SCIENTIFIC REPORT

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Purpose of the visit

The major objective of the project was the analysis and design of a Beam-Forming Network (BFN) using a combination of simple unit cells made of lumped elements. The research activity is based on the preliminary works on tensor transmission lines by G. Gok, and A. Grbic in [1]. Beam-forming networks provide the desired phase an amplitude distribution in antenna arrays. Therefore, BFN are the key element in multi beam or steering antennas.

Typical examples of BFNs are: Blass [2], Nolen [3], or Butler matrices [4], and Rotman lenses [5]. All these solutions offer beam-scanning or multiple-beam capabilities by selecting one or several input ports. Classical BFNs are transmission line based with bulky layouts, especially at low frequencies. Lumped-element structures could represent very attractive solutions for applications that require size reduction.

In particular the unit cell proposed (shown in Fig. 1) can be used as directional coupler with arbitrary output phase and amplitude distributions and as cross-over with arbitrary output phase distribution. In the following we present the analysis and design of an arbitrary directional coupler. The same approach may be used for designing cross-overs. Finally, the combination of cross-overs and arbitrary directional couplers may be used in compact Butler matrixes.

Description of the work carried out during the visit

1. Network analysis

The configuration of the analyzed unit cell is presented in Fig. 1. Using Kirchhoff's voltage and current laws and considering the network as a four port network (see Fig. 1 for port definition), its Y-matrix representation can be found as

$$Y_{11} = Y_{33} = \frac{2Z_1 + Z_3}{2Z_1(Z_1 + Z_3)} + \frac{1}{Z_2} + \frac{1}{Z_4} + \frac{1}{Z_5}, \tag{1.a}$$

$$Y_{12} = Y_{21} = Y_{34} = Y_{43} = -\left(\frac{1}{2(Z_1 + Z_3)} + \frac{1}{Z_2} \right), \tag{1.b}$$

$$Y_{13} = Y_{31} = -\left(\frac{Z_3}{2Z_1(Z_1 + Z_3)} \right), \tag{1.c}$$

$$Y_{14} = Y_{41} = Y_{23} = Y_{32} = -\left(\frac{1}{2(Z_1 + Z_3)} + \frac{1}{Z_4} \right), \tag{1.d}$$

$$Y_{22} = Y_{44} = \frac{Z_1 + 2Z_3}{2Z_3(Z_1 + Z_3)} + \frac{1}{Z_2} + \frac{1}{Z_4} + \frac{1}{Z_6}, \tag{1.e}$$

$$Y_{24} = Y_{42} = -\left(\frac{Z_1}{2Z_3(Z_1 + Z_3)} \right). \tag{1.f}$$

where Z_1, Z_2, Z_3, Z_4, Z_5, Z_6 are the impedances of the unit cell (see Fig. 1).

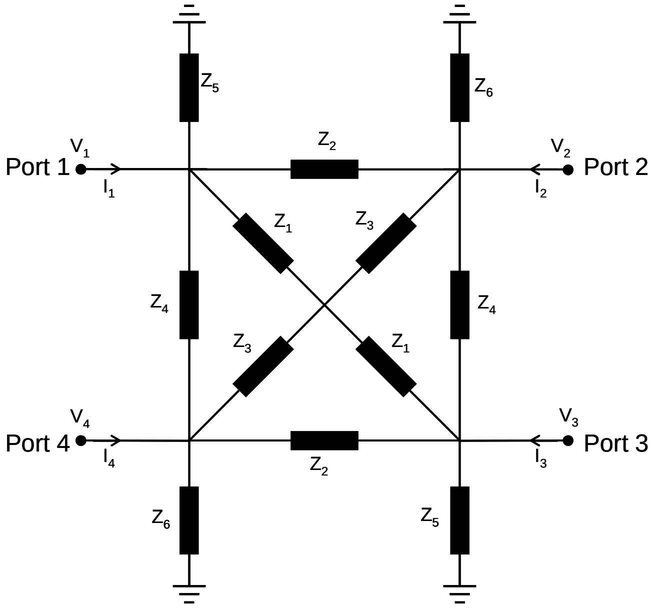

Fig. 1. Schematic of the unit cell proposed.

