ESF Research Networking Programme NEWFOCUS
Final scientific report

Project Title

Analysis of focusing reflector systems based on complex conical beams

Applicant

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1. PURPOSE OF THE VISIT

The main objective of this research project was to further develop the recently introduced method for analysing reflector antenna systems based on a combination of Physical Optics and a propagation model built around a new type of wave objects called complex conical beams. The preliminary investigation performed in previously conducted research had confirmed the potential of the new hybrid method, which was shown to be significantly faster than pure Physical Optics and comparably accurate. It has been shown that expressing the radiated field in terms of higher complexity wave objects is a well-suited approach for high-frequency problems, because it enables a significant reduction of the number of unknowns in the system.

The work on beam formulations, the most important outcome of which are the complex conical beams, has been an ongoing research topic for several years. At the current stage, the project was concerned with extending the capabilities of the developed analysis method, by including more reflector shapes, e.g. paraboloidal or flat (previously only ellipsoidal reflectors were considered), by investigating modified CCB formulations which offer faster computation times, and so on. Also of interest was the possibility of incorporating CCB analysis into other analysis workflows, so as to extend also the range of possible applications of the developed analysis model.

2. DESCRIPTION OF THE WORK CARRIED OUT DURING THE VISIT

During the exchange period at the University of Siena the research was focused, as mentioned, mostly on studying refelector system analysis via CCB expansion. However, part of the time was also dedicated to studying pattern synthesis techniques for designing planar leaky wave antennas. The latter is a topic very closely related to metasurface antennas research, which has also been of interest to and supported by NewFocus (e.g. NewFocus grant n.3703: "Variable metasurface based lens antennas: numerical and experimental evaluation"). Therefore, this part of the report is divided in two parts: one related to reflector antenna systems and beam analysis, and the other concerned with preliminary investigations in the field of pattern synthesis for planar leaky wave antennas, i.e. metasurface antennas.

2.1. Work Related to Analysis of Reflector Systems Using CCB's

In previous research, a new propagation model called complex conical beams (CCB) has been introduced [1], [2]. By using Generalized Pencil of Functions method (GPOF) [3] and Fourier series expansion, it was possible to reduce the radiation integral to a double summation, significantly accelerating the computation of propagation. The use of GPOF method ensures a low number of elements in the sum, due to its adaptive nature. The basic formulation of CCB's (type A) starts with the scalar radiation integral in spectral domain:

$$I'(x,y,z) = \frac{1}{8\pi^2 j}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{F}\left(k_x,k_y\right)\frac{e^{-j\left(k_x x+k_y y\right)-jz\sqrt{k^2-k_x^2-k_y^2}}}{\sqrt{k^2-k_x^2-k_y^2}}dk_x dk_y \tag{1}$$

where $\tilde{F}\left(k_x,k_y\right)$ denotes the spectrum of the aperture field, and $I'(x,y,z)$ is the radiated spatial domain potential away from the aperture. By switching to cylindrical coordinates, introducing the Fourier series in the spectral azimuthal variable α and subsequently applying the GPOF approximation on the obtained Fourier coefficients, the double integral from Eq. (1) is replaced by this double summation

$$I(\rho,\phi,z) = \sum_{n}\sum_{m=1}^{M} a_{mn} W_n(\rho,\phi,z+jb_{mn}) \tag{2}$$

where W_n denotes the *n*-th order complex conical beam, and is defined by

$$W_n\left(\rho,\phi,\tilde{z}\right) = e^{jn\phi}\int_{0}^{\infty}\frac{e^{-j\tilde{z}\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}}\mathrm{J}_n\left(\rho k_\rho\right)k_\rho dk_\rho\,. \tag{3}$$

