.2.

2 Details of the Work carried out

2.1 Analytics Results

Let's consider the fermionic particles in a Bose-Fermi mixture as impurities immersed in a bosonic condensate [2]. In these conditions we can describe the system within the Bogoliubov approach in the presence of impurities. The treatment assumes most of the bosonic particles to be in the condensate, suitably described by a macroscopic wavefunction ψ_i, and includes the first quantum corrections. The shape of the condensate wavefunction ψ_i is determined by replacing the bosonic operators $\hat{b}_i$ with a classical complex field ψ_i and imposing the (classical) energy of the system to be minimal. This is equivalent to imposing the condensate wavefunction to satisfy the Gross-Pitaevskii (GP) equation.

If there are no fermions in the system the condensate wavefunction is translationally invariant, i.e. $\psi_i = n_0$, and the GP equation $\frac{\partial H_B^0}{\partial (\psi_i^0)^*} = 0$ is given by[5, 6]

$$-J_B \sum_{j \in nn_i} \sqrt{\bar{n}_0} + U_{BB} \bar{n}_0 \sqrt{\bar{n}_0} = \mu_B \sqrt{\bar{n}_0} \tag{4}$$

where $\mu_B = U_{BB}\bar{n}_0 - J_B z$ fixes the actual value $\bar{n}_0$ of the unperturbed conden-

[2] Both the fraction of fermions and the Hamiltonian parameters in the mixture can be easily tuned to a regime where these conditions are fulfilled (see [3, 4]).

sate density and the corresponding energy is given by[3] $(H)_{n_0=\bar{n}_0} = -N_s \frac{U_{BB}}{2} \bar{n}_0^4$ In presence of impurities (assumed to be static for simplicity at this stage and described by the classical distribution function f_i) instead the GP equation becomes

$$-J_B \sum_{j \in nn_i} \bar{\psi}_j - \mu_B \bar{\psi}_i + U_{BB} |\bar{\psi}_i|^2 \bar{\psi}_i + U_{BF} \bar{\psi}_i f_i = 0 \tag{5}$$

where the impurity distribution acts a source term on the condensate wavefunction. It is clear that the presence of impurities breaks the translational invariance of the lattice. Moreover we also realized, since the GP equation is nonlinear in the field ψ_i, that it is also not possible to obtain a generic solution corresponding to a generic f_i, since the Green function method used in Ref. [7] and implicitly in Ref. [3] does not apply anymore[4]. We however demonstrated that the GP energy corresponding to $\bar{\psi}_i$ which solves the GP equation assumes in general the same espression of the unperturbed case, provided one replaces $\bar{n}_0$ with $\bar{\psi}_i$, i.e.

$$(H)_{\psi_i = \bar{\psi}_i} = -\frac{U_{BB}}{2} \sum_i |\psi_i|^4 \tag{6}$$

The Bogoliubov approach corresponds then to further expanding the bosonic operators as $\hat{b}_i = \psi_i + \hat{\theta}_i$, keeping only the leading order terms in the quantum fields, i.e. only terms involving the condensate. Within these approximations, one obtains the following Hamiltonian for the bosonic part $\hat{H} \approx H + \hat{H}_\theta$ where

$$\hat{H}_\theta = -J_B \sum_{<i,j>} \hat{\theta}_i^\dagger \hat{\theta}_j - \mu_B \sum_i \hat{\theta}_i^\dagger \hat{\theta}_i + 2U_{BB} \sum_i |\bar{\psi}_i|^2 \hat{\theta}_i^\dagger \hat{\theta}_i + \frac{U_{BB}}{2} \sum_i (\hat{\theta}_i^\dagger \bar{\psi}_i)^2 + (\hat{\theta}_i \bar{\psi}_i^*)^2 \tag{7}$$

The Bogoliubov modes are now *inhomogeneus* because of the impurities, even though they depend on the impurity distribution only through the condensate wavefunction ψ_i. The Hamiltonian $\hat{H}_\theta$ is however quadratic and can be diagonalized using a generalized Bogoliubov transformation (see Ref. [8] for the explicit form of the Bogoliubov-De Gennes equations), obtaining

$$\hat{H}_\theta = \sum_\nu{}' \hbar \omega_\nu \hat{\beta}_\nu^\dagger \hat{\beta}_\nu + \Delta E \tag{8}$$

where ΔE is the quantum correction to the classical ground state energy. According to the Bogoliubov prescription therefore, the Bogoliubov modes are not directly coupled to the fermionic density, since the Bogoliubov Hamiltonian $\hat{H}_\theta$ does not depend explicitly on f_i, though $\bar{\psi}_i$ clearly depends on f_i. Moreover, as already noted in Ref. [9] in the context of BEC in presence of disorder, is at least not justified (if not incorrect) within the Bogoliubov approach to neglect the effect of the impurities on the condensate wavefunction and directly expand the bosonic operators around the *unperturbed* condensate wavefunction as in Refs. [10].