2. Directional coupler synthesis

The Y-matrix of a directional coupler, represented as the four-port network, is given by

$$[Y]=\frac{jY_0}{\beta\sin\theta}\begin{pmatrix}\beta\cos\theta & 0 & -1 & \alpha \\ 0 & -\beta\cos\theta & \alpha & -1 \\ -1 & \alpha & \beta\cos\theta & 0 \\ \alpha & -1 & 0 & -\beta\cos\theta\end{pmatrix}, \tag{2}$$

where Y_0 is the characteristic admittance of the ports ($Y_0 = 1/Z_0$). α and β represent the output amplitudes on the through and coupled ports, respectively; θ is the output phase of coupled port when port 1 is excited. The phase of the coupled port when port 4 is fed is called ϕ in the followings. To guarantee the network to be lossless two conditions have to be satisfied: i) $\alpha^2+\beta^2 = 1$, ii) $\theta+\phi = \pm180^\circ+n360^\circ$, with $n = 0, 1 \ldots$ Note that the angle θ has to be chosen different to $n180^\circ$ ($n = 0, 1\ldots$).

Equating expressions (1) and (2), six equations in six unknowns (the components of the network) can be written. In this way it is possible to express the impedances of the network as function of the desired output amplitudes and phases:

$$Z_1 = Z_3 = -jZ_0\frac{\beta\sin\theta}{4}, \tag{3.a}$$

$$Z_2 = jZ_0\beta\sin\theta, \tag{3.b}$$

$$Z_4 = jZ_0\frac{\beta\sin\theta}{1+\alpha}, \tag{3.c}$$

$$Z_5 = jZ_0\frac{\beta\sin\theta}{1-\alpha-\beta\cos\theta}, \tag{3.d}$$

$$Z_6 = jZ_0\frac{\beta\sin\theta}{1-\alpha+\beta\cos\theta}. \tag{3.e}$$

These results show that the network of Fig. 1 can be designed to behave as a directional coupler with generic output phases and amplitudes providing the lossless conditions i) and ii) are satisfied.

Description of the main results obtained

To verify equations (3), the unit cell of Fig. 1 has been reproduced and simulated with the commercial software Agilent Advanced Design System (ADS). The realized structure is a directional coupler, with: $\alpha = 1/\sqrt{3}$, $\beta = \sqrt{2/3}$, $\theta = 60°$, $\phi = 120°$. The characteristic impedance is equal for all the ports and it is fixed to 50 Ω. From equations (3) it is possible to find the impedances of the structure, and then, fixing the operating frequency, the reactive components of the network. The values of such components, at the central frequency f_0 = 1 GHz, are: C_1 = 18.006 pF, L_2 = 5.6270 nH, C_3 = 18.006 pF, L_4 = 3.5674 nH, L_5 = 390.72 nH, L_6 = 6.7722 nH. In Fig. 2 the amplitude of the S-parameters correspondent to matching (S_{11} and S_{44}) and isolation between the input ports (S_{14}) are plotted as a function of frequency. They present a -10dB reflection/isolation bandwidth of about 10%. In Fig. 3 the output amplitudes are shown and compared with the theoretical values of α and β. The simulated values, at the central frequency, correspond exactly to the nominal ones. The output phases are reported in Fig. 4, and again their values at the central frequency are in perfect agreement with the theoretical phases, θ for S_{13}, ϕ for S_{42}, and 0° for S_{12} and S_{43}.

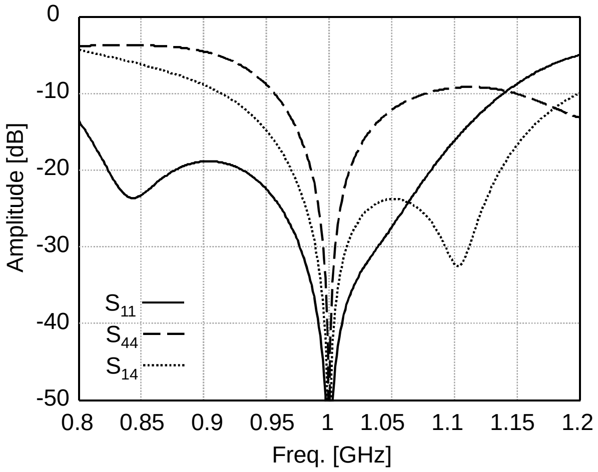

Fig. 2. Matching (S_{11} and S_{44}) and isolation (S_{14}) for the directional coupler under analysis.