The integral in (3) possesses an analytical solution, albeit a recursive one. It has been demonstrated in [5] that a (*n*+1)-th order CCB can be computed from (*n*-1)-th CCB as

$$W_{n+1} = e^{j2\phi}W_{n-1} - 2je^{j(n+1)\phi}\frac{\partial}{\partial\rho}I_n^{(g)}, \tag{4}$$

where $\frac{\partial}{\partial\rho}I_n^{(g)}$ is the rho-derivative of the so-called Gradshteyn integral [6]

$$\frac{\partial I_n^{(g)}}{\partial \rho} = (-j)^{n+1} \frac{k\pi}{8} \frac{\rho}{r} \Big[\mathrm{J}_{n/2+1}(d^-)\,\mathrm{H}^{(2)}_{n/2}(d^+) - \mathrm{J}_{n/2-1}(d^-)\,\mathrm{H}^{(2)}_{n/2}(d^+) + \mathrm{J}_{n/2}(d^-)\,\mathrm{H}^{(2)}_{n/2-1}(d^+) - \mathrm{J}_{n/2}(d^-)\,\mathrm{H}^{(2)}_{n/2+1}(d^+) \Big], \tag{5}$$

with $d^- = -\frac{1}{2}k(r-z)$ and $d^+ = \frac{1}{2}k(r+z)$.

In subsequent research, this propagation model was included in the Physical Optics analysis of reflector systems [4], [5], and the new model was successfully verified for the case of one and two ellipsoidal reflectors. However, the fact that in order to compute one *n*-th order CCB, one has to calculate all lower-order CCB's (W_0 thru W_{n-1}), coupled with the fact that the computation of Bessel and Hankel functions ($\mathrm{J}_{n/2}$ and $\mathrm{H}^{(2)}_{n/2}$ in (5)) is already a recursive process, has meant that the new procedure, although significantly faster than PO, did not achieve the desired gain in speed.

The work on alternative and potentially faster CCB formulations has in part been covered by this grant. It has been noticed that by artificially introducing a $\left(k_\rho/k\right)^{|n|}$ factor in the integrand in (3), one arrives at a similar but different integral, which too possesses an analytical solution.

The CCB's of Type B are then defined as

$$\Psi_n\left(\rho,\phi,z\right) = e^{-jn\phi} \int_0^\infty \frac{e^{-jz\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}} J_n\left(k_\rho \rho\right) \left(\frac{k_\rho}{k}\right)^{|n|} k_\rho \, dk_\rho \tag{6}$$

while the analytical solution of (6) becomes simply

$$\Psi_n\left(\rho,\phi,\tilde{z}\right) = e^{-jn\phi} k \sin^n \tilde{\theta}\, \mathrm{h}^{(2)}_n\left(k\tilde{r}\right) \tag{7}$$

where $\mathrm{h}^{(2)}_n$ is the spherical Hankel function of the second kind and the complex angle $\tilde{\theta}$ is calculated from complex coordinate $\tilde{z}$ and real coordinates *x* and *y* ($\rho^2 = x^2 + y^2$):

$$\sin^n \tilde{\theta} = \frac{\rho^n}{\left(\tilde{z}^2 + \rho^2\right)^{n/2}}. \tag{8}$$

The introduced $\left(k_\rho/k\right)^{|n|}$ factor in the starting integral has to be compensated for by the reciprocal term $\left(k/k_\rho\right)^{|n|}$. This extra term is attributed to the starting field spectrum in (1), which means that a separate GPOF expansion has to be performed for every order *n*, for a *modified* *n*-th order spectrum, slowing down that part of the analysis. However, for a moderate number of expansion points, the GPOF expansion is a very fast process and the resulting overall procedure is still significantly faster for Type B beams than for the original Type A beams.

The other problem which had to be solved regarding the modified beam formulation is the fact that the reciprocal term $\left(k/k_\rho\right)^{|n|}$ actually apparently introduced a *n*-th order pole in the subintegral function. As has been discovered, though, this pole is cancelled by a *n*-th order zero, which appears in the starting field spectrum $\tilde{G}\left(k_\rho, \alpha\right)$ when it is converted to polar coordinates. However, for numerical reasons, one cannot evaluate the radiation integral from $k_\rho = 0$ to $k_\rho = k_0$; the origin has to be omitted.