The simultaneous solution of the GP+Bogoliubov-De Gennes equations provides the explicit form of the condensate wavefunction and of the Bogoliubov modes for a given impurity distribution. Work in the direction of extending the

[3] N_s is the number of lattice sites.

[4] This is particularly crucial since the fermions are in general dynamic impurities and their distribution function is not fixed *a priori*.

treatment to the case of dynamic impurities, where the fermionic distribution function f_i is replaced by the quantum operator $\hat{n}_i$, is currently under way. Though feasible in principle, we are anyway not specifically interested here in the solution of this problem (which is somehow already discussed in literature (e.g. in Ref. [9] in the context or disordered BEC)). Our focus was instead in understanding what is the connection between the Bogoliubov approach above and an effective descriptions in terms of a suitable Holstein model, since this connection would allow an explicit link between the physics of Bose-Fermi mixtures and the Holstein polaron.

To this aim suppose now we start again from the Bose-Fermi Hubbard Hamiltonian and apply a prescription analogous to the one used in solid state physics to describe the lattice vibrations in terms of phonons. The bosonic operator are written as sum of a C-number (the equilibrium position of the ion in a solid) + a quantum part representing the displacement operators with respect to the equilibrium position. Following a similar procedure for the operator $\hat{b}_i$ which creates a bosonic atoms at the lattice site i would correspond to separate the "unperturbed" condensate (in the absence of impurities) and keep only the small quantum fluctuation around them to the lowest order, i.e. $\hat{b}_i \approx \sqrt{n_0} + \hat{\theta}_i$. This would result in the following hamiltonian $\hat{H} \approx H_0 + H^0_{\hat{\theta}} + \hat{H}_{lin}$ where

$$H_0 = -N_s \left[\frac{U_{BB}\bar{n}_0^2}{2}\right] + U_{BF}\bar{n}_0 \sum_i f_i \qquad \text{(0-th order)} \tag{9}$$

$$H^0_{\hat{\theta}} = -J_B \sum_{<i,j>} \hat{\theta}_i^\dagger \hat{\theta}_j - \mu_B \sum_i \hat{\theta}_i^\dagger \hat{\theta}_i + 2U_{BB}\bar{n}_0 \sum_i \hat{\theta}_i^\dagger \hat{\theta}_i + \tag{10}$$

$$\frac{U_{BB}}{2}\bar{n}_0 \sum_i (\hat{\theta}_i^\dagger)^2 + \hat{\theta}_i^2 \tag{11}$$

$$\hat{H}_{lin} = U_{BF}\sqrt{\bar{n}_0} \sum_i [\hat{\theta}_i^\dagger + \hat{\theta}_i] f_i \tag{12}$$

By using now the *unperturbed* Bogoliubov transformation from $\hat{\theta}_i$ to $\hat{\beta}_k$, one can introduce the unperturbed Bogoliubov modes which are perfectly analogous to the phonons in condensed matter, obtaining $\hat{H} \approx H_0 + H^0_{\hat{\beta}} + \hat{H}_{lin}$, where

$$\hat{H}^0_\beta = {\sum_k}' \hbar\omega_k \hat{\beta}_k^\dagger \hat{\beta}_k + \Delta^0 E$$

and

$$\hat{H}_{lin} = \sum_i {\sum_{\mathbf{k}\in FBZ}}' \hbar\omega_{\mathbf{k}} [M_{i,\mathbf{k}}\beta_{\mathbf{k}} + M^*_{i,\mathbf{k}}\beta^\dagger_{\mathbf{k}}] f_i \tag{13}$$

$$M_{i,\mathbf{k}} = U_{BF}\sqrt{\frac{\bar{n}_0 \epsilon^*_{\mathbf{k}}}{N_s(\hbar\omega_{\mathbf{k}})^3}} e^{i\mathbf{k}\cdot\mathbf{R}_i} \tag{14}$$

