SCIENTIFIC REPORT ON THE EXCHANGE TRAVEL GRANT
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visit from IRE NASU, Kharkiv, Ukraine to National Institute of Telecommunications, Warsaw, Poland

Topic: "Scattering of three-dimensional circular Gaussian beam by a double-periodic slab of isotropic or anisotropic material in quasi-static approximation"

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The purpose of the visit was a collaborative efforts around the research into modeling of the scattering of 3-D Gaussian beam with the circular symmetry of the field profile incident on a double-periodic anisotropic (gyrotropic) slab using the method of plane wave spectrum analysis and the method of integral functionals in quasi-static approximation. The interest in this study is explained by applications of the quasi-optical transmission line of hollow oversized waveguide type using Gaussian beams propagation at millimeter (mm) and sub-mm wavelength range because of low losses and possibility of development of all necessary components and circuits. Double-periodic slabs of anisotropic materials like ferrite placed in a quasi-optical transmission line are of greater practical interest because they may lead to mm and sub-mm wave filter devices with band gaps controlled by varying an external magnetic field.

Description of the work carried out during the visit. According to the proposal, I have been working on a quasi-static solution of the problem related to the diffraction of circular three-dimensional Gaussian beam by a double-periodic slab made of anisotropic material.

The geometry of the beam scattering problem is schematically shown in Fig. 1. 3-D Gaussian beam with the circular symmetry of the field profile is incident at a double-periodic gyrotropic slab with a thickness h. The beam waist is chosen to be located at a distance d away from the origin along the incident beam axis. The field of the incident Gaussian beam in the region $-d<z<0$ is expressed in the form of double infinite integral of plane waves [1] as follows:

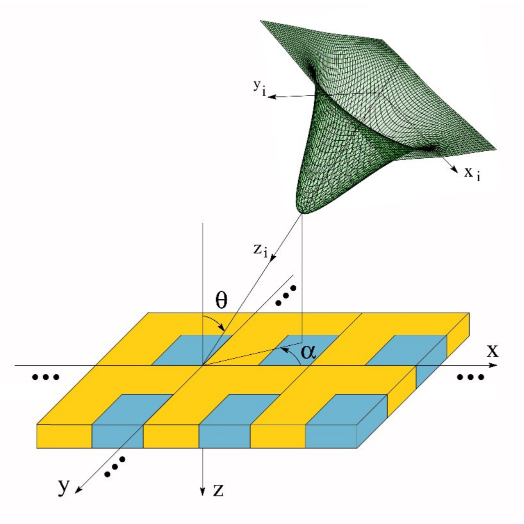


Fig. 1. The geometry of the beam scattering problem.

$$F(x,y,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Phi(k_x,k_y)\exp\left[i(k_x x+k_y y+k_z z)\right]\times$$
$$\times\exp\left[i(k_x\sin\theta+k_z\cos\theta)d\right](\cos\theta+\frac{k_x}{k_z}\sin\theta)dk_x dk_y \quad (1)$$

The spatial-spectrum function $\Phi(k_x,k_y)$ of obliquely incident ($0^0<\theta<90^0$) Gaussian beam can be analytically derived from the Fourier transform of the field profile $F(x_i,y_i,-d)=F_0\exp[-(x_i^2+y_i^2)/w_0^2]$ at the beam waist as

$$\Phi(k_x,k_y)=\frac{F_0 w_0^2}{4\pi}\exp\left\{-\frac{w_0^2}{4}\left[\left(k_x\cos\theta-k_z\sin\theta\right)^2+k_y^2\right]\right\} \quad (2)$$