To apply the beams in the analysis of electromagnetic problems, they have to be vectorised. As was the case with Type A conical beams, here also the chosen approach was to interpret the scalar radiation integral in (1) as components of the vector potential [5], from which the radiated field can be obtained via differentiation

$$\mathbf{E} = \hat{x}\left(\frac{1}{\varepsilon}\frac{\partial F_y}{\partial z}\right) - \hat{y}\left(\frac{1}{\varepsilon}\frac{\partial F_x}{\partial z}\right) + \hat{z}\left(\frac{1}{\varepsilon}\left(\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x}\right)\right), \tag{9}$$

where $F_{x,y}\left(\rho, \phi, z\right) = \sum_{n,m} a_{mn}^{x,y} \Psi_n\left(\rho, \phi, z + j b_{mn}^{x,y}\right)$ is a sum of complex conical beams. Some comparisons of results obtained with the two CCB formulations will be presented in Sec. 3 of this report.

The other aspect of CCB analysis which was given some consideration is concerned with the extension of capabilities of the global reflector system analysis method based on CCB's. Until present, only ellipsoidal reflectors have been included [1], [5]. When shaping the beam though, other reflector geometries are also necessary, most often paraboloidal or flat. The mathematical derivations required to introduce new reflector shapes into the analysis procedure, although fairly straightforward *per se*, are somewhat lengthy and may require special attention from case to case. The process includes finding intersection points and corresponding radii of curvature of the reflector in tangential and sagittal planes, and detecting the reflected beam direction and the location of beam waist after reflection, as illustrated in Fig.1. In some cases, if the reflected beam results well-collimated, additional consideration is necessary to decide where to put the auxiliary plane for restarting the (modular) process of beam expansion. This extension was successfully undertaken for paraboloidal reflectors, and the method can now be used to analyse also often employed Gregorian reflector systems, which are composed of a combination of ellipsoidal and paraboloidal reflectors.

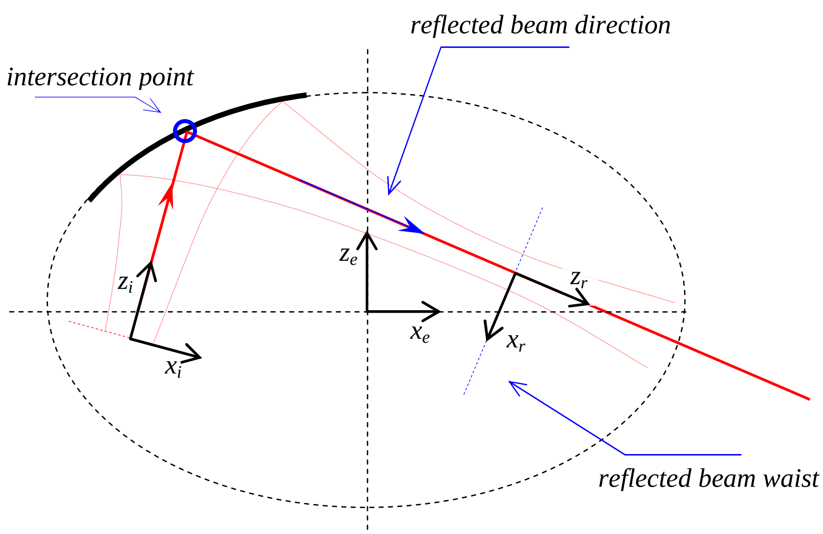

Figure 1. Parameters for describing the interaction of a beam with a reflector

2.2. Work Related to Pattern Synthesis

The efforts in this segment were undertaken in the context of metasurface antennas. A metasurface is a thin metamaterial layer characterized by unusual reflection properties of plane waves and/or dispersion properties of surface/guided waves [6]. Impenetrable metasurfaces are composed of a dense periodic texture of small elements printed on a grounded slab, with or without shorting vias. The printed metallic texture creates particular impedance boundary conditions on the surface of the slab, which can convert a surface wave into a leaky wave and effectively become a highly directive planar antenna [7]. The radiation pattern of such an antenna can be controlled by altering the metallization pattern on the surface of such an antenna, i.e. its surface impedance.

