Report: Exchange Grant 'Statistical inversion for binary asteroid orbits'

Dagmara Oszkiewicz[1], Daniel Hestroffer[2]
[1] University of Helsinki, Department of Physics,
P.O. Box 64, FI-00014 Helsinki, Finland
[2] IMCCE, UMR CNRS 8028, Paris Observatory,
77 Av. Denfert Rochereau, F-75014 Paris, France

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1 Purpose of the visit

The purpose of the visit was to develop a fully non-linear, statistical orbit computation for visual binary asteroids.

2 Description of the work carried out during the visit and results

First, a novel Markov chain Monte-Carlo method orbit computation for binary asteroids was developed. The method is based on sampling model parameters (seven parameters in total: three R.A.s, three Decs. and the orbital period) from the Thiele-Innes algorithm. The resulting sample orbits are sampled according to the target probability density. The resulting orbits can be directly used in other problems, such as for example ephemeris prediction or outlier detection.

Second, the method was applied to the example binary asteroids from the transneptunian region (example binary cases taken from the Lowell Observatory webpages). The new method works well for both well- and poorly- constrained cases.

Third, the method was implemented in Java and is now included in the Gaia processing pipeline, DU457. The code can be used within the Gaia processing pipeline or as a standalone software. The code included the MCMC algorithm and test cases.

2.1 Short description of the method

The novel method combines the Thiele-Innes algorithm with Markov chain Monte-Carlo sampling procedure [1]. To sample the possible orbital solutions for a given binary asteroid we make use of the Metropolis-Hastings (M-H) algorithm. First, from the whole set of N observations $\psi_i = (x_i, y_i)$ made at observation times t_i (where $i = 1.., N$) we randomly select three observations from the same tangent plane and a staring orbital period P. We refer to those seven parameters as the sampling parameters and denote by $\mathbf{S} = (x_1, x_2, x_3, y_1, y_2, y_3, P)$. From the three selected observations and the period we compute a starting orbital elements $\mathbf{P} = (a, e, i, \Omega, \omega, P, \mathcal{M} = (m_1 + m_2))$ using the Thiele-Innes method. Once a starting orbit have been computed we start Markov chain Monte-Carlo (MCMC) sampling of the parameters $\mathbf{S}$ by adding random deviates to the selected three observations and the orbital period.

In practice at each iteration t in a chain a new candidate sampling parameters are proposed using so-called proposal densities. In particular we make use of Gaussian proposal densities for all the seven sampling parameters. For the cartesian coordinates we use proposal densities that are centered around the last accepted sampling parameters in the chain and the size of the proposal density is proportional to the observational noise: $x_i^{(c)} \propto N(x_i^{(t-1)}, \sigma_{x_i})$, $y_i^{(c)} \propto N(y_i^{(t-1)}, \sigma_{y_i})$ (where $i = 1, 2, 3$) for x and y coordinates respectively. For the orbital period we use a normal distribution that is centered around the last accepted period $P^{(c)} \propto N(P^{(t-1)}, \sigma_P)$. The size of the proposal density for the orbital period σ_P is the only parameter to be tuned in the method, but in general in most of the cases, an educated guess of the size of that parameter can be made. If the correlations between the different observations are known, they could also be utilized as in the proposal density.

Once a new candidate sampling parameters have been generated $\mathbf{S}^{(c)} = (x_1^{(c)}, x_2^{(c)}, x_3^{(c)}, y_1^{(c)}, y_2^{(c)}, y_3^{(c)}, P^{(c)})$ the acceptance coefficient a_r is computed as:

$$a_r = \frac{p_p(\mathbf{P}^c)}{p_p(\mathbf{P}^{t-1})} \frac{\mid J^{t-1} \mid}{\mid J^c \mid}, \tag{1}$$

where $J^{(c)}$ and $J^{(t-1)}$ are the Jacobians from the sampling parameters to orbital parameters for the candidate and the last accepted orbit respectively (see the appendix for the analitical formulas). $\mathbf{P}^{(c)}$ and $\mathbf{P}^{(t-1)}$ are the p.d.f.s. for the candidate and the last accepted orbit respectively. Next the candidate parameters are accepted or rejected based on Metropolis-Hastings criteria:

$$
\begin{aligned}
&\text{If } \mathrm{a_r} \geq 1, \quad \text{then } \mathbf{P}_\mathrm{t} = \mathbf{P}^\mathrm{c}. \\
&\text{If } \mathrm{a_r} < 1, \quad \text{then } \begin{cases} \mathbf{P}_t = \mathbf{P}^c, \text{ with probability } \mathrm{a_r}, \\ \mathbf{P}_t = \mathbf{P}_{t-1}, \text{ with probability } 1 - \mathrm{a_r}. \end{cases}
\end{aligned} \tag{2}
$$