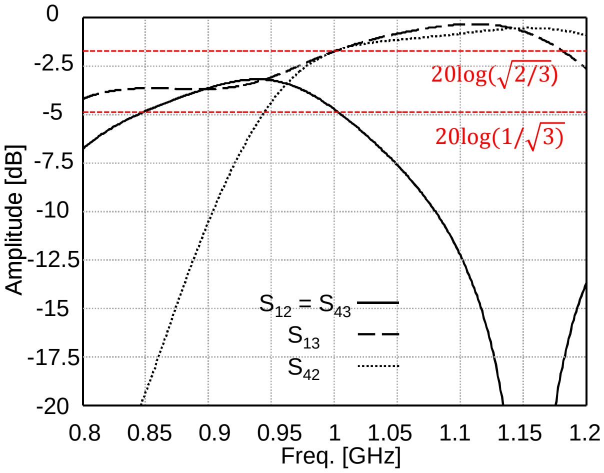

Fig. 3. Output amplitudes for the directional coupler under analysis. The red lines represent the expected values of α and β.

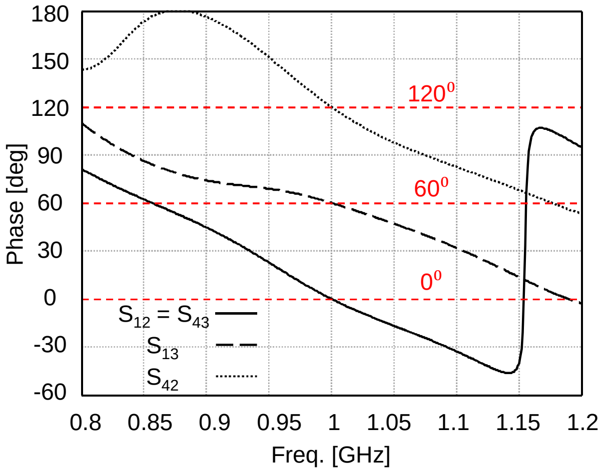

Fig. 4. Output phases for the directional coupler under analysis. The red lines represent the theoretical values of the output phases.

Conclusion

A lumped-element directional coupler with arbitrary output amplitudes and phases has been presented. Design equations are derived from the coupler's Y-matrix representation. A possible implementation was introduced and the ADS simulations show that the results at the central frequency correspond to the theoretical ones. Reflection/isolation bandwidths on the order of 10% are shown. The application of the network as a building block for a Butler matrix was shown.

Project publications

1. E. Gandini, A. Grbic, M. Ettorre, R. Sauleau, “Lumped-element unit cell for designing beam forming networks”, accepted at the international conference 6th European Conference on Antennas and Propagation (EuCAP 2012) that will be held in Prague on March 26th-30th 2012.

References

[1] G. Gok, and A. Grbic, “Tensor transmission-line metamaterials,” *IEEE Trans. Antennas Propag.*, vol. 58, no. 5, pp. 1559-1566, May. 2010.

[2] J. Blass, “Multidirectional antenna - A now approach to staked beams,” *IRE Int. Convention Record*, pt. 1, vol. 8, pp. 48-50, 1960.

[3] N. J. G. Fonseca, “Printed S-band 4X4 Nolen matrix for multiple beam antenna applications,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 6, pp. 1673-1678, Jun. 2009.

[4] J. Butler, and R. Lowe, “Beam forming matrix simplifies design of electronically scanned antennas,” *Electronic Design*, pp. 170-173, Apr. 1961.

[5] W. Rotman, and R. F. Turner, “Wide-angle microwave lens for line source applications,” *IEEE Trans. Antennas Propag.*, vol. AP-11, no. 11, pp. 623-632, Nov. 1963.