The key issue in the design of such antennas is finding an efficient and accurate way of passing from the desired pattern to the surface impedance distribution on the surface of the antenna. So far, the alternate projections method has been used for achieving this. However, this requires multiple iterations of simulating the entire antenna structure by a full-wave method, which, due to the inherent anisotropy in the antenna structure, becomes a cumbersome and time-consuming process.

During this exchange period, some time was also dedicated to this issue and a new approach has been proposed, whereby the planar antenna is only considered as a radiating aperture with a given surface impedance distribution. The impedance distribution is directly related to the radiation pattern since it affects the electric and magnetic surface currents and their ratio. The procedure starts from an approximate distribution of these currents (which radiate an approximate radiation pattern) [8]:

$$\vec{E}(\vec{r}) = \frac{jk\eta}{4\pi}\iint\limits_S \left[\left(-\vec{J}_s(\vec{r}') + \left(\vec{J}_s(\vec{r}')\cdot\hat{R}\right)\hat{R}\right) - \frac{1}{\eta}\left(\vec{M}_s(\vec{r}')\times\hat{R}\right)\right]\cdot\frac{e^{-jkR}}{R}dS'\,. \qquad (10)$$

The currents J and M provide the initial approximate distribution of surface impedance, given that they are interrelated

$$\vec{M}_s = \bar{\bar{Z}}_s\cdot\left(\hat{n}\times\vec{J}_s\right). \qquad (11)$$

Upon some mathematical manipulations, the radiation integral in (10) can be rewritten as

$$\begin{aligned}\vec{E}(\vec{r}) = -\frac{jk}{4\pi}\iint\limits_S &\Big[\hat{\theta}\left(J_\rho\left(\eta\cos\theta + Z_{\phi\phi}\right) - J_\phi Z_{\phi\rho}\right)\\ &+ \hat{\phi}\left(J_\phi\left(\eta - Z_{\rho\rho}\cos\theta\right) - J_\rho Z_{\rho\phi}\cos\theta\right)\Big]\cdot\frac{e^{-jkR}}{R}dS'\end{aligned}\,. \qquad (12)$$

where $Z_{\phi\phi}$, $Z_{\rho\rho}$, $Z_{\phi\rho}$ and $Z_{\rho\phi}$ are the components of the surface impedance matrix.

The new idea proposed during the exchange visit suggests to introduce an additional perturbational factor $\bar{\bar{\Delta}}$ into (12) which will serve to modify the surface impedance distribution so as to obtain the right ratio of electric and magnetic currents and radiate the desired radiation pattern. The preliminary investigations have shown that by constructing the $\bar{\bar{\Delta}}$ matrix as

$$\bar{\bar{\Delta}} = \begin{bmatrix}\Delta_{\rho\rho} & \Delta_{\rho\phi}\\ \Delta_{\rho\phi} & 1\end{bmatrix} \qquad (13)$$

it may be possible to obtain the final $\bar{\bar{Z}}_s$ matrix by solving an integral equation, without the need to simulate the whole antenna structure. Investigations in this field continue.

3. DESCRIPTION OF THE MAIN RESULTS OBTAINED

3.1. Work Related to Analysis of Reflector Systems Using CCB's

The first example is illustrated in Fig. 2, which shows a comparison of a radiation pattern calculated with CCB's of Type A and Type B. The source is a TM_{mn} circular waveguide mode, which has been chosen because it allows a simple construction of the reference solution. One can see that both types of beams attain the same level of accuracy with a similar number of terms. However, the calculation of Type B beams is much simpler.