In practice if the new orbit produces a better fit to the full observational data set, it is always accepted. If it produces a worse fit, it is accepted with the probability equal to a_r. The sampling is repeated until a large enough number of orbits have been obtained. After the sampling is complited convergence diagnostics has to be performed to insure that the stationary distribution was reached and to test for the lenght of burn-in period (the time required for the chain to reach the stationary). The burn-in period corresponds to the time before the stationary distribution was reached. The orbital solutions sampled during the burn-in period are removed from the final solution. The obtained distributions of the orbital parameters reflect the properties of orbital element uncertainties. Full details of the method can be found in [1].

2.2 Example results

An example results obtained from the method we present in Fig. 2. The figure presents distribution of orbital elements for the binary asteroid 1998 WW_{31}. The astrometric data contained 14 data points leading to quite well constrained solution. The results obtained with the new method fit the values obtained by [2].

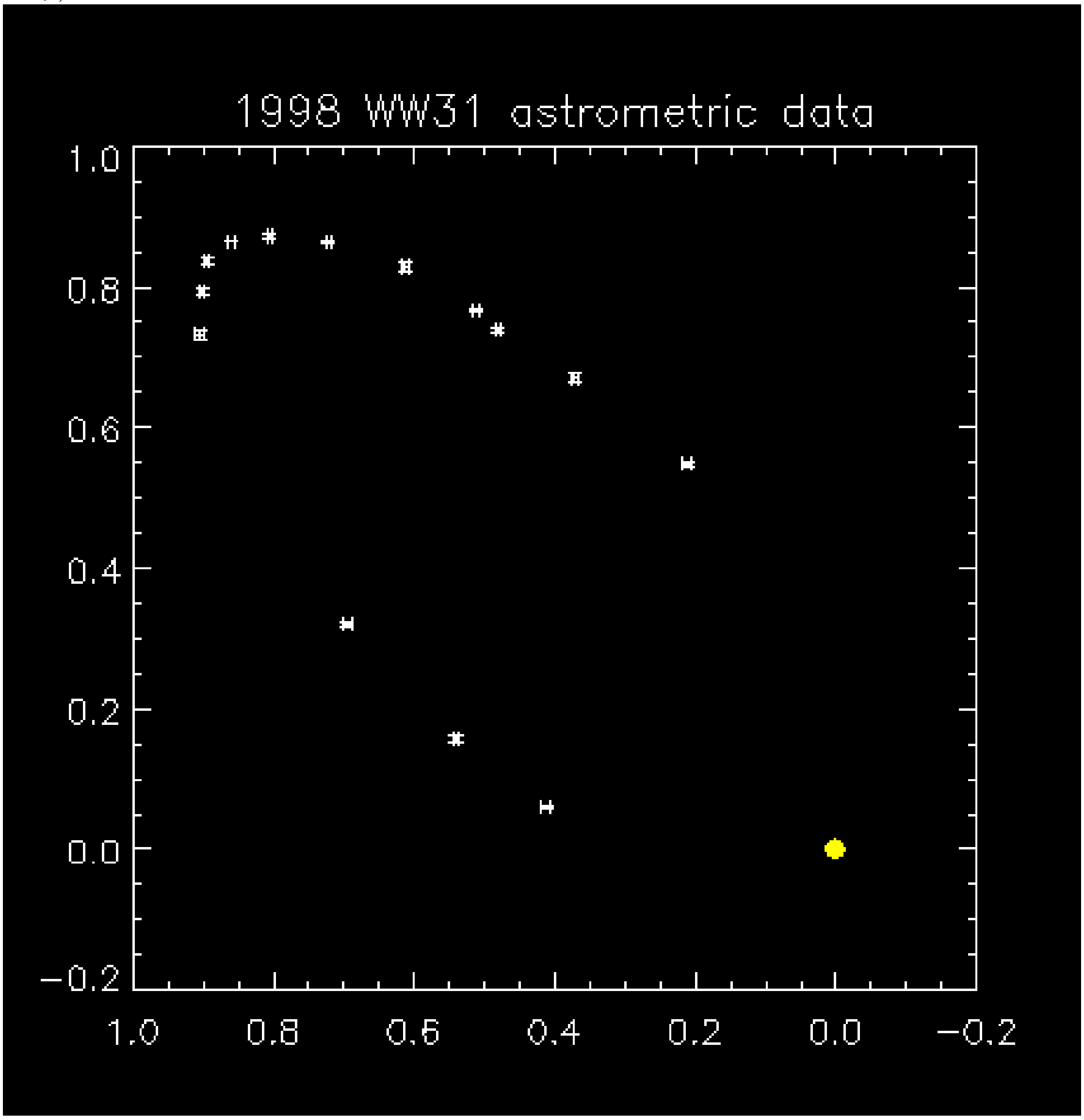
3 Projected publications/articles

The exchange visit will result in a publication "Markov chain Monte-Carlo orbit computation for binary asteroids" by Oszkiewicz D.A., Hestroffer D., and Davis P., currently in preparation. The draft is already prepared for submission.

4 Future work

The method will be further automated and adjustment will be made to fully fit the Gaia pipeline requrement. At this point the method allows only elliptic orbits, but a hyperbolic option could still be included. The code could also be further modified to include perturbations, such as for example those arriving from J_2 gravity field component of the primary.

Figure 1: Astrometric data for the binary asteroid 1998 WW_{31} (Lowell Observatory)

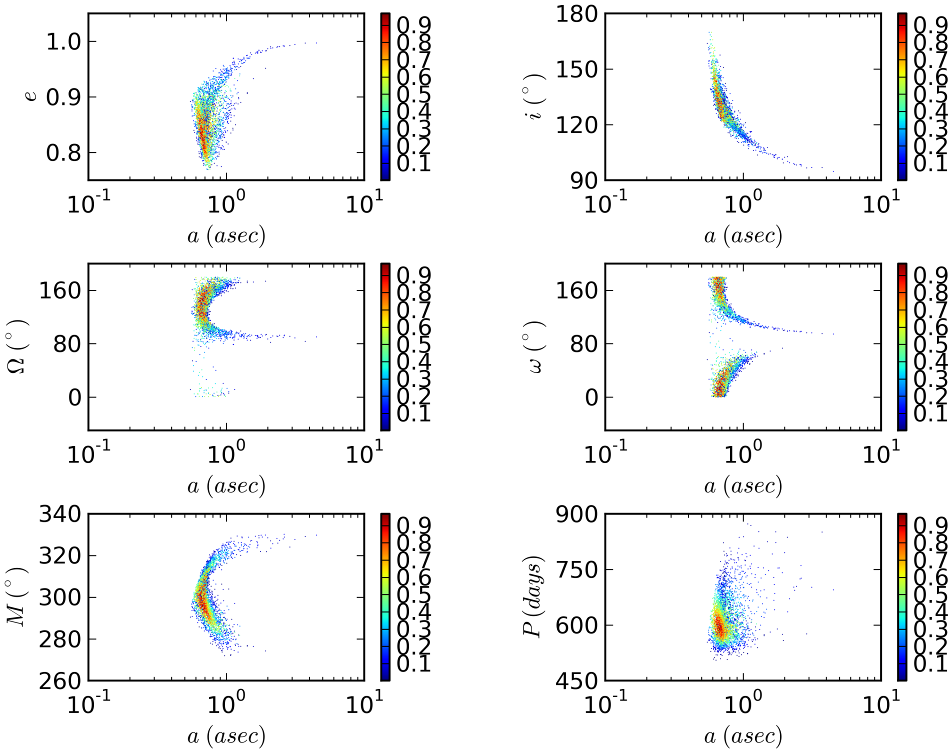


References

[1] DA. Oszkiewicz, D. Hestroffer, and P Davis. Markov chain monte-carlo orbit computation for binary asteroids. *in prep.*, 2012.

[2] C. Veillet, J. W. Parker, I. Griffin, B. Marsden, A. Doressoundiram, M. Buie,

Figure 2: Orbital element distribution for a binary asteroid 1998 WW_{31}



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