The incident field at the beam waist is taken to be linearly polarized, F is E_y for the perpendicular polarization or H_y for the parallel polarization, and w_0 is the beam waist radius. The beam is assumed to be monochromatic and quasi-optical under the condition $k_0 w_0=2\pi(w_0/\lambda_0)>>1$. Here k_x, k_y and k_z are wavenumbers in the x, y, z directions, respectively, and $k_x^2+k_y^2+k_z^2=k_0^2$, where k_0 is the free-space wavenumber. In order to use the reflection and transmission coefficients for a gyrotropic slab in the derivation of the reflected and transmitted Gaussian beam fields we cannot use the partial plane waves of only one polarization. Therefore, each plane wave component presented in the Fourier plane-wave spectrum representation should be decomposed into two polarization components, parallel and perpendicular to the plane of incidence. Thus, x- and y-electric field components of the reflected and transmitted Gaussian beam for the perpendicularly polarized Gaussian beam incidence ($\perp$) are separated into two parts, $E_y^{ref(tr)}=E_{y\perp}^{ref(tr)}+E_{y\|}^{ref(tr)}$, $E_x^{ref(tr)}=E_{x\perp}^{ref(tr)}+E_{x\|}^{ref(tr)}$. Note that the x-component of the scattered field appears because of the Faraday rotation if the beam impinges on the gyrotropic slab.

Due to interaction with a gyrotropic slab, the vector $\vec{E}^{inc}$ of the incident plane wave undergoes a rotation and a change of its magnitude. As a result, x- and y- components of reflected plane wave in the perpendicularly polarized ($\perp$) incidence on a gyrotropic slab are obtained as follows:

$$E_{x\perp}^{ref}(x,y,z) = R_{x\perp}^{ref}(x,y,z)E_{x\perp}^{inc}(x,y,z) - R_{y\perp}^{ref}(x,y,z)E_{y\perp}^{inc}(x,y,z),$$
$$E_{y\perp}^{ref}(x,y,z) = R_{y\perp}^{ref}(x,y,z)E_{x\perp}^{inc}(x,y,z) + R_{x\perp}^{ref}(x,y,z)E_{y\perp}^{inc}(x,y,z), \quad (3)$$

where $R_{x\perp}^{ref}$, $R_{y\perp}^{ref}$ are the plane wave reflection coefficients from the gyrotropic slab for the azimuth angle α=0, and $E_{x\perp}^{inc}$ and $E_{y\perp}^{inc}$ are *x*- and *y*- components of the incident plane wave amplitude.

Since a Gaussian beam is composed of a continuous spectrum of plane waves and its overall behavior is treated as a linear superposition of all the plane wave components, we can apply (3) when deriving the expressions for *x*- and *y*-components of the reflected Gaussian beam field. Taking into consideration that

$$E_{\perp}^{inc}(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{-k_xk_y}{k_x^2+k_y^2}\vec{x}+\frac{k_x^2}{k_x^2+k_y^2}\vec{y}\right)\Phi(k_x,k_y)\exp\left[i(k_xx+k_yy+k_zz)\right]\exp\left[i(k_x\sin\theta+k_z\cos\theta)d\right]\left(\cos\theta+\frac{k_x}{k_z}\sin\theta\right)dk_xdk_y \quad (4)$$

and introducing the cylindrical coordinate system (ρ, α, z) instead of rectangular one (x, y, z), i.e. $k_x = k_\rho\sin\alpha$, $k_y = k_\rho\cos\alpha$, and $k_z = \sqrt{k_0^2 - k_\rho^2}$, we obtain *x*- and *y*-components of the reflected beam field for the case of perpendicular polarization beam incidence as follows:

$$E_{x\perp}^{ref}(k_\rho,\alpha,z) = -\frac{1}{4\pi}\int_0^{\infty}\int_0^{2\pi}\left[\sin\alpha\cos\alpha\, R_{x\perp}^{ref}(k_\rho)+\sin^2\alpha\, R_{y\perp}^{ref}(k_\rho)\right]\exp\left[-\frac{1}{4}(k_\rho\sin\alpha\cos\theta-\sqrt{k_0^2-k_\rho^2}\sin\theta)^2-\frac{1}{4}k_\rho^2\cos^2\alpha\right]\times$$
$$\times\exp\left[i(k_\rho\sin\alpha\, x + k_\rho\cos\alpha\, y - \sqrt{k_0^2-k_\rho^2}z)\right]\exp\left[i(k_\rho\sin\alpha\sin\theta + \sqrt{k_0^2-k_\rho^2}\cos\theta)d\right]\left(\cos\theta+\frac{k_\rho\sin\alpha}{\sqrt{k_0^2-k_\rho^2}}\sin\theta\right)k_\rho d\alpha\, dk_\rho,$$

