Pisa, May 17th, 2012

Scientific Report

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This is the final report for the work that I have done in Paris (during the period March, April 2012).

The aim of my visit was to find new applications of the method of forcing with generalized side conditions, presented in [4], and build on ideas from [3], and to generalize this method.

This method amounts to applying a *pure (generalized) side conditions* poset to a finite working part. The latter is then used to build a new object of size $\aleph_2$ in the generic extension. The main novelty of the method is the *pure (generalized) side conditions* poset, that consists of finite $\in$-chains of models of size $\aleph_0$ or $\aleph_1$ closed under intersection (i.e. if $\mathcal{M}_p$ is one such chain and M and N are two models belonging to it, then also $M \cap N$ is a model belonging to $\mathcal{M}_p$). The properness of this poset with respect to models of size $\aleph_0$ and $\aleph_1$ guarantees the preservation of cardinals below $\aleph_3$.

For what concern further applications, I gave a new proof of how to force an ω_2-Souslin tree with finite conditions, using generalized side conditions.

For what concern the possibile generalization, the work is still in progress. Indeed what I menage to show is that a straightforward generalization of the *pure (generalized) side conditions* poset that would allow models of three different types (i.e. models of size $\aleph_0$, $\aleph_1$ and $\aleph_2$) closed under intersection, cannot be used to preserve cardinals. As a matter of fact this "three types models" poset cannot be shown to be proper for models of size $\aleph_0$ and $\aleph_1$. Moreover if assumed to be proper for all models involved, then it is contradictory.

Nevertheless there are promising suggestions on how to deals with these problems in a recent paper by Gitik ([1]) and in a well known work of Mitchell ([2]). Then, in the last weeks I started reading and understanding these papers, in order to generalize the *pure side condition* poset.

References

[1] Moti Gitik: "A certain generalization of SPFA to higher cardinals" on http://www.math.tau.ac.il/ gitik/somepapers.html

[2] Bill Mitchell: "Adding closed unbounded subsets of ω_2 with finite conditions" *Notre Dame Journal of Formal Logic*, 46(3), 2005, pp.357–371.

[3] Itay Neeman: "Forcing with side conditions". http://www.math.ucla.edu/ ineeman/ basis theorem". http://arxiv.org/abs/1012.0596

[4] Boban Veličković and Giorgio Venturi: "Proper forcing remastered". To appear in *Appalachian Set Theory* (Cummings, Schimmerling, eds.), LMS lecture notes series. Available at http://www.math.cmu.edu/ eschimme/Appalachian/Index.html