One can easily realize that this Hamiltonian expression is pretty different from the Bogoliubov approach above. Indeed they correspond to different orders of approximation since the equilibrium position of the expansion is not affected by the impurities at this level, while it was in the GP approach in Eq. 5. If

one sticks to the static impurities case with $J_F = 0$, the bosonic and fermonic parts in the expressions above can be made indipendent with a suitable change of basis, corresponding to use the Lang-Firsov (LF) transformation [11]

$$\hat{H}_\beta \quad \xrightarrow{LF} \hat{H}_\beta - \hat{H}_{lin} + V \tag{15}$$

$$\hat{H}_{lin} \quad \xrightarrow{LF} \hat{H}_{lin} - 2V \tag{16}$$

$$V \quad = \sum_{i,j} V_{i,j} f_i f_j \quad \text{where} \quad V_{i,j} = {\sum_{\mathbf{k}\in FBZ}}' \hbar\omega_{\mathbf{k}} M_{\mathbf{k}}^2 \cos\left[\mathbf{k}\cdot(\mathbf{R}_i - \mathbf{R}_j)\right] \tag{17}$$

The Lang-Firsov transformed Hamiltonian has therefore no coupling between phonons and fermions, while the latters now interact through a density-density interparticle potential V. In the final expression the Bogoliubov modes are not affected by the impurities, contrarily to the Bogoliubov approach above. We demonstrated however that, at least in the static case, the two treatments give the same result whenever one includes the additional condition

$$\alpha = \frac{U_{BF}(U_{BB}n_0)^{\frac{D}{2}-1}}{(2J_B)^{\frac{D}{2}}} \ll 1$$

on the Bose-Fermi Hubbard Hamiltonian. Under this condition, which constraints the model parameter and is in principle independent of the validity of the Bogoliubov approach, the GP equation can be linearized and $\hat{b}_i \approx \sqrt{n_0} + \delta_i + \hat{\theta}_i$ where $\delta_i \ll \sqrt{n_0}$ is the distorsion of the condensate wavefunction induced by the impurities. Since the corresponding GP equation for δ_i (already derived in Ref. [3] for the static impurities case)

$$\frac{\sum_{j\in nn_i}(\delta_j - \delta_i)}{l^2} - \frac{2U_{BB}n_0}{J_B l^2}\delta_i = \frac{U_{BF}\sqrt{n_0}}{J_B l^2} f_i \tag{18}$$

is now *linear*, it allows for a generic solution using the Green function method and it is easier to generalize to the case of dynamic impurities. By formally solving the equation, one obtains that, in the static case, the (linearized) Bogoliubov approach including the condition $\alpha \ll 1$ is identical to write down a Holstein Hamiltonian for the Bose-Fermi mixture. This suggest therefore that, under suitable constraints over the model parameters, the Bose-Fermi mixture are well described by this simplified model for electron-phonon interaction in condensed matter. The extension to the dynamic case, where $f_i \to \hat{n}_i$ and $J \neq 0$, of this formal equivalence it currently under investigation since there is no analogous of the Lang-Firsov transformation for finite J in order to decouple bosonic and fermionic degrees of freedom. In this respect, a numerical validation of the results obtained with different Hamiltonians would be very useful to assess the equivalence of the two treatment in the general case which defies a simple analytical solution. See the discussion about the numerics in Sec. 2.2.

In conclusion, we demostrated analytically, at least in the static case with $J_F = 0$, that the Bogolioubov approach to the bosonic component in a Bose-Fermi mixture is equivalent to a Holstein model only in a suitable parameter regime, which is smaller then the regime of validity of the Bogoliubov approach itself. Indeed it is necessary to include the additional condition $\alpha \ll 1$ on the Hamiltonian parameters, which allows to linearize the GP equation and makes the Bogoliubov spectrum unaffected by the fermionic impurities.