$$(5)$$

$$E_{y\perp}^{ref}(k_\rho,\alpha,z) = -\frac{1}{4\pi}\int_0^{\infty}\int_0^{2\pi}\left[\sin^2\alpha\, R_{x\perp}^{ref}(k_\rho)-\sin\alpha\cos\alpha\, R_{y\perp}^{ref}(k_\rho)\right]\exp\left[-\frac{1}{4}(k_\rho\sin\alpha\cos\theta-\sqrt{k_0^2-k_\rho^2}\sin\theta)^2-\frac{1}{4}k_\rho^2\cos^2\alpha\right]\times$$
$$\times\exp\left[i(k_\rho\sin\alpha\, x+k_\rho\cos\alpha\, y-\sqrt{k_0^2-k_\rho^2}z)\right]\exp\left[i(k_\rho\sin\alpha\sin\theta+\sqrt{k_0^2-k_\rho^2}\cos\theta)d\right]\left(\cos\theta+\frac{k_\rho\sin\alpha}{\sqrt{k_0^2-k_\rho^2}}\sin\theta\right)k_\rho d\alpha\, dk_\rho$$

Then using Maxwell's equations and performing all derivations by analogy with $E_{x\perp}^{ref}$ and $E_{y\perp}^{ref}$, we obtain the parallel polarized $(\|)$ components $E_{x\|}^{ref}$ and $E_{y\|}^{ref}$ as follows:

$$E_{x\|}^{ref}(k_\rho,\alpha,z) = \frac{1}{4\pi}\int_0^{\infty}\int_0^{2\pi}\left[-\cos^2\alpha\, R_{y\|}^{ref}(k_\rho)+\sin\alpha\cos\alpha\, R_{x\|}^{ref}(k_\rho)\right]\exp\left[-\frac{1}{4}(k_\rho\sin\alpha\cos\theta-\sqrt{k_0^2-k_\rho^2}\sin\theta)^2-\frac{1}{4}k_\rho^2\cos^2\alpha\right]\times$$
$$\times\exp\left[i(k_\rho\sin\alpha\, x+k_\rho\cos\alpha\, y-\sqrt{k_0^2-k_\rho^2}z)\right]\exp\left[i(k_\rho\sin\alpha\sin\theta+\sqrt{k_0^2-k_\rho^2}\cos\theta)d\right]\left(\cos\theta+\frac{k_\rho\sin\alpha}{\sqrt{k_0^2-k_\rho^2}}\sin\theta\right)k_\rho d\alpha\, dk_\rho,$$

$$E_{y\|}^{ref}(k_\rho,\alpha,z) = \frac{1}{4\pi}\int_0^{\infty}\int_0^{2\pi}\left[\cos^2\alpha\, R_{x\|}^{ref}(k_\rho)+\sin\alpha\cos\alpha\, R_{y\|}^{ref}(k_\rho)\right]\exp\left[-\frac{1}{4}(k_\rho\sin\alpha\cos\theta-\sqrt{k_0^2-k_\rho^2}\sin\theta)^2-\frac{1}{4}k_\rho^2\cos^2\alpha\right]\times \quad (6)$$
$$\times\exp\left[i(k_\rho\sin\alpha\, x+k_\rho\cos\alpha\, y-\sqrt{k_0^2-k_\rho^2}z)\right]\exp\left[i(k_\rho\sin\alpha\sin\theta+\sqrt{k_0^2-k_\rho^2}\cos\theta)d\right]\left(\cos\theta+\frac{k_\rho\sin\alpha}{\sqrt{k_0^2-k_\rho^2}}\sin\theta\right)k_\rho d\alpha\, dk_\rho.$$