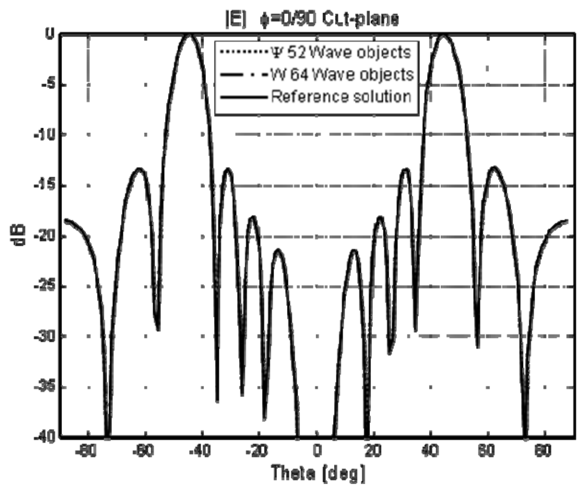

Figure 2. Total radiated electric field magnitude of a TM_{44} *circular waveguide mode, sampled on a sphere of radius 300λ (E-plane).*

Regarding the extension of capabilities of the beam-based reflector system analysis, Fig. 3 shows the geometry and the calculated radiation pattern of a dual-reflector system composed of an ellipsoidal subreflector and a paraboloidal main reflector. The size of the main reflector is over 50λ in diameter. The reference solution has been computed in the industry-standard EM simulator GRASP (TM). Results show excellent agreement .

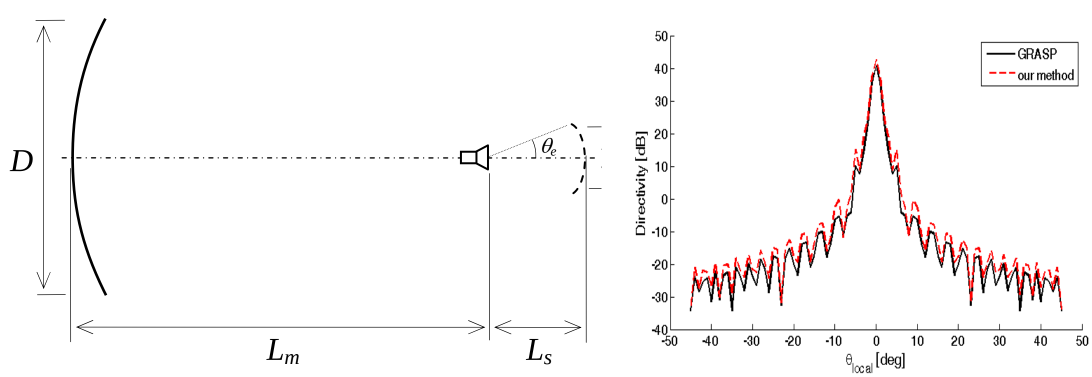

Figure 3. Total radiated electric field magnitude of a TM_{44} *circular waveguide mode,*

3.2. Work Related to Pattern Synthesis

This research is still at a very early stage, i.e. formulation and feasibility study of the proposed mathematical model, so no visual results are available at present time.

4. FUTURE COLLABORATION WITH HOST INSTITUTION

As mentioned in the introduction, the work on beam formulations and other approaches to analysing electrically large (i.e. high-frequency) antenna systems has been a joint research venture of University of Siena, Italy, and University of Zagreb for several years. This collaboration has proved to be fruitful and successful for both parties, resulting in several scientific publications and one Ph.D. co-mentored by members of both institutions. The continuation of this collaboration is seen as valuable and important and will be incented by both involved universities. The research will focus not only on further expansion of beam analysis capabilities, but also on other topics of common interest, like pattern synthesis described in this report. Funds permitting, one more short-term visit to University of Siena is also planned for 2012.

5. PROJECTED PUBLICATIONS / ARTICLES RESULTING OR TO RESULT FROM THE GRANT

One to two conference papers covering the research undertaken and partially funded by this grant are expected in 2012 and 2013. In those papers, ESF NewFocus support will be duly acknowledged. For the time being, no journal papers are planned.

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