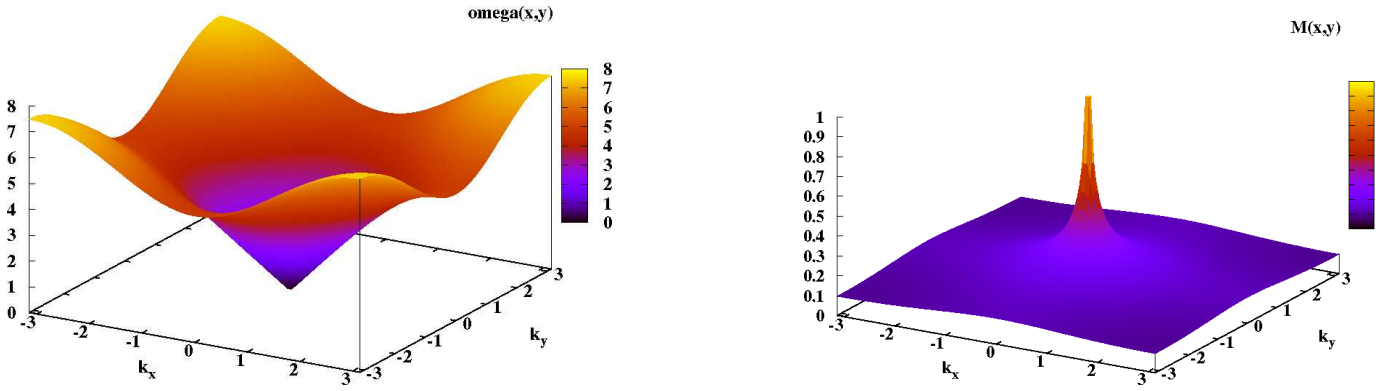

Figure 1: Typical shape of the Bogoliubov spectrum $\omega_{\mathbf{k}}$ (Left Panel) and of the matrix elements $M_{\mathbf{k}}$ (Right Panel) for $D = 2$ $(J_B = 1)$.

2.1.1 From dispersive to Einstein modes

Stimulated by the fact that for practical reasons most of the investigations of the Holstein model in the literature deal with phononic modes only in Einstein approximation, we also investigated the possibility of reducing the Bogoliubov modes to an equivalent Einstein mode. However, as one can immediately realize by looking at Fig. 1, the Bogoliubov modes are intrinsically acoustic, since the unperturbed Bogoliubov dispersion is given by [6] $\hbar\omega_{\mathbf{k}} = \sqrt{\epsilon^*_{\mathbf{k}}(\epsilon^*_{\mathbf{k}} + 2U_{BB}n_0)}$ and therefore $\hbar\omega_{\mathbf{k}} \to 0$ for $k \to 0$ [5] . The reduction to Einstein modes is better suited for the case of optical phononic bands where there is a finite minimum energy for the phonons. Since the presence of soft modes is expected to strongly affect the physics of the system, it is not clear to us if one can find a suitable Einstein mode able to mimick the dispersive modes results in the whole parameter regime. We realized however that, at least at strong coupling, a possible strategy is to compare systems with the same value of the polaron shift, which is supposed to be the relevant energy scale in this regime. This still leaves space for a suitable choice of the equivalent coupling g between the effective Einstein mode and the fermion to replace the matrix elements $M_{i,\mathbf{k}}$. We are currently investigating if this assumption can be extended to smaller coupling regimes, expecially since this would also strongly reduce the computational effort to simulate the dynamics in the numerics, due to the exponential reduction of the relevant phononic Hilbert space size.

2.2 Numerics

As already outlined in the original proposal, most of the ongoing experiment devised to study polaronic effects within Bose-Fermi mixtures are intrinsically dynamic in character, the easiest setup being the expansion of the fermionic cloud in the optical lattice once the trapping potential is removed. Therefore in order to directly simulate the experimental results, we developed a computer

[5] $\epsilon^*_{\mathbf{k}} = \epsilon_{\mathbf{k}} + zJ_B = 2J_B \sum_{i=1}^{D}(1 - \cos(k_i l)) \geq 0$ is the (shifted) single particle spectrum in tight-binding approximation

code which is able to address the real-time dynamics of bosonic and fermionic degrees of freedom simultaneously. Moreover, we are going to use the same code in the static case to complement the prediction coming from the analytic approach also in regimes where an explicit solution is not available and further approximations are needed.