Here, $R_{x\perp}^{ref}(k_\rho)$, $R_{y\perp}^{ref}(k_\rho)$ and $R_{x\|}^{ref}(k_\rho)$, $R_{y\|}^{ref}(k_\rho)$ are the *x*- and *y*- components of the Fresnel reflection coefficients of the perpendicularly polarized $(\perp)$ and parallel polarized $(\|)$ plane wave incident on a gyrotropic slab when azimuth angle α equals zero. Thus,

$$
\begin{aligned}
E_y^{ref}(k_\rho,\alpha,z) &= E_{y\perp}^{ref}(k_\rho,\alpha,z)+E_{y\parallel}^{ref}(k_\rho,\alpha,z)=\frac{1}{4\pi}\int_0^{\infty}\int_0^{2\pi}\Big[-\sin\alpha\cos\alpha\Big(R_{y\perp}^{ref}(k_\rho)-R_{y\parallel}^{ref}(k_\rho)\Big)+\frac{1}{2}\Big(R_{x\perp}^{ref}(k_\rho)-R_{x\parallel}^{ref}(k_\rho)\Big)-\\
&-\frac{\cos 2\alpha}{2}\Big(R_{x\perp}^{ref}(k_\rho)-R_{x\parallel}^{ref}(k_\rho)\Big)\Big]\exp\Big[-\frac{1}{4}(k_\rho\sin\alpha\cos\theta-\sqrt{k_0^2-k_\rho^2}\sin\theta)^2-\frac{1}{4}k_\rho^2\cos^2\alpha\Big]\times\\
&\times\exp\Big[i(k_\rho\sin\alpha\, x+k_\rho\cos\alpha\, y-\sqrt{k_0^2-k_\rho^2}z)\Big]\exp\Big[i(k_\rho\sin\alpha\sin\theta+\sqrt{k_0^2-k_\rho^2}\cos\theta)d\Big]\times\left(\cos\theta+\frac{k_\rho\sin\alpha}{\sqrt{k_0^2-k_\rho^2}}\sin\theta\right)k_\rho d\alpha\, dk_\rho
\end{aligned}
\qquad (7)
$$

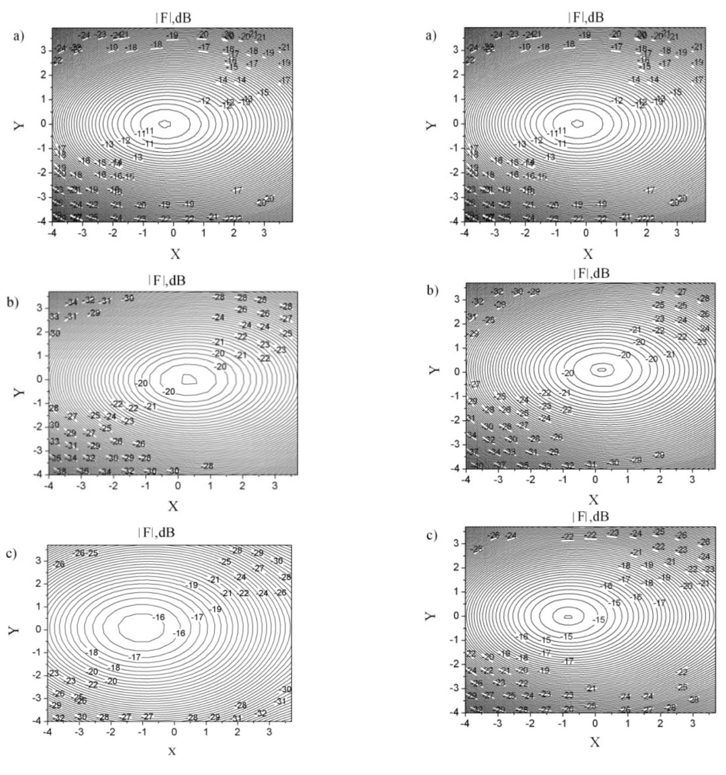
Expression for $E_x^{ref}(k_\rho,\alpha,z)$ can be easily deduced from that of $E_y^{ref}(k_\rho,\alpha,z)$ by using substitutions; $R_{y\perp}^{ref}(k_\rho)=R_{x\perp}^{ref}(k_\rho)$, $R_{y\parallel}^{ref}(k_\rho)=R_{x\parallel}^{ref}(k_\rho)$, $R_{x\perp}^{ref}(k_\rho)=-R_{y\perp}^{ref}(k_\rho)$, $R_{x\parallel}^{ref}(k_\rho)=-R_{y\parallel}^{ref}(k_\rho)$.

Making certain transformations and integrating with respect to α like in [2], we obtain the components of the reflected in free space 3-D Gaussian beam field as a sum of single integrals with respect to k_ρ.. The x- and y-components of the transmitted beam field in the form of a sum of single integrals with respect to k_ρ is determined by analogy. The estimation of the scattered field coefficients for a plane wave incident onto a uniform gyrotropic layer is an independent problem. It has been solved using a novel numerical method in the frequency domain method that formulates a set of volume integro-differential equations for the equivalent electric and magnetic polarization currents of the layer in vectorial form (so-called "method of integral functionals") [3,4].

Fig.2 shows the simulation of Gaussian beam field components reflection from uniform gyromagnetic slab for parallel-polarized and perpendicular-polarized Gaussian beam incidence at z=0. The Gaussian beam oscillating frequency is 110GHz. The beam with the waist radius w_0=0.69cm impinges on the gyromagnetic slab having the thickness of 4.76mm at an angle θ=45^0. One can see the lateral shifts of the peaks of the beam cross-sections for x- and y-components of the reflected beam fields with respect to the peak of the incident beam cross-section. This phenomenon can be explained by the fact that an angle of the incidence of the partial wave presented in the Fourier plane wave spectrum representation with maximal reflectivity doesn't coincide with the incidence angle of Gaussian beam.

Fig.3 demonstrates the simulation of Gaussian beam field components reflection from uniform and perforated gyromagnetic slabs with the same parameters of permeability tensor. The double-periodic slab is perforated with square apertures. The filling factor of the unit cell is S=0.81.

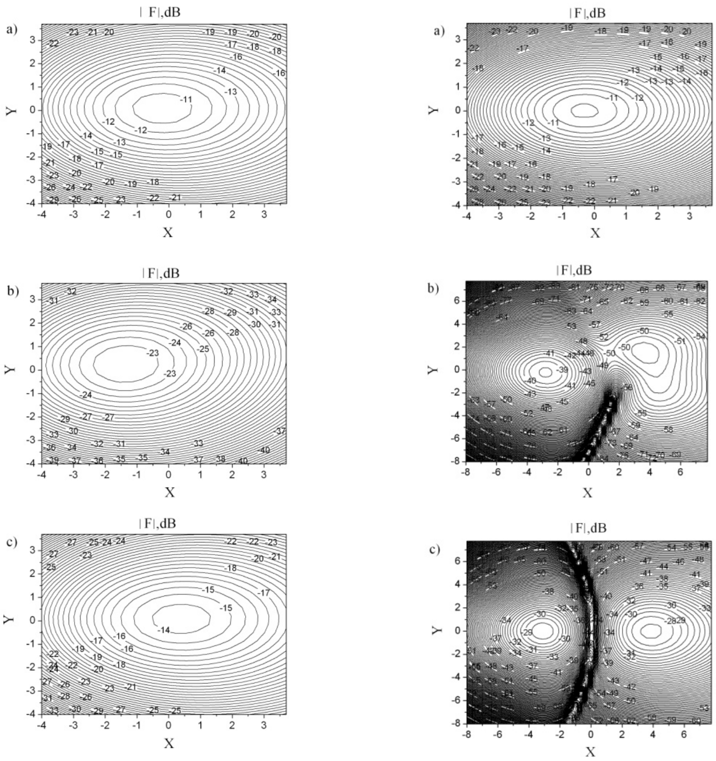
The scattering coefficients for perforated slab have been calculated in quasi-static approximation when wavelength is much larger than the period of scattering structure. The figure for y-component of Gaussian beam reflected from perforated slab (Fig.3(c)) demonstrates two-lobe profile with central power significantly suppressed. This phenomenon is caused by the existence of full-pass angle for y-component of transmission coefficient for parallel-polarized plane wave incidence at θ=47^0 in case of perforated slab (Fig.4 (b)) and coincidence this angle with the angle of Gaussian beam incidence while x-component of the transmission coefficient is zero. As a result, x-component of Gaussian beam reflected from perforated slab is negligibly small (Fig.3(b)) whereas the behavior of y-component (Fig.3(c)) is similar to one of corresponding component reflected from dielectric slab at the Brewster angle incidence [1, 2].



perpendicular-polarized incidence **parallel-polarized incidence**

Fig 2. Contour plots of
(a). y-component of the incident beam field (in dB)
(b). x-component of the reflected beam field (in dB)
(c). y-component of the reflected beam field (in dB); f=110GHz, w_0=0.69cm, d=18cm, ε_r=2.25, μ_r= μ_z=0.998, μ_g=10, h=0.476cm.

parallel-polarized incidence



uniform slab (S=1) **perforated slab (S=0.81)**

Fig 3. Contour plots of
(a). *y*-componen of the incident beam field (in *dB*)
(b). *x*-componen of the reflected beam field (in *dB*)
(c). *y*-componen of the reflected beam field (in *dB*);
f=110GHz, w_0=0.69cm, d=18cm, ε_r=2.25, $\mu_r= \mu_z$=0.998, μ_g=10, h=0.1736cm, $\theta=47^0$.

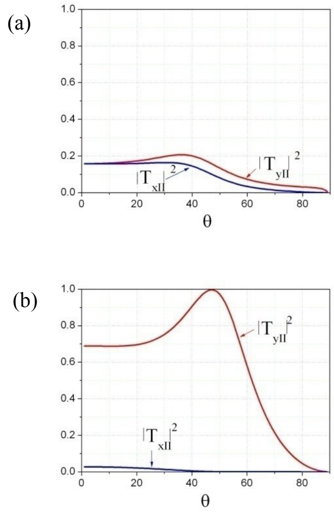


Fig.4. Dependence of plane wave power fractions transmitted through gyromagnetic slab with parameters of permeability tensor $\mu_r= \mu_z$=0.998, μ_g=10, and ε_r=2.25 on an angle of incidence when the incident plane wave is parallel-polarized.
(a). uniform slab
(b). perforated slab, S=0.81
f=110GHz, h=0.1736cm.

References

[1.] Q. Li, R.J. Vernon, "Theoretical and experimental investigation of Gaussian beam transmission and reflection by a dielectric slab at 110 GHz," *IEEE Trans. Antennas Propag*., vol. 54, no. 11, pp. 3449–3457, 2006.

[2.] V.V. Yachin, T.L. Zinenko, and V.K. Kiseliov, "Diffraction of a 3-D Gaussian Beam with Circular Symmetry of the Spatial Field Distribution by Penetrable Screens," *Telecommunications and RadioEngineering*, vol.71, no.8, pp. 677 – 691, 2012.

[3.] V.V. Yachin, K. Yasumoto, "Method of integral functionals for electromagnetic wave scattering from a double-periodic magnetodielectric layers," *J. Opt. Sc. America*, vol. 24, no. 11, pp. 3606-3618, 2007.

[4.] V.V. Yachin, T.L. Zinenko, V.K. Kiseliov, "Quasi-static approximation for plane wave scattering by two-periodic gyrotropic layer," *Radio Physics and Radio Astronomy*, vol. 14, no 2, pp. 121-149, 2009 (in Russian).

<u>Publications based on the project work:</u>

Conference papers

1. T.L.Zinenko, V.V. Yachin , M. Marciniak, V.K. Kiseliov, "Scattering of a three-dimensional Gaussian beam with circular cross section from a gyrotropic slab," *Proc. Int. Conf. Mathematical Methods in Electromagnetic Theory (MMET-12)*, Kharkov, Ukraine, QO-II-3, Aug. 2012.

Journal papers

1. T.L.Zinenko, V.V. Yachin , M. Marciniak, V.K. Kiseliov, "Analytic solution for plane wave scattering from double-periodic gyromagnetic slab in quasi-static approximation.", *J. Opt. Soc. Am,* in preparation.
2. T.L.Zinenko, V.V. Yachin , M. Marciniak, V.K. Kiseliov, "Scattering of three-dimensional circular Gaussian beam by a double-periodic slab of gyromagnetic material in quasi-static approximation", *J. Opt. Soc. Am,* in preparation.