Our method of choice is to use *exact* diagonalization on a finite cluster. While the method does not have any limitation in principle with respect to the Hamiltonian parameters, since it is expected to be accurate in any regime of coupling, the main drawback is given by finite-size effects due to having a finite Hilbert space. This problem, which is already present in purely fermionic systems, is further complicated by the presence of phonons, whose number-per-mode is not bounded by the Pauli principle. In order to cope with the rapid increase of the Hilbert space size with the cluster size, we used the Lanczos algorithm [12], which allows to compute the extremal eigenvalues of large sparse matrices with relatively small effort compared to *brute-force* exact diagonalization. While this method is usually employed to deal with zero or very low temperature properties of physical systems in equilibrium, it can be used in principle also to study the real-time evolution of a given initial configuration [13], and is therefore suitable to be used to describe cloud expansions.

The code at the moment is implemented to deal with the Holstein Hamiltonian in presence of Einstein (i.e. not dispersive) phonons. As already mentioned above, since in principle the number n of phonons in each mode is unbounded, we must proceed to a truncation procedure in the boson number per-mode. The nature of the problem and the strength of the parameters significantly affect the convergence rate of the truncated expansion. With a partial use of symmetry, it turns out that with $N = N_\uparrow + N_\downarrow$ fermions on L lattice sites, and interacting with M boson modes (each i-th mode with truncated occupation M_i), the size of the basis set in the Fock space is

$$N_{tot} = \left(\begin{array}{c} L \\ N_\uparrow \end{array} \right) \left(\begin{array}{c} L \\ N_\downarrow \end{array} \right) \prod_{i=1}^{M} (M_i + 1) \tag{19}$$

On a ordinary workstation, the Lanczos method is able to deal with large matrices up to several millions of elements. This would fix the maximum size of the system we can address. For a single fermion the largest size we can address is strongly bounded from the maximum number of phonons-per-mode allowed. For example, allowing a phononic mode per lattice site and up to 2 phonons per mode the maximum cluster size addressable is rapidly saturating for $L = 5, 6$. However, at the moment we are also considering different procedures to improve the code efficiency in dealing with the phononic Hilbert space. For example in presence of dispersive phonons which have different energy in different modes, an energy cut-off Λ would be much more efficient, since soft modes are more likely to be created then hard modes.

The code is fully functional at the moment and we are going to test it against known results for the polaron problem in order to understand the relevance of finite-size effects in different parameter regimes. In order to give the reader an intuitive idea about how the Lanczos algorithm works in the practice, we briefly sketch it below.

2.2.1 Lanczos method for temporal evolution

Suppose we wish to determine the time evolved state

$$|\Phi_t\rangle = e^{-iHt}|\Phi_0\rangle = \sum_{k=0}^{\infty} \frac{(-it)^k}{k!}|v_k\rangle, \quad |v_k\rangle = H^k|\Phi_0\rangle. \tag{20}$$

According to the Lanczos method [12], the space spanned by the vector sequence $\{|v_k\rangle\}$ is also spanned by the orthonormal Lanczos sequence $\{|V_k\rangle\}$, obtained by building the Lanczos sequence starting from $|\Phi_0\rangle \equiv |V_0\rangle$. The associated expression for the Hamiltonian H is tridiagonal in the $\{|V_k\rangle\}$ basis and we can write:

$$|\Phi_t\rangle = \sum_{k=0}^{M_L} |V_k\rangle \, \langle V_k|e^{-iH_L t}|V_0\rangle, \quad |V_0\rangle = |\Phi_0\rangle, \tag{21}$$

with $M_L \leq M$. We have renamed H by H_L to make explicit that we are in the full Lanczos basis. Now, the basic approximation is to truncate the Lanczos chain (sequence) $\{|V_k\rangle\}$ to a maximum vector $|V_K\rangle$ ($K < M_L$), and similarly to consider H_L into this truncated subspace, i.e. $H_L \to H_L^{(K)}$. Naturally, $H_L^{(K)}$ still maintains its tridiagonal structure. Under this approximation we can write

$$|\Phi_t\rangle = \sum_{k=0}^{K} |V_k\rangle \, \langle V_k|e^{-iH_L^{(K)} t}|V_0\rangle. \tag{22}$$

The key point is that such procedure is useful when i) K its significantly smaller than the dimension M of the full space, and ii) when K is within the range of numerical diagonalization procedures but M is not, which applies to large scale problems. In this case, we can now numerically diagonalize $H_L^{(K)}$; and inserting a complete (in the truncated K-space) set of eigenstates $|\lambda\rangle$ of $H_L^{(K)}$:

$$|\Phi_t\rangle = \sum_{k=0}^{K} |V_k\rangle \, \left[\sum_{\lambda} \langle V_k|\lambda\rangle e^{-i\epsilon_\lambda t}\langle\lambda|V_0\rangle\right]. \tag{23}$$

If, in the original basis , $|\Phi_0\rangle = \sum_m c_m(0)|m\rangle, |\Phi_t\rangle = \sum_m c_m(t)|m\rangle$, we easily find:

$$c_m(t) = \sum_{k=0}^{K} \langle m|V_k\rangle \, \left[\sum_{\lambda} \langle V_k|\lambda\rangle e^{-i\epsilon_\lambda t}\langle\lambda|V_0\rangle\right], \quad |V_0\rangle = |\Phi_0\rangle, \tag{24}$$

where $\langle m|V_k\rangle$ is a $M \times (K+1)$ matrix formed by projecting in the original basis $\{|m\rangle\}$ the states $\{|V_k\rangle\}$ generated during the Lanczos recursion.

3 Conclusions and Perspectives

During my stay in Lund, we have developed two complementary tools in order to better understand the connection between Bose-Fermi mixtures experiments and the Holstein polaron problem in condensed matter. On one side we have shown analytically in the case $J_F = 0$ that only by adding the additional condition $\alpha \ll 1$ (which allow to linearize the GP equation) the Bogoliubov approach to the bosonic component of the mixture results in an Hamiltonian identical to the

Holstein model. On the other side we have developed a computer code which allows to deal simultaneously with bosonic and fermionic degrees of freedom in a finite cluster. The code is fully functional and can address both systems in thermal equilibrium at low or zero temperature and also real-time dynamics starting from a given initial configuration. We are at the moment involved in testing the code against known results for the Holstein model, in order to quantify finite-size effects, and in extending the analytical approach to the case of dynamic impurities.

We have already envisaged in the early future, by combining the two methods we have developed, to publish theoretical results which will be directly comparable with cloud expansion experiments in Bose-Fermi mixtures. Conversely we should be able to use the inverse mapping in order to obtain accurate estimates for the polaronic crossover in the Holstein model from ultracold gases experiments. The research visit has been extremely fruitful and we are grateful to the ESF-POLATOM for the opportunity given us to strengthen the collaboration between Lund University and the SISSA in Trieste.

References

[1] T. Holstein, Annals of Physics **8**, 325 (1959); ibid. **8**, 343 (1959).

[2] R. Jördens, N. Strohmaier, K. Günter , H. Moritz, and T. Esslinger, Nature **455** 204 (2008); U. Schneider, L. Hackermüller, S. Will, Th. Best, I. Bloch, T. A. Costi, R. W. Helmes, D. Rasch, and A. Rosch, Science **322**, 5907 (2008).

[3] A. Privitera and W. Hofstetter, Phys. Rev. A **82**, 063614 (2010).

[4] Th. Best, S. Will, U. Schneider, L. Hackermüller, D. van Oosten, I. Bloch, and D.-S. Lühmann, Phys. Rev. Lett. **102**, 030408 (2009).

[5] K. Burnett, M. Edwards, C.W. Clark, and M. Shotter, J. Phys. B **35**, 1671 (2002).

[6] D. van Oosten, P. van der Straten, and H. T. C. Stoof, Phys. Rev. **A** 63, 053601 (2001).

[7] M. Bruderer, A. Klein, S.R. Clark, and D. Jaksch, New Journal of Physics **10**, 033015 (2008).

[8] A. L. Fetter, Annals of Physics, **70**, 67 (1972).

[9] C. Gaul and C. A. Müller, Phys. Rev. A **83**, 063629 (2011).

[10] K. Huang and H.-F. Meng, Phys. Rev. Lett. **69**, 644 (1992); M. Kobayashi and M. Tsubota, Phys. Rev. B **66**, 174516 (2011).

[11] I. J. Lang and Y. A. Firsov, Zh, Eksp. Teor. Fiz. **43**, 1843 (1962).

[12] Golub G H and Van Loan C F 1996 *Matrix computation* Chapter 9 Third edition

[13] C. Verdozzi and C-O. Almbladh, arXiv:0